In 1955, British Nash et al. published the study on reflective air spaces (reflective insulation assemblies) that reviewed heat transfer aspect of reflective insulation, calculated heat resistance value and introduced reflective insulation assemblies that are similar to those of today. Around this time, National Bureau of Standards (Robinson et al. 1954, Robinson et al. 1957) and Robinson and Powell (1956) of the US published two important study reports. The report of 1954 discussed a series of hot box measurement on reflective insulation assemblies. The measure data became the base of thermal resistances values of plane reflective air space of ASHRAE Handbook of Fundamentals in 1972 and another Handbook published after that.
In 1983, Yarbrough proposed analytically how to calculate thermal resistance of single reflective air space (Method 1) by using the existing 309 test data of Robinson and Powell and applying Method of Least Squares with air space thickness l, temperature difference across the space Δt, mean temperature across the air space t_m as variables for the study of Oak Ridge National Laboratory.
In 1991, Desjarlais and Yarbrough did further heat resistance test related with reflective insulations and established more heat resistance data that helped confirm the initial NBS measurement result and extended heat resistance value calculation scope for reflective insulations. They also proposed the mathematical method (Method 2) that calculate heat resistance value by using dimensionless relations that used Nusselt number(N_u) on five major heat flow cases (*Horiz.+up, 45° Slope+up, Vertical, 45°Slope+Down, Horiz.+Down*, Refer to Table 1).
As the proposed Method 1 and Method 2 above calculate numerical value through different analytical methods, the difference of heat resistance value of reflective insulations from both methods needs to be observed. This study developed Microsoft Visual Basic Function on both methods (Method 1 and Method 2) that calculate the thermal resistance value of single reflective air space installed with reflective insulations and compared it with the 875 test data suggested by ASHRAE Fundamental to see how big the heat resistance value is by both methods.

This study was conducted as below.
First, it performed theoretic review on the two methods that calculate the thermal resistance value of single reflective air space for buildings installed with reflective insulations and developed Microsoft Visual Basic Function respectively that estimates thermal resistance value of plane reflective air space.
Second, the results of each function were compared with the thermal resistance value of single reflective air space (875 test data) that the 2013 ASHRAE Fundamentals Chapter 26.3 Table 3 compared to verify the validity of the two methods.
Third, it analyzed the strengths and weaknesses of the two methods based on above and suggested the way to calculate proper thermal resistance value of reflective insulations.

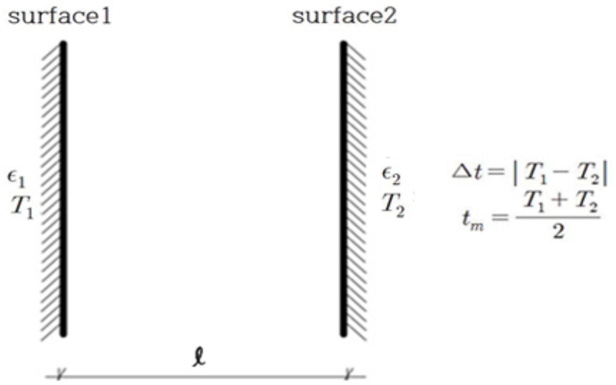


Figure 2. *Model of reflective air space (air space thickness l , emittance and surface temperature on boundaries of both sides ε_1, ε_2, T_1, T_2)*

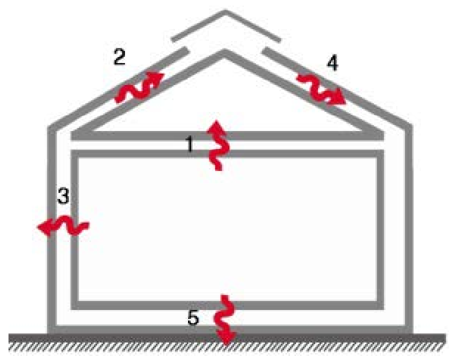
The process of heat transfer caused by air space's conduction and convection is more complicated than process of radiation heat transfer. h_c (heat transfer coefficient for conduction-convection) can be defined with the function related with four variables as shown at formula (6).

$$h_c = f(l,\ t_m, \Delta t,\ Heat\ Flow\ Direction) \qquad (6)$$

l : air space thickness
Δt : temperature difference across the air space
Heat Flow Direction : Case by position of air space and flow direction

Table 1. *Case of Heat Flow Direction*

Case	Position of Air Space	Flow Direction
1	Horizontal	Up ↑
2	45° Slope	Up ↗
3	Vertical	→
4	45° Slope	Down ↘
5	Horizontal	Down ↓



h_c can be calculated by the following methods.

h_c calculation of Method 1

h_c is analytically calculated using formula (7) with application of air space thickness (l), temperature difference across the air space (Δt) and five cases' conditional heat transfer coefficient test result to the least square fitting. $h_{c(50)k}$ here is a dependent variable and means the heat transfer coefficient at the mean temperature across the air space of 10˚C (50°F).

$$h_{c(50)k} = \sum_{i=1}^{4}\left[\left(\sum_{i=0}^{3}\alpha_{ijk}l^{i}\right) + a_{i4k} / l\right]\Delta t^{i-1} \qquad (7)$$

h_c calculation of Method 2

To calculate h_c, Method 2 uses formula (8) that applies dimensionless Nusselt number (N_u) and thermal conductivity of air (k_{air}) as variables.

$$h_c = N_u \cdot k_{air} / l \quad (8)$$

$$k_{air} = 0.0003053 \times t_m + 0.1575 \quad (9)$$

where k_{air} is thermal conductivity of air ($Btu \cdot inch / ft^2 h$°F).

The formula to quantity heat transfer caused by conduction-convection is the polynomial of Nusselt Number (N_u) expressed as Grashoff Number (G_r) and can be written as below formulas (10) to (13). The constants used here apply Table 3's $(G_r)_{0\sim a_3} \sim a_3$ constants depending on five types of heat flow direction.

$$log_{10}(N_u) = 0 \ (log_{10}(Gr) \le \ log\_10(Gr)_0) \quad (10)$$

$$\log_{10}(\mathrm{N_u}) = [\mathrm{a_2}\log_{10}(\mathrm{Gr}) - \log_{10}(\mathrm{Gr})_0]^2 + [\mathrm{a_3}\log_{10}(\mathrm{Gr}) - \log_{10}(\mathrm{Gr})_0]^3$$

$$(\log_{10}(\mathrm{Gr})_0 \le \log_{10}(\mathrm{Gr}) \le \log\_10(\mathrm{Gr})_1) \quad (11)$$

$$\log_{10}(\mathrm{N_u}) = \mathrm{a} + \mathrm{b}\log_{10}(\mathrm{Gr}) + \mathrm{c}\log_{10}(\mathrm{Gr})^2$$

$$(\log_{10}(\mathrm{Gr})_1 \le \log_{10}(\mathrm{Gr}) \le \log\_10(\mathrm{Gr})_2) \quad (12)$$

$$log_{10}(Gr) = log_{10}(\Delta T \times l^3) + 3.4146 - 0.4359 \times 10^{-2} t_m + 0.36441 \times 10^{-5} {t_m}^2 \quad (13)$$

Table 3. *Constants used to calculate N_u of Method 2*

case	$(G_r)_0$	$(G_r)_1$	$(G_r)_2$	*a*	*b*	*c*	a_2	a_3
1	2.5	3.49	6.80	-0.8620	0.2912	-3.8630E-04		
2	2.5	4.14	6.90	-0.3000	0.0381	2.4090E	1.5679E	-3.4369E
3	2.5	4.35	7.20	-2.2234	0.6784	-3.0280E-02	1.6481E-03	2.3528E-02
4	3.5	4.68	7.82	-1.3771	0.2989	5.9300E-03	1.2676E-03	8.7056E-02
5	2.5	5.09	7.50	1.4959	-0.6080	6.3590E-02	6.5572E-03	2.7129E-04

RESULTS & DISCUSSION

To compare with the thermal resistance value of reflective insulation (875 test data) that the 2013 ASHRAE Fundamentals Chapter 26.3 Table 3 showed, the verification conditions of Method 1 and 2 were listed at Table 4. The comparison was presented by distributing x-y chart by placing ASHRAE table value at x and calculation result values from Method 1 or 2 at y with 875 data whose value of variable items is the same as Table 4. When a straight line of perfect 45 degree angle is created, it can be considered that the values from the concerned method and ASHRAE calculation are the exact match.

Table 4. *Variable items and variable values to verify calculation result of Method 1 and 2*

ASHRAE variable items	ASHRAE variable values
1. Position of Air Space	Case 1, Case 2, Case 3, Case 4, Case 5
2. Flow Direction	
3. Mean Temp	32.2, 10.0, -17.8, -45.6℃
4. Temp. Diff	5.6, 11.1, 16.7K
5. Air Space Thickness	3, 20, 40, 90, 143mm
6. Effective Emittance	0.03, 0.05, 0.2, 0.5, 0.82

The difference between the ASHRAE Table value and calculation value of Method 1 showed that several values are distributed while the 875 results are straight as shown at the figure 3. When this was expressed with trend of regression equation of the first straight line, R^2 value was presented relatively low at 0.742 with $y = 0.9378x + 0.0406$. On the other hand, the distribution of ASHRAE Table value and calculation value of Method 2 was straight as shown at the figure 4 and the trend of regression equation of the first straight line was $y = 1.0046x + 0.0037$ with R^2 value at 0.9648. This means that Calculating thermal resistance value of single reflective air space with Method 2 was more approximate to the ASHRAE table value than Method 1.

However, the bottom left of figure 4 showed that there is a region where distribution is inconsistent. This region has the case condition of 3, 4 or 5 and the air space thickness l is within 13 or 20mm. When this part was confined for comparing with ASHRAE Table values, R^2 value dropped to 0.6903 with the trend of regression equation of the first straight line at $y = 1.0761x + 0.0206$ as the figure 5 showed. On the other hand, when the case condition is 3, 4 or 5 and the air space thickness l is within 13 or 20mm at Method 1, R^2 value was 0.9835 with the trend of regression equation of the first straight line at $y = 1.0451x + 0.0055$ as the figure 5 showed.

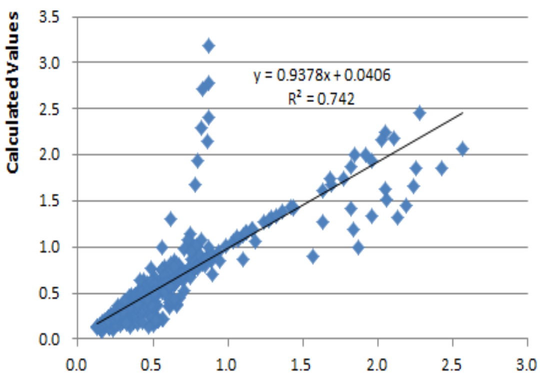


Figure 3. *Comparison between ASHRAE Table value and calculation value of Method 1* $(m^2 \cdot K/W)$

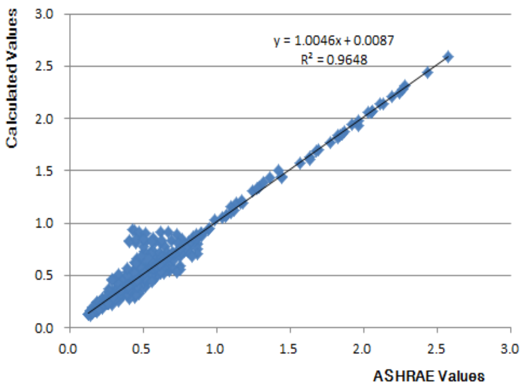


Figure 4. *Comparison between ASHRAE Table value and calculation value of Method 2* $(m^2 \cdot K/W)$

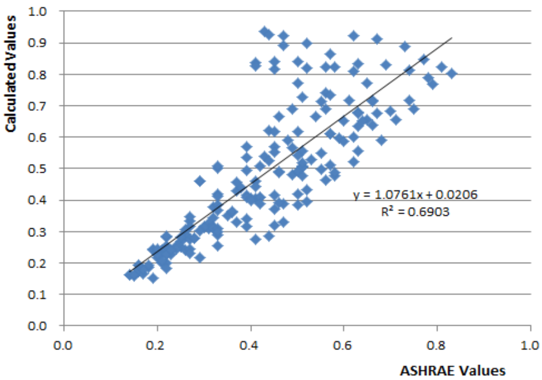


Figure 5. Result comparison under the condition of Heat Flow Direction 3, 4, 5 and air space thickness 13 and 30mm of Method 2 ($m^2 \cdot K/W$)

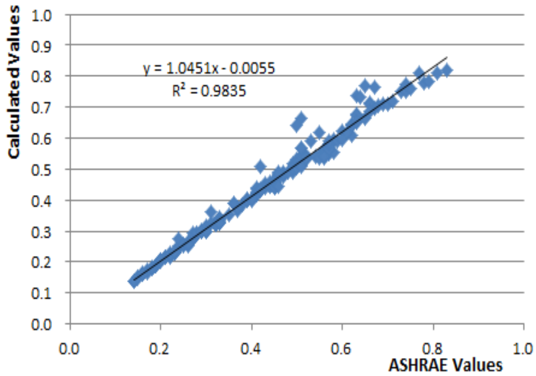


Figure 6. Result comparison under the condition of Heat Flow Direction 3, 4, 5 and air space thickness 13 and 30mm of Method 1 ($m^2 \cdot K/W$)

CONCLUSION AND IMPLICATIONS

This study developed Microsoft Visual Basic Function on both methods (Method 1 and Method 2) that calculate the thermal resistance value of single reflective air space for buildings installed with reflective insulations and compared it with the 875 test data suggested by ASHRAE Fundamental to see how big the thermal resistance value is by both methods. The following is the result.

1. When the calculation results of Method 1 and 2 were compared with the ASHRAE Fundamental Table value, Method 2 was more consistent with the ASHRAE Fundamental Table value than Method.
2. However, Method 1's result was found more consistent with it than Method 2's when the case condition was 3, 4 or 5 and air space thickness l was within 13 or 20mm.
3. Therefore, applying Method 2 is desirable to calculate the thermal resistance value of plane reflective air space for buildings installed with reflective insulations while Method 1 is when the case condition is 3, 4 or 5 as well as the air space thickness l is within 13 or 20mm. If Method 1 and 2 are selectively applied, the trend of regression equation with ASHRAE Fundamental Table value is $y = 1.0046x + 0.0002$ as the figure 7 shows and the R^2 value is 0.994. It can be expected from this that the thermal resistance value of reflective insulations can be calculated with high reliability.

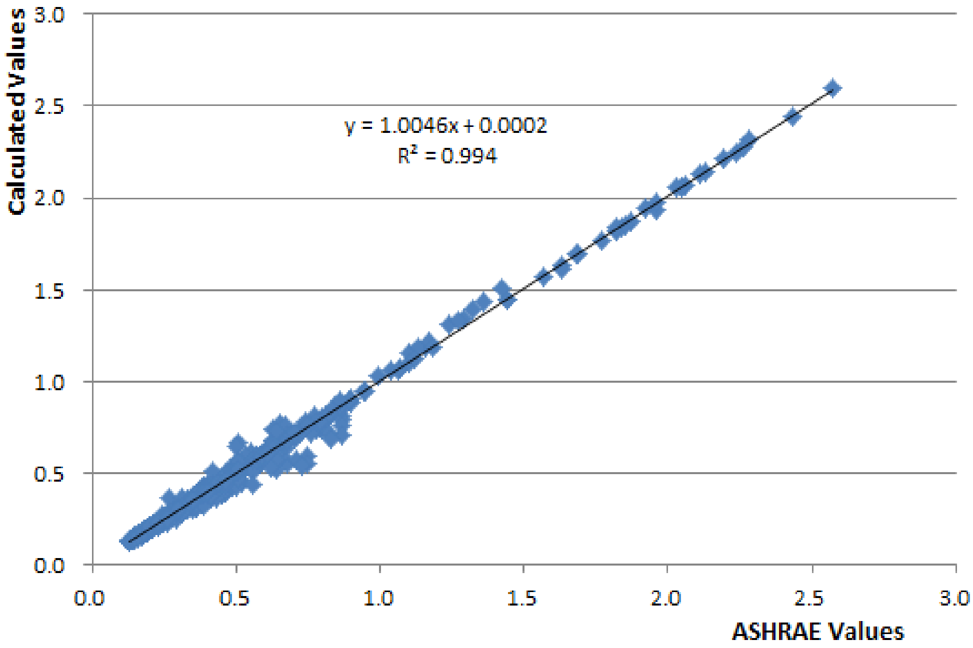


Figure 7. *Result value comparison by Method 1 + Method 2* ($m^2 \cdot K/W$)

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