A NEW LOCAL PRESSURE LOSS COEFFICIENT MODEL OF DUCT TEE JUNCTION APPLIED IN TRANSIENT SIMULATION OF HVAC AIR-SIDE SYSTEM

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ABSTRACT
Most of the local pressure loss coefficient (LPLC) models for duct fittings used in heating ventilation and air conditioning (HVAC) air-side system transient simulations are the simplified ones. The LPLCs are defined as a constant value at the rated flow condition or as a polynomial function of flow ratio (or velocity ratio). However, literature reviews give the evidence that the LPLC is affected by the inlet Reynold number, which makes the usability of the two simplified LPLC models (Constant-value model, Polynomial function model) questionable. To figure out the influence of these simplifications, this study took the diverging tee junction as an example. First we used CFD calculations to generate a new LPLC dataset and trained a data-driven model from it with the method of feature weighted support vector regression combined with particle swarm optimization (PSO-FWSVR). Then we compared this new model with the polynomial model in the LPLC calculation on the test set. The mean absolute percentage error of new model is less than 3%, which is much better than polynomial model. At last we applied all the three LPLC models in a simple air-side system simulation to analyze the impact of LPLC simplifications on the simulated operation condition of a system. It is found that the constant-value LPLC model can cause a large error of the simulation results of system flow conditions. It is too simplified to reflect the dynamic behavior of air distribution system correctly. And the result from polynomial model is comparatively closer to PSO-FWSVR model, but the bias may be unacceptable when simulating a system with high control precision.

KEYWORDS
HVAC air-side system; Transient simulation; local pressure loss coefficient; uniform design; feature weighted SVR

1 INTRODUCTION
High indoor comfort with low energy consumption is always the common goal of HVAC engineers. Among the several main subsystems of HVAC system, the air-side system is the terminal subsystem that serves the occupant more directly than any others. A well-controlled air-side system is beneficial to achieve the goal. So many researchers are dedicated to the development of advanced control strategies (Xu et al. 2009). Before used in practice, these strategies are often put through simulation trials. There are several commonly used HVAC transient simulation platforms such as TRNSYS, Simulink and Dymola. And many delicate component models are developed based on them. When we are conducting a transient simulation of air-side HVAC system, the balance between fan pressure and ductwork resistance should be considered carefully. This balance will

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determine the flow condition of the air distribution system including the total air flow rate and the flow allocation among air branches and finally the dynamic response of every single terminal zone supplied by one air branch. As a result, the resistance property of the component models is crucial for correct simulation of an HVAC air-side system. And as an inconspicuous component, the resistance property of duct fittings is often neglected or greatly simplified.

Yan Chen et al. (2014) developed a Simulink block library of dynamic HVAC component models, in which the modules of junctions like flow splitting and merging are included. The pressure loss at these junctions are defined as the product of a constant value and the square of mass flow rate of air. The same constant-value assumption is also adopted by the Modelica Building Library (2009), which is developed by LBNL. As regards to TRNSYS, the flow split type in its official type library doesn't contain resistance property. As a result, some researchers developed their own fitting types. Meiwei Liu et al. (2012) built a mathematical model to reflect the hydraulic characteristics of the VAV (variable air volume) air-side system and defined the LPLC of the tee junction as a quadratic polynomial function of the flow ratio of downstream branch to upstream. The above literatures present two LPLC models for tee junctions with different degrees of simplification, i.e. constant-value model and polynomial model. The question comes: How will this two models function in simulating the dynamic behavior of air-side system? Will the simplification to models have any influence on the accuracy of simulation?

If one wants to build a new fitting resistance model or select proper parameters for known models, the existing LPLC tables are the fundamental dataset right off the top of his head. The local pressure loss coefficient (LPLC) of duct fitting is a historical research topic which could date back to 1950s. Most of the early researches were done through experiment, including the works of ASHRAE (2009), D.S. Miller (1971) and some other researchers from the former Soviet Union. However, since the experimental method and the geometry details for a same type of fitting, like tee junction, are different from each other, there are differences among the LPLC results from different experiments. In 1970s, Construction Ministry of China published a technical handbook (1999) for duct and fittings, which gave many geometry recommendations of fittings and corresponding LPLCs tables based on experiment results. On the other hand, the LPLC tables for many types of fittings can also be found in the chapter of duct design in ASHRAE Handbook of Fundamentals (2009). However, Shao L et al. (1995) stated that some of the LPLCs for junctions in ASHRAE handbook may have large errors because of the negligence of Reynold number. And the technical handbook from China suffers the same problem as well. To generate the LPLCs data of duct fittings, CFD has been approved to be a convenient and sufficiently accurate way besides experiment. Since the end of last century many researcher tried to apply different turbulent models to calculate the LPLCs of different type of fittings (Mumma S et al. 1997, Gan G H et al. 2000, Liu Wei et al. 2012), which verified the applicability of CFD method in LPLC calculation.

From all the existing LPLC dataset and some related literatures, we know that LPLC is mainly effect by its geometry factors, the real time flow allocation and upstream flow condition. An ideal LPLC model should be capable of simultaneously reflecting the influence of all these factors. And in the meantime, the model should be used easily and run quickly. The geometry parameters are set by users ahead of simulation and the flow condition variables are transmitted among different modules during simulation. Unfortunately, this kind of model has not been found in any available relevant literatures. So in this study we took the diverging smooth tee junction as an example to describe the process of a new LPLC model development. Then it was compared with the two simplified LPLC models (polynomial and constant-value model) to validate the improved accuracy and to figure out the deviation brought by junction resistance simplifications.

2 METHODOLOGY

2.1 Calculation methods for LPLCs and duct friction pressure loss coefficient

LPLC was calculated using the following equations,
For blow-through branch:

$$\xi_b = \frac{\left(\frac{1}{2}\rho V_1^2 + P_{s1}\right) - \left(\frac{1}{2}\rho V_2^2 + P_{s2}\right) - L_u \Delta P_{f,u} - L_b \Delta P_{f,b}}{\frac{1}{2}\rho V_1^2} \quad (1)$$

For vertical branch:

$$\xi_v = \frac{\left(\frac{1}{2}\rho V_1^2 + P_{s1}\right) - \left(\frac{1}{2}\rho V_3^2 + P_{s3}\right) - L_u \Delta P_{f,u} - L_v \Delta P_{f,v}}{\frac{1}{2}\rho V_1^2} \quad (2)$$

The subscripts 1, 2 and 3 refer to the pressure measuring section of upstream, blow-through and vertical branch respectively. $\frac{1}{2}\rho V^2$ refers to dynamic pressure and P_s the static pressure. And $L_u \Delta P_{f,u}$、$L_b \Delta P_{f,b}$、$L_v \Delta P_{f,v}$ means the duct friction pressure loss of upstream duct, blow-through and vertical branch respectively. The duct friction pressure loss coefficient was calculated using the Moody equation as follow and the roughness height of K was determined by straight duct resistance experiment.

$$\lambda = 0.0055\left[1 + (20000\frac{K}{d} + \frac{10^6}{Re})^{\frac{1}{3}}\right] \quad (3)$$

2.2 Case design process for CFD calculation

Since the existing LPLC database might have some flaws, we had to generate a new set of LPLC data first. We chose to use CFD to calculate the LPLCs of the objective junctions, since it's more economic than the experimental way. And we adopted uniform design method to arrange the CFD cases. As regards to the dependent and independent variables, the LPLCs of blow-through and vertical branch became the dependent variable naturally. On the other hand, according to the influence factors in some existing LPLC tables, we firstly chose the section area ratio AR_b and AR_v and flow ratio FRb, where the subscript of b and v means the ratio of blow-through and vertical branch to the upstream respectively. In addition, since Shao L et al. (1995) state that Reynold number Re also has an impact on LPLC, we added the upstream inlet velocity V to reflect the influence of Re.

China's state and local departments of construction have published a few manufacture specification standards of duct fittings. In one of the local standard of DBJT05-104 (1999), the section area ratio of smooth tee junction should meet the following limits.

$$1 \le AR_b + AR_v \le 1.16$$
$$AR_b \ge 0.5, \quad (4)$$
$$AR_v \le 0.65$$

Based on the dimension recommendation in the standard, 6 combinations of AR_b and AR_v could be concluded in total as shown in Tab. 1. Since there exists a linear constraint between AR_b and AR_v, we designed a tri-variable case set for each level of AR_b and used the multi-level uniform design table of U^6 ($6^{2\times2}$) to arrange the cases, where AR_v was set in the corresponding two levels and FR_b and V in 6 levels within [0.1, 0.9] and [4, 12] respectively as listed in Tab. 1. Then the geometry models for CFD calculation were built according to the values of AR_b and AR_v, and the boundary conditions were set according to the values of FR_b and V. Finally, LPLCs of both blow-through and vertical branches were calculated using Eq. (1) and (2) with two decimal places reserved.

Tab. 1 *Uniform design case set*

Case No.	AR_b	AR_v	FR_b	V (m/s)	Case No.	AR_b	AR_v	FR_b	V (m/s)	Case No.	AR_b	AR_v	FR_b	V (m/s)
1	0.50	0.5	0.1	5.6	7	0.63	0.4	0.1	5.6	13	0.80	0.25	0.1	5.6
2		0.63	0.26	8.8	8		0.5	0.26	8.8	14		0.3	0.26	8.8

3		0.5	0.42	12	9		0.4	0.42	12	15		0.25	0.42	12
4		0.63	0.58	4	10		0.5	0.58	4	16		0.3	0.58	4
5		0.5	0.74	7.2	11		0.4	0.74	7.2	17		0.25	0.74	7.2
6		0.63	0.9	10.4	12		0.5	0.9	10.4	18		0.3	0.9	10.4

2.3 PSO-FWSVR of LPLC data

After the CFD results of LPLCs were obtained, the corresponding feature weighted support vector regression model with particle swarm optimization (PSO-FWSVR) could be built. Here are some basics of FWSVR. When the correlation degree with the dependent variable y_i varies among the different features of the independent variable x_i, applying the same feature weight to all of the features during regression can cause undesirable prediction accuracy. Then one can use a diagonal matrix$A = diag(\gamma_1, \gamma_2 \dots \gamma_i \dots), i = 1,2 \dots n$, where $\gamma_i \in [0,1]$ representing the weight of the i'th feature, to assign different weight to each feature. And the unknown parameters of the final feature weighted SVR function as Eq. (5) are estimated by minimizing the following dual problem as Eq. (6) and Eq. (7), where C is the regularization parameter.

$$f(x) = \sum_{i=1}^{n}(a_i^* - a_i)K(Ax_i, Ax) + b \quad (5)$$

$$min\frac{1}{2}\sum_{i,j=1}^{n}(a_i - a_i^*)(a_j - a_j^*)(\varphi(Ax_i)\cdot\varphi(Ax_j)) + \sum_{i=1}^{n}a_i(\varepsilon - y_i) + \sum_{i=1}^{n}a_i^*(\varepsilon + y_i) \quad (6)$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n}(a_i - a_i^*) = 0 \\ a_i, a_i^* \in [0, C] \end{cases} \quad (7)$$

During K-fold CV process, we used PSO with linear dynamic inertia weight to simultaneously determine the SVR parameters of C and γ and the weights of each feature of independent variable to build the PSO-FWSVR model of LPLCs. And the detailed modelling process is described by taking the vertical branch LPLC as an example and it is analogous for the blowthrough branch LPLC. And we used LIBSVM, a Matlab SVR toobox developed by Chih-Chung Chang et al.(2011), to finish the K-fold CV process and model training work.

We chose area ratios AR_b, AR_v, upstream inlet velocity V and the velocity ratio of vertical branch to upstream duct VR_v as the features of independent variable. The reason why VR_v was used instead of flow ratio was that some researcher mentioned the correlation between LPLC and velocity ratio shew more regularity than the one of flow ratio, which was also validated during trials. So the weight of VR_v was set to 1 while the weights of others were altered by PSO. And the CV error was used as the fitness function. Before regression the values of VR_v and V were linearly normalized as Eq. (8).

$$x_i' = \frac{x_i - x_{min}}{x_{max} - x_{min}}, \quad (8)$$

To test the prediction accuracy of SVR model, we chose 2 sets of data as test set. Group A (25 points) was the full permutation results of LPLCs of a tee junction whose AR_b and AR_v were both 0.5, when FRb and V were set in 5 levels each within their own range. It was used to test the model performance when predicting the LPLCs of a junction with a certain dimension under different combinations of FR_b and V. Group B (5 points) was the LPLC results of junctions with the rest 5 AR_b and AR_v combinations under random FR_b and V. This one was used to test the performance for junctions of different dimensions. And the prediction performance on the test set was evaluated by mean square error (MSE), correlation coefficient R^2 and mean absolute percentage error (MAPE), see Eq. (9) and (10).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left|\xi_{CFD,i} - \xi_{SVR,i}\right|^2, \quad (9)$$

$$MAPE \quad \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\xi_{CFD,i} - \xi_{SVR,i}}{\xi_{CFD,ave}}\right| \times 100\% \quad (10)$$

Where $\xi_{CFD,ave}$ is the mean value of the CFD LPLCs of the test set.

2.4 Comparison with the traditional LPLC models

And at last we compared this new LPLC model with the two traditional models in two stages. First we compared the results of both polynomial model and PSO-FWSVR model with the CFD results to verify the improvement of accuracy when calculating the LPLC of an individual junction. Here the constant-value model was left out since it keeps the LPLC unchanged. To build the polynomial model, we did a univariate cubic polynomial regression for both blow-through and vertical branch LPLCs with the same training set using velocity ratio as the independent variable.

And secondly we built up a simple ductwork system to demonstrate the influence of the three different LPLC models upon the simulated results of system operation condition on a system level. We built a simple duct system with three outlets, connected by several straight ducts and 2 tee junctions and driven by a constant speed fan on the Simulink platform, shown in Fig. 1. And the three tee junction LPLC models were used one by on. Here, three dampers with hydraulic diameter of 315mm, 200mm and 200mm were put at outlet A, B and C respectively. The rated flow of them were 3000CMH and 2000CMH. And the relationship of flow, openness and resistance was described as a 2D table, from which the pressure loss for each combination of flow and openness could be looked up and the data was from a damper manufacturer. Then the capacity of the fan was determined according to the total flow summed up from all outlets and corresponding pressure loss the ductwork consumed. The curve of fan flow and static pressure was defined as a quadratic polynomial function. For ducts, the friction property was determined by Moody equation as Eq. (3) and the length of each duct was tuned to achieve the flow balance between branches at rated condition. For constant model, the tee junction LPLC value at rated flow condition was chosen as the one kept constant during simulation. And here the rated LPLC value was set as the same as the one of PSO-FWSVR model. Simulation period was set as 200s. During the simulation, the openness of damper at outlet B was defined as a ramp function shown in Fig. 1, which starts to reduce at 50s from 100% to 30% in the following 150s. While the openness of damper A and C were kept at 100% throughout the simulation process. And the flow conditions and the LPLCs of the two junctions were monitored for subsequent comparison.

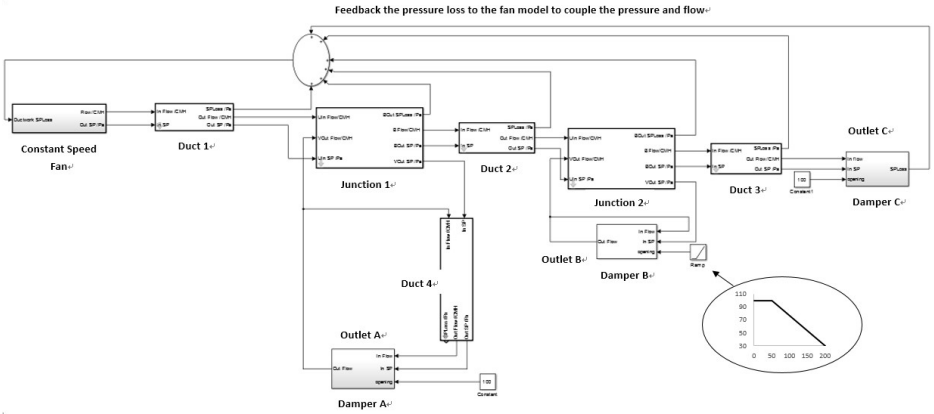


Fig. 1 *Demonstration of simple duct system driven by constant speed fan*

3 RESULTS AND DISCUSSION

3.1 LPLCs results of PSO-FWSVR model

We firstly trained the PSO-FWSVR$_{UD}$ model for vertical branch LPLC using the 18-points uniform design results. And the final optimized feature weights, SVR parameters and the results of MSE, R^2 and MAPE are listed in Tab. 2.

Tab. 2 *Final feature weights, SVR parameters and MSE, R^2 and MAPE of PSO-FWSVR$_{UD}$ for vertical branch LPLC*

Feature weights	（C, γ）	MSE	R^2	MAPE/%
（1, 0, 1, 0.018）	（100, 1.647）	0.0570	0.9728	16.84

(Note: the figures in (1, 0, 1, 0.018) represent the weights of AR_b, AR_v, VR_v and V successively.)

As listed in Tab. 2, the prediction R2 is as high as 0.973. However, a non-ignorable fact is that although the R^2 of PSO-FWSVR$_{UD}$ is very high, MAPE is 16.84% which is still too high to accept. To fix this, we decided to supplement some more data points with small value of LPLCs to feed the model with more information about this region. And on the other hand same amounts of points from the big value zone were also taken into the training set to keep the data balance. For the whole of 6 kinds of junctions, there are 6×6 = 36 supplemental data points.

Then the 36 supplemental points together with the original 18 uniform design points were used to train the PSO-FWSVR$_{UD+Sup}$ model. And the final feature weights, SVR parameters and accuracy indexes this time are listed in Tab. 3. We can see that with the supplemental points, the MSE and R^2 are improved and more importantly the MAPE is remarkably reduced from 16.84% to 2.76%. We can also find that the optimized feature weights this time has changed. The weight of AR_v is no longer zero and the one of AR_b decreases from 1 to 0.583. The influence of upstream velocity also becomes stronger. From the variation of MAPE, we can draw a conclusion that the targeted supplemental points do have a positive effect on determining proper feature weights and corresponding SVR parameters with which the PSO-FWSVR model can fit all the test points well enough throughout the available range.

Tab. 3 *Final feature weights, SVR parameters and accuracy indexes of PSO-FWSVR$_{UD+Sup}$ for vertical branch LPLC*

Feature weights	（C, γ）	MSE	R^2	MAPE/%
（0.583, 0.375, 1, 0.072）	（100, 9.965）	0.0016	0.9982	2.76

(Note: the figures of (0.583, 0.375, 1, 0.072) have the same meaning as the notation of Tab. 2.)

The PSO-FWSVR model for blow-through branch LPLC was built as the same procedure as for the vertical branch. And the corresponding results of it are listed in Tab. 4, which shows a consistently good prediction performance.

Tab. 4 *Final feature weights, SVR parameters and accuracy indexes of PSO-FWSVR$_{UD+Sup}$ for blow-through branch LPLC*

Feature weights	（C, γ）	MSE	R^2	MAPE/%
（0.164, 0.623, 1, 0.193）	（100, 1.637）	0.00016	0.9993	2.46

(Note: the figures of (0.164,0.623,1,0.193) represent the weights of AR_b, AR_v, VR_b (velocity ratio of blow-through branch to upstream) and V successively.)

3.2 Comparison with the traditional LPLC models

3.2.1 Individual junction level comparison

The MAPEs of vertical and blow-through branch LPLCs results of polynomial model on the test set are 27.93% and 11.25%, respectively. The comparison results are shown in Fig. 2. Apparently, the prediction results of PSO-FWSVR model is closer to CFD results for almost every test data point compared with polynomial model. It is obvious that using the velocity ratio of downstream branch to upstream as the only independent variable to predict LPLC is not enough to reflect the highly-nonlinear relationship of LPLC with its influencing factors. And the vertical branch LPLC suffers more seriously than blowthrough branch.

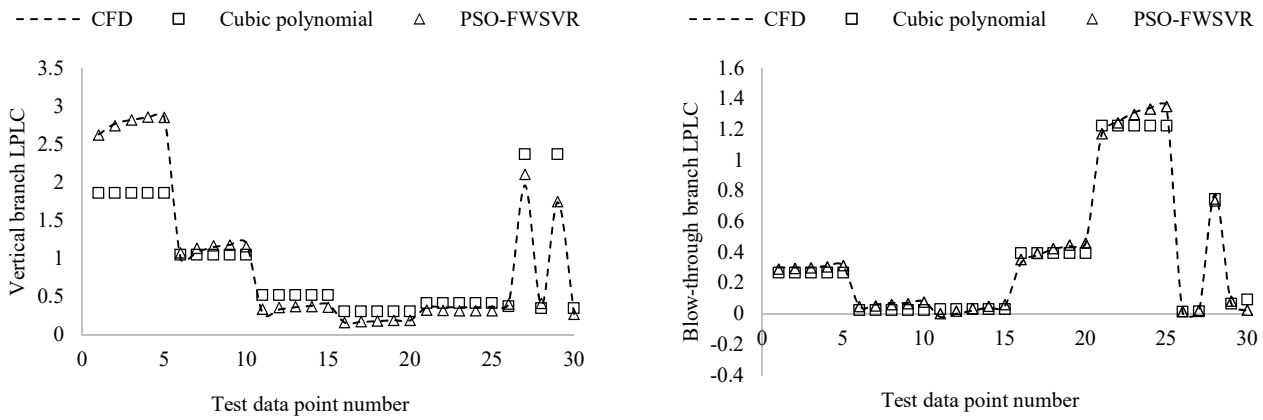


Fig. 2 *LPLC prediction performance comparison between cubic polynomial model and PSO-FWSVR model*

3.2.2 System level comparison

As to the system-level comparison, the results of system total flow using different LPLC models are shown in Fig. 3. And from now on, we treat the results of PSO-FWSVR model as the baseline since it's the most complex and accurate LPLC model from the comparison results for individual junction. Compared with the baseline, polynomial results have a stable bias, which is less than 150CMH, while results of constant-value model start to deviate from baseline after 50s and end up with a bias more than 700CMH, i.e. nearly 15% of baseline total flow.

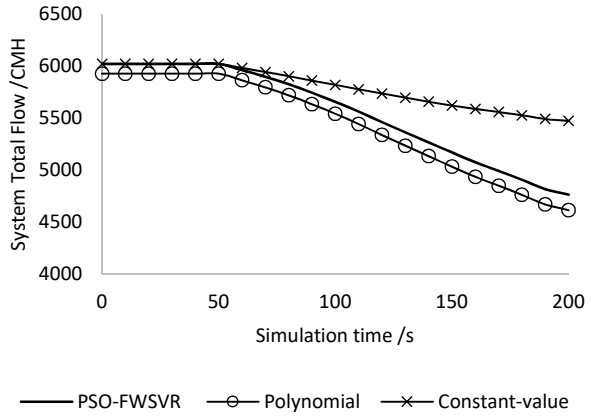


Fig. 3 *System total flow comparison among three LPLC models*

Let us take a detailed look into the flow conditions and corresponding LPLCs at the two junctions of these three cases and try to find out some differences worth notice. First we compared the results of polynomial model with the baseline. Fig. 4 and Fig. 5 shows the flow condition and LPLCs results of junction 1, respectively. When damper B starts closing, more air is forced into outlet A and the total flow decreases. In Fig. 4 the biggest difference between polynomial result and baseline is that the increasing rate of polynomial vertical flow of junction 1 is not as rapid as the baseline. And the bias of polynomial vertical flow from baseline is about -150CMH, i.e. -5% of the baseline vertical flow. It could be explained by the corresponding change of LPLCs in Fig. 5. The vertical LPLC of polynomial model rises faster than the baseline and the relative error of it rises from 20.2% at the beginning to 59.6% at the end of simulation.

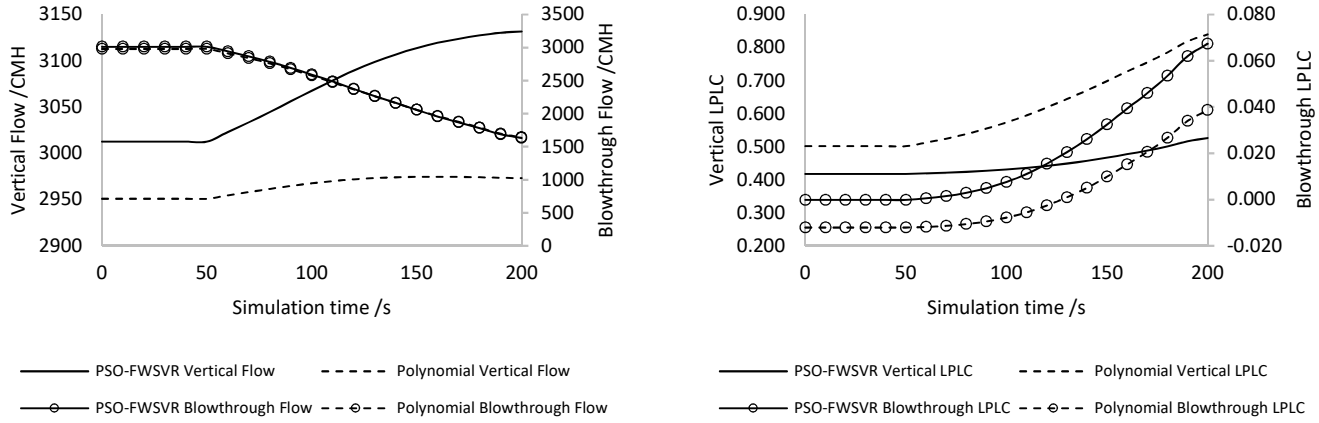
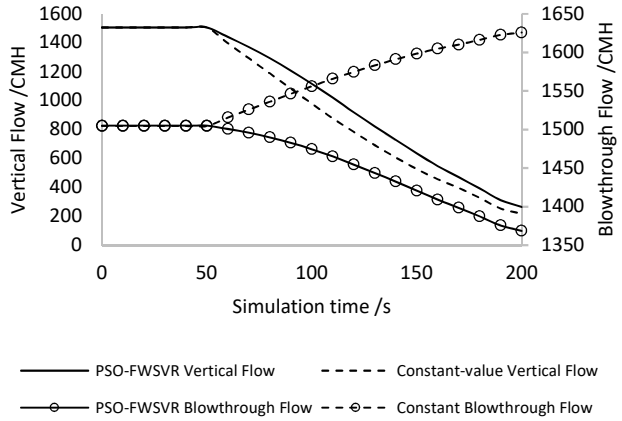


Fig. 4 *Comparison of flow condition of junction 1 between polynomial and the baseline model*

Fig. 5 *Comparison of LPLCs of junction 1 between polynomial and the baseline model*

Then we compared the result of constant-value model with the baseline. Fig. 6 and Fig. 7 give the results of flow conditions and LPLCs of junction 2. In Fig. 6 the blowthrough flow of constant-value model increases while the baseline blowthrough flow decreases, which is totally two different direction. The reason why the baseline blowthrough flow goes down is that the baseline blowthrough LPLC grows greatly and becomes big enough to prevent more air being forced into outlet C as we can see in Fig. 7. It's obvious that using the constant-value LPLC model brings bigger deviation than polynomial model does and what is worse is that under some circumstances it may lead to a totally opposite dynamic behavior of flow condition.



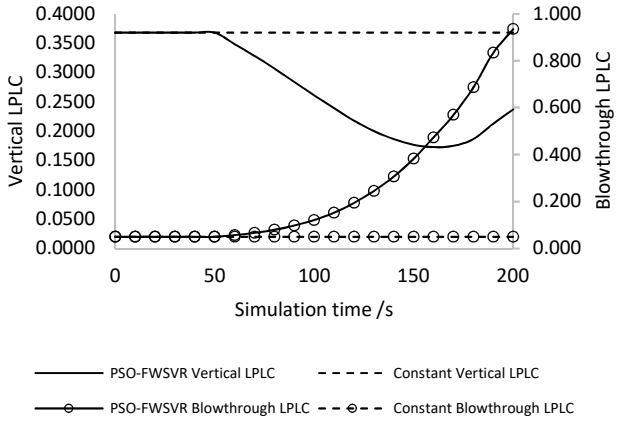


Fig. 6 *Comparison of flow condition of junction 2 between constant-value and the baseline model*

Fig. 7 *Comparison of LPLCs of junction 2 between constant-value and the baseline model*

From the above comparison and analysis among the results of the three models on the system level, we could firstly conclude that the constant-value LPLC model is basically unusable since the variation from the fixed value of rated condition could be very big, which could result in more than 10% error of system total flow and a totally different dynamic response of branch flow. So if one needs to reflect the mutual influence of adjacent terminal dampers correctly, the constant-value LPLC model should be off his list. As regards to the polynomial model, it seems that the difference from the baseline is not so big, i.e. only 5% error of branch flow in the above demo. However if the simulated system demands a high control precision, say precisely controlled constant temperature and humidity conditioner, the error of polynomial model could be unacceptable then.

4 CONCLUSION

This study used the method of PSO-FWSVR to build a new LPLC model for smooth diverging tee junction. Then the accuracy of PSO-FWSVR and polynomial model was compared on the level of individual junction. At last a model of a small air-side system was built to demonstrate the function of the former two models as well as the constant-value LPLC model on the system level. The main conclusions are as follows.

1) In the application of LPLCs model development, it is efficient to use the uniform design method for CFD case arrangement and adopt PSO-FWSVR for data fitting. One can obtain an accurate LPLC model with relatively less CFD calculation work.
2) PSO-FWSVR model can accurately calculate the LPLC for junctions with different dimensions under different conditions of upstream velocity and flow allocation, which is much better than polynomial model for individual junction.
3) On the system level, the constant-value assumption for LPLCs of junctions is too simplified to reflect the dynamic behavior of air-side system correctly. And the polynomial LPLC model using velocity ratio as its uni-independent variable might be used in the simulation of most common HVAC air-side system but more care should be taken when simulating some precisely controlled system.

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