A Theoretical Framework for The Integration of a Green Roof Model in ESP-r

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ABSTRACT

Green roofs are increasingly used in low energy buildings because of their potential in reducing energy use, improving thermal comfort, mitigating heat island effect and managing storm water. An accurate evaluation of a green roof is important in order to assess the energy and carbon impact from introducing them in the building fabric. It has been found that the currently available tools are either stand-alone programs or are not fully accounting for the dynamic interactions of heat and moisture exchanges in green roof layers. The objective of this paper is to outline the theoretical framework of on-going research that aims to integrate a green roof model within the ESP-r whole building energy simulation program. The green roof model is based on the finite volume approach to simultaneously solve energy and moisture balance equations within control volumes, which represent and couple the various layers of the green roof. In line with the ESP-r structure, the Crank-Nicolson scheme of discretised equations is used. These are solved locally so that the results can be used in the integrated whole building simulation engine of ESP-r.

KEYWORDS

Green roofs; Coupling thermal and moisture balances; Finite volume; Whole building energy simulation.

INTRODUCTION

There are numerous references giving details of constructions and guidelines for green roofs (FLL 2002, GRO 2011, Grant 2006 and Newton et al 2007).

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Green roof usage, as part of a sustainable construction, is emerging in many parts of the world. This is expected to continue due to its numerous benefits which include reducing urban heat island effect, promotion of biodiversity, reduced energy and storm-water management. The opportunity to expand vegetated surfaces to urban landscapes is particularly interesting for relatively land-deprived cities such as Singapore, Hong Kong and Toronto (Li et al. 2010). However there are challenges in constructing and maintaining a soil vegetation medium over the roof. The energy performance prediction is critical in decision making situations. The unfamiliar physics around the plants pose additional challenges to building practitioners.

This paper is concerned with the development of a green roof model that is based on a finite volume approach and the integration of this model within the ESP-r whole building energy simulation program (2011). In the development of the green roof model, three layers of green roofs are considered for analysis: the vegetation layer, soil layer and roof support layer. As elaborated further in subsequent sections of this paper, the vegetation layer includes two control volumes: canopy leaves and canopy air (Figure 1). The soil layer is subdivided into three control volumes: upper soil, mid soil and lower soil (Figure 1). The roof support layer is considered only as a boundary layer for the lower soil volume and it will be treated by the existing simulation engine in ESP-r.

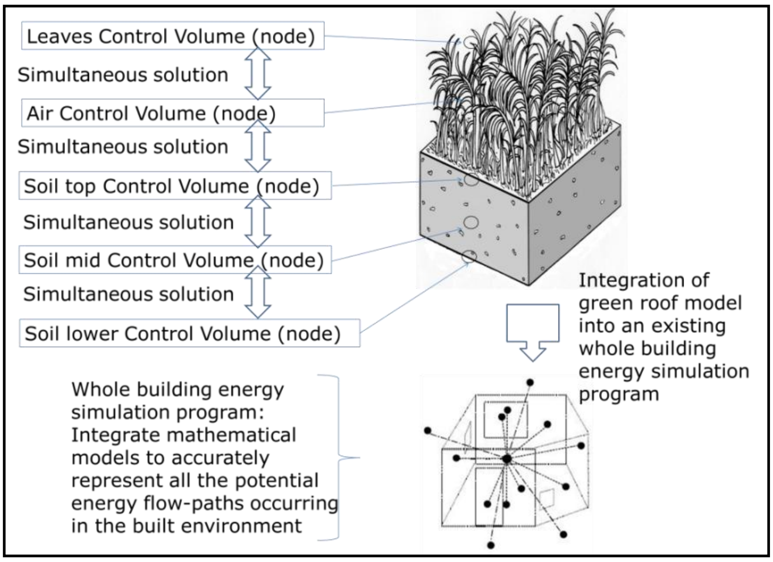

Figure 1. *Green roof modelling approach*

The development of the green roof model requires the definition and the coupling of the energy and moisture exchanges within control volumes and between the control volumes of the model. The next section will summarise the basic equations associated with the five control volumes of the canopy and soil layers.

ENERGY AND MOISTURE BALANCES

This section summarises the fundamental equations for thermal and moisture exchanges for each of the five control volumes that define the proposed green roof model. The equations are sourced from well referenced publications (Del Barrio 1998, Sailor 2008, Bittelli et al 2008, Smith 2000) and are listed in Table 1.

***Table 1** Green Roof thermal and moisture equations summary*

No	Equations	Variable list
	Canopy plant volume	**T_a/T_p/T_{su}/T_{sm}/T_{sl}** Temperatures [℃] of five control volumes; **ρ_p-/C_p** leaf density [kg/m^3]/ sp heat [J/kgK]; **d/L** leaf thickness/canopy height [m]; **LAI** leaf area index [-]; **$\Phi_{rad,sol}$ / $\Phi_{rad,long}$** short /long wave radiation heat received at canopy plant [W/m^2]; **Φ_s** incident solar radiation at the top of canopy [W/m^2]; **$\Phi_{conv,p-a}$/$\Phi_{conv,a-su}$ /$\Phi_{conv,a-\alpha}$** convective heat exchange between plant and canopy air/canopy air and soil/ canopy air and outside air [W/m^2]; **$\Phi_{trans,p-a}$** transpiration heat loss from plant to air [W/m^2]; **τ_s** transmittance; **ρ_α/ρ_g** reflectance; **h_r,sky/ h_r,su**
1	$1.1\rho_p C_p dLAI \frac{dT_p}{dt} = \phi_{rad,sol} + \phi_{rad,long} + \phi_{conv,p-a} + \phi_{trans,p-a}$	
2	$\phi_{rad,sol} = [1-\tau_s] - (1-\tau_s)\rho_\alpha(1+\tau_s\rho_g)\phi_s$	
3	$\phi_{rad,long} = h\_r,sky(T_{sky} - T_p) + h\_r,su(T_{su} - T_p)$	
4	$\phi_{conv,p-a} = -2LAI\frac{\rho Cp}{r_e}(T_p - T_a)$	
5	$\phi_{trans,p-a} = -2LAI\frac{\rho Cp}{\gamma(r_e + r_i)}(e_p - e_a)$	
	Canopy air volume	
6	$\rho_a Cp_a L\frac{dT_a}{dt} = \phi_{conv,a-p} + \phi_{conv,a-su} + \phi_{conv,a-\alpha}$	
	Soil upper layer	
7	$\rho_{su}C_{su}\frac{dT_{su}}{dt} = A_{su}\tau_{ca}\phi_s + \phi_{lw,sky} + \phi_{lw,p} + \phi_{conv,a-su} - \phi_{evap,su-a} - \lambda\frac{dT}{dz}$	
8	$\phi_{evap,su-a} = \frac{\rho_a Cp_a}{\gamma(r_v + r_s)}(e_{su} - e_a)$	
	Equations, 9 and 10 are written in discretised form because they involve non-linear differentials: Soil mid layer (discretised) volume	
9	$\rho_{sm}C_{sm}\frac{T_{sm}^{t+\Delta t} - T_{sm}^{t}}{\Delta t} = \frac{1}{SM}\left[\frac{\lambda_{sm}T_{sm}^{t} - \lambda_{su}T_{su}^{t}}{SU}\right]$ $-\frac{1}{SM}\left[\frac{\lambda_{sl}T_{sl}^{t} - \lambda_{sm}T_{sm}^{t}}{SL}\right] - Lv\left[\frac{Jvp_{sm}^{t} - Jvp_{su}^{t}}{SM}\right]$	
	Soil lower layer (discretised) volume	
10	$\rho_{sl}C_{sl}\frac{T_{sl}^{t+\Delta t} - T_{sl}^{t}}{\Delta t} = \frac{1}{SL}\left[\frac{\lambda_{sl}T_{sl}^{t} - \lambda_{sm}T_{sm}^{t}}{SM}\right] -$ $\frac{1}{SL}\left[\frac{\lambda_x T_x^{t} - \lambda_{sl}T_{sl}^{t}}{X}\right] - Lv\left[\frac{Jvp_{sl}^{t} - Jvp_{sm}^{t}}{SL}\right]$	

	Variable list (contd) **e_a/e_p /e_{su}** air/saturated/ soil surface vapour pressures [Pa]; γ thermodynamic constant gamma; **τ_{ca}** transmittance of canopy layer; **A_{su}** fraction of radiation absorption at soil surface; **$\Phi_{evap,su-a}$** evaporation heat loss from soil to air [W/m^2]; **$\Phi_{lw,sky}$/ $\Phi_{lw,p}$** long wave radiation received at soil from sky/plant [W/m^2]; **r_v/r_s** aerodynamic/ soil surface resistances to mass transfer [s/m]; **C_{su}/C_{sm}/C_{sl} volumetric specific heat of soil layers [J/m^{3}K]** **SU/SM/SL/X** layer thickness soil layers upper/mid/lower /roof [m]; **λ** soil thermal conductivity [W/mK]; **Jvp** isothermal vapour flux[kg/m^{2}s]; **Lv** latent heat of vaporisation of water [J/kg].	linearized radiation coefficients [W/m^{2}K]; **ρ_a / Cp_a** air density [kg/m^3]/ specific heat [J/kgK]; **r_e/r_i** air/stomatal resistances to sensible heat transfer [s/m];

The above equations of Table 1 are amongst the most classic ones used in the literature. However, alternative relationships could be also found in a number of other research areas such as plant physiology (Lambers et al 2008), environmental biophysics (Campbell 1998), agricultural meteorology (Bishnoi 2007, Feddes et al. 2004) and soil-plant-atmosphere studies (Hansen et al 1990, Jansson et al 2004). The development of the first green roof model in ESP-r will use the relationships described in this section while it is also expected that future versions of the model will incorporate additional algorithms which the user will be able to choose from.

The green roof model that is derived by the above equations of Table 1 will need to be integrated within the ESP-r program by using the ESP-r's numerical approach for simultaneously solving all the control volumes of a building model. The next section is giving a general description of this numerical approach in ESP-r.

INTEGRATED SIMULATION PROCEDURE IN ESP-r

ESP-r is an open-source building performance energy modelling software capable of simulating the energy and mass flows within the building systems (Clarke 2001). The fundamental physical processes associated with a building are represented as governing equations in control volumes. The equations represent mass, energy and momentum balances. The couplings between domains are achieved by passing information between them in every time step.

The finite difference control volume approach in ESP-r can be summarised into three steps (Beausoleil-Morrison 2000):

1. Discretisation of building into control volumes, which consist of air volumes (of rooms), construction elements layers (walls, windows, etc.) and plant components (HVAC units).
2. Heat balance equations are developed for each control volume. These differential equations link to space (neighbouring control volumes) and time (time steps of simulation). The differential equations are subsequently discretised for numerical solution.

3. The equations, for example for the thermal domain, are simultaneously solved to obtain the temperatures in and heat flows across all nodes for a given point in time. The procedure is repeated in the next time step at which time the present state point values become known from the previous time step values. Within each time step, an iteration process of solving the relevant equations is done until the difference with the known boundary conditions become acceptably small.

The equations within a control volume are post-processed for solution in the following way (Kelly 2001):

1. The control volume equation is discretised, that is the differential term is replaced by a forward difference expression of explicit nature. Explicit means they are expressed in terms of present time step values (superscript t in the right side terms in equation numbers 9 and 10 in Table 1)
2. An implicit form of the same equation is created by changing the right hand side terms to future time step values (the superscripts t will now be replaced with t+Δt)
3. The implicit expression is multiplied by α (0 < α <1), the explicit expression multiplied by (1 - α) and the two added together to give one equation. ESP-r is using the Crank-Nicolson scheme where the weighing factor α is given a value of 0.5, which makes the solution unconditionally stable.
4. The equation is re-arranged in a specific manner to obtain three types of coefficients: 'self-coupling coefficients', are those associated with nodal property (temperature in the case of heat exchange equation) of control volume in consideration, 'cross coupling coefficients' are those associated with the nodal property (e.g. temperatures in the case of heat exchange equation) of the neighbouring control volumes and 'right-hand side coefficients' are those remaining of known values. This equation is suitable for use in an overall matrix solution procedure in ESP-r. An example of a matrix format for a 3-control volume system (nodes i, j and k) is shown below in Figure 3 (nodes l, m and n are surroundings that define the variable boundary conditions to this system).

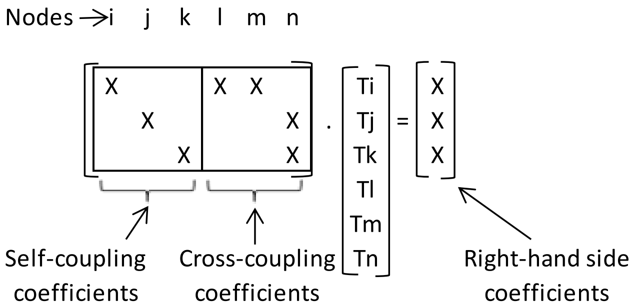

Figure 3. *Solution matrix showing coefficients*

FRAMEWORK FOR INTEGRATION OF GREEN ROOF MODEL IN ESP-r

The expansion of the equation that represents the canopy plant volume into a format that could be used within the ESP-r simulation engine is given as an example in this section. The steps are the same for the rest of the control volumes but are not presented here due to space limitations. The explicit form of equation for the canopy plant, from equation number 1 in Table 1 is:

$$
\begin{aligned}
&1.1\rho_p C_p dLAI\left[\frac{T_p^{t+\Delta t}-T_p^t}{\Delta t}\right]=\left\{\left[1-\tau_s\right]-\left(1-\tau_s\right)\rho_\alpha\left(1+\tau_s\rho_g\right)\right\}\phi_s^t+h\_r,sky^t\left(T_{sky}^t-T_p^t\right)+\\
&h\_r,su^t\left(T_{su}^t-T_p^t\right)-2LAI\frac{\rho Cp}{r_e^t}\left(T_p^t-T_a^t\right)-2LAI\frac{\rho Cp}{\gamma\left(r_{e+i}^t\right)}\left(e_p^t-e_a^t\right)
\end{aligned}
\qquad (11)
$$

The implicit form of the equation is:

$$
\begin{aligned}
&1.1\rho_p C_p dLAI\left[\frac{T_p^{t+\Delta t}-T_p^t}{\Delta t}\right]=\left\{\left[1-\tau_s\right]-\left(1-\tau_s\right)\rho_\alpha\left(1+\tau_s\rho_g\right)\right\}\phi_s^{t+\Delta t}+h\_r,sky^{t+\Delta t}\left(T_{sky}^{t+\Delta t}-T_p^{t+\Delta t}\right)+\\
&h\_r,su^{t+\Delta t}\left(T_{su}^{t+\Delta t}-T_p^{t+\Delta t}\right)-2LAI\frac{\rho Cp}{r_e^{t+\Delta t}}\left(T_p^{t+\Delta t}-T_a^{t+\Delta t}\right)-2LAI\frac{\rho Cp}{\gamma\left(r_{e+i}^{t+\Delta t}\right)}\left(e_p^{t+\Delta t}-e_a^{t+\Delta t}\right)
\end{aligned}
\qquad (12)
$$

Multiplying equation (11) by (1-α), equation (12) by α and adding them together:

$$
\begin{aligned}
&\left[\frac{1.1\rho_p C_p dLAI}{\Delta t}+\alpha h\_r,sky^{t+\Delta t}+\alpha h\_r,su^{t+\Delta t}+\alpha 2LAI\frac{\rho Cp}{r_e^{t+\Delta t}}\right]T_p^{t+\Delta t}+\left[\alpha h\_r,sky^{t+\Delta t}\right]T_{sky}^{t+\Delta t}+\\
&\left[\alpha 2LAI\frac{\rho Cp}{r_e^{t+\Delta t}}\right]T_a^{t+\Delta t}=\left[\frac{1.1\rho_p C_p dLAI}{\Delta t}-\left(1-\alpha\right)h\_r,sky^t-\left(1-\alpha\right)h\_r,su^t-\left(1-\alpha\right)2LAI\frac{\rho Cp}{r_e^t}\right]T_p^t\\
&+\left[\left(1-\alpha\right)h\_r,sky^t\right]T_{sky}^t+\left[\left(1-\alpha\right)2LAI\frac{\rho Cp}{r_e^t}\right]T_a^t\\
&+\left(1-\alpha\right)\left\{\left[1-\tau_s\right]-\left(1-\tau_s\right)\rho_\alpha\left(1+\tau_s\rho_g\right)\right\}\phi_s^t+\alpha\left\{\left[1-\tau_s\right]-\left(1-\tau_s\right)\rho_\alpha\left(1+\tau_s\rho_g\right)\right\}\phi_s^{t+\Delta t}\\
&+\left(1-\alpha\right)h\_r,su^tT_{su}^t+\alpha h\_r,su^{t+\Delta t}T_{su}^{t+\Delta t}-\\
&\left[\left(1-\alpha\right)2LAI\frac{\rho Cp}{\gamma\left(r_{e+i}^t\right)}e_p^t+\alpha 2LAI\frac{\rho Cp}{\gamma\left(r_{e+i}^{t+\Delta t}\right)}e_p^{t+\Delta t}\right]+\left[\left(1-\alpha\right)2LAI\frac{\rho Cp}{\gamma\left(r_{e+i}^t\right)}e_a^t+\alpha 2LAI\frac{\rho Cp}{\gamma\left(r_{e+i}^{t+\Delta t}\right)}e_a^{t+\Delta t}\right]
\end{aligned}
\qquad (13)
$$

The first terms on the left side and right side of the equation give the self-coupling coefficients (future and present values respectively) of the nodal temperature under consideration. The next two terms on the left side and right side give the cross-coupling coefficients of the connected nodes. This equation can be written in a shorter way as:

$$
a_{12}T_p^{t+\Delta t}+a_{11}T_{sky}^{t+\Delta t}+a_{13}T_a^{t+\Delta t}=b_{12}T_p^t+b_{11}T_{sky}^t+b_{13}T_a^t+c_{11} \qquad (14)
$$

All the elements of this equation form the first row of a matrix set of equation, which can be solved by standard matrix solving procedures such as Gaussian elimination steps. The complete matrix equation relevant to green roof model nodal temperatures is given below:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} & a_{35} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} & 0 \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} & a_{57} \end{bmatrix} \bullet \begin{bmatrix} T_{sky}^{t+\Delta t} \\ T_{p}^{t+\Delta t} \\ T_{a}^{t+\Delta t} \\ T_{su}^{t+\Delta t} \\ T_{sm}^{t+\Delta t} \\ T_{sl}^{t+\Delta t} \\ T_{x}^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & 0 & 0 \\ 0 & b_{22} & b_{23} & b_{24} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & b_{34} & b_{35} & 0 & 0 \\ 0 & 0 & 0 & b_{44} & b_{45} & b_{46} & 0 \\ 0 & 0 & 0 & 0 & b_{55} & b_{56} & b_{57} \end{bmatrix} \bullet \begin{bmatrix} T_{sky}^{t} \\ T_{p}^{t} \\ T_{a}^{t} \\ T_{su}^{t} \\ T_{sm}^{t} \\ T_{sl}^{t} \\ T_{x}^{t} \end{bmatrix} + \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \\ c_{41} \\ c_{51} \\ c_{61} \\ c_{71} \end{bmatrix}$$

A similar matrix has also been completed for moisture transport in all the five control volumes, which is of the form:

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 & 0 & 0 & 0 \\ 0 & m_{22} & m_{23} & m_{24} & 0 & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} & m_{35} & 0 & 0 \\ 0 & 0 & 0 & m_{44} & m_{45} & m_{46} & 0 \\ 0 & 0 & 0 & 0 & m_{55} & m_{56} & 0 \end{bmatrix} \bullet \begin{bmatrix} e_{o}^{t+\Delta t} \\ e_{p}^{t+\Delta t} \\ e_{a}^{t+\Delta t} \\ \psi_{su}^{t+\Delta t} \\ \psi_{sm}^{t+\Delta t} \\ \psi_{sl}^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} & 0 & 0 & 0 & 0 \\ 0 & n_{22} & n_{23} & n_{24} & 0 & 0 & 0 \\ 0 & 0 & n_{33} & n_{34} & n_{35} & 0 & 0 \\ 0 & 0 & 0 & n_{44} & n_{45} & n_{46} & 0 \\ 0 & 0 & 0 & 0 & n_{55} & n_{56} & 0 \end{bmatrix} \bullet \begin{bmatrix} e_{o}^{t} \\ e_{p}^{t} \\ e_{a}^{t} \\ \psi_{su}^{t} \\ \psi_{sm}^{t} \\ \psi_{sl}^{t} \end{bmatrix} + \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \\ p_{51} \\ p_{61} \end{bmatrix}$$

The state variables of this last matrix relationship are vapour pressures e [Pa] for nodes in air and water potentials ψ [Pa] for nodes in soil.

CONCLUSION

Green roofs are increasingly being used in sustainable building sector and their realistic simulation is an important element of design. The modelling of green roof constructions is challenging because it involves complex heat and moisture processes in the plant and soil layers of the roof. The process for integrating a control volume green roof model within the ESP-r's simulation engine was presented in this paper. All the coefficients required for the model implementation in ESP-r have been sourced and compiled. It is found that that the modular and integrated whole building energy simulation platform of ESP-r is an ideal environment for implementing such model and future work of this research intends to develop the required program for the practical use of the model by the ESP-r users.

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