The ESP System: Towards a New Generation of Building Energy Analysis Program

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Summary

This paper characterises the current generation of building energy models and identifies the essential features of the next generation. The recently developed UK system, ESP, is then described as a program which exemplifies some of these new generation features.

Introduction

In recent years several building energy modelling systems have reached the concluding phase of their development. Utilizing near state-of-the-art simulation techniques, these systems have fought to replace the traditional techniques which rely on many empirical relationships and which are usually confined to the steady state. At the present time a growing user community - of practitioners, researchers and educationalists - is struggling to find ways to cost-effectively apply these systems in practice.

Whilst this new technology offers sophisticated modelling capabilities, a number of deficiencies can still be identified which will restrict future adaptations to satisfy the needs of an increasingly more demanding user base. For example, many of the systems have an inflexible software structure forced on them by the now outdated machine environment within which they were conceived. There is an obvious lack of commonality in relation to system I/O, thermal network representations, component linking protocols, numerical solution techniques, operational interfaces, software management, documentation, results display, graphical capabilities and the degree to which the application knowledge is bound to the system code. More worryingly, perhaps, there is no consensus statement on the long term objectives of model formulation and the mechanism by which new theories can be appended as they emerge.

Many of these problems have been addressed within a research and development project funded by the UK Science and Engineering Research Council: to develop, test and document a dynamic energy model capable of the simulation of the heat and mass transfers within combined building and plant systems when constrained to conform to control action.

A Characterisation of Existing Systems

From a thermal modelling viewpoint, a building and its plant can be viewed as a complex electrical network: regions possessing capacitance are linked by resistances representing conductive, convective, advective and radiative flowpaths. Heat injections - analogous to a current source - are then applied to represent boundary flux exchanges. Note that in thermal problems there is usually no source of temperature, or voltage by analogy. This means that a thermal modelling problem is of lower order than is usually found in complex electrical circuit design. Figure 1 demonstrates the flowpaths which may be considered as candidates for inclusion in any model.

In energy modelling, the sophistication of a model is determined by the way in which the equivalent electrical network is treated. Often some portion is neglected completely, time invariant values may be assigned to one or more of the state variables, or simplifying boundary conditions may be assumed. In broad terms existing techniques will fall into one of five categories: steady state, simple dynamic, response function, numerical and electrical analogue. Each category is concerned, at its own level, to satisfy the first and second laws of thermodynamics but, as modelling sophistication diminishes, so many of the active flowpaths are degraded or ignored and the method becomes indicative, not predictive, application limited, not general, and of low integrity vis-a-vis the real world (Clarke 1984).

The extant dynamic systems for building energy analysis are based either on response function methods or on numerical methods in finite difference (or volume) form. For this reason, only these methods are included in what follows.

The response function approach

Inherent in this approach are many assumptions. For example, the building system is assumed to be linear and to possess heat transfer properties which are time invariant. The calculation technique is piecemeal: response factors are used to obtain construction response to climatic stimuli, weighting factors or energy balance techniques are then used to achieve zone response by invoking superimposition. The cooling and heating loads to result are then used as the basis for separate, and usually steady state, calculations addressing the building's systems and central plant. Modelling integrity is low because of this decoupling and its effect on control system modelling.

The strength of the approach is in its ability to provide rapid analytical solutions for the fabric transient conduction problem. It is possible, by so specifying system boundary conditions, to solve the partial differential heat equation, which governs the flow of heat within the building fabric, to provide the means of modelling the dynamic response of a building. Two main branches of the method exist: time domain and frequency domain formulations. In both cases the starting point is the same. For any homogeneous element (within some multi-layered construction) three relationships are of interest.

The change of temperature with distance:

$$\frac{\partial\theta(x,t)}{\partial x} = -\frac{1}{k}q(x,t) \qquad (1)$$

where $\theta(x,t)$ is the temperature at point x at time t (C), k is the material conductivity $(W\ m^{-1}C^{-1})$, $q(x,t)$ is the heat flux at point x and time t $(W\ m^{-2})$, and $\partial/\partial x$ is the differential coefficient with respect to the space dimension.

The change of flux with distance:

$$\frac{\partial q(x,t)}{\partial x} = -\rho C\frac{\partial\theta(x,t)}{\partial t} \qquad (2)$$

where ρ is the material density $(kg\ m^{-3})$, C is the material specific heat $(J\ kg^{-1}C^{-1})$, and $\partial/\partial t$ is the differential coefficient with respect to the time dimension.

And a combination of equations 1 and 2; the Fourier Field Equation which governs the flow of heat in solids:

$$\frac{\partial^2\theta(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial\theta(x,t)}{\partial t} \qquad (3)$$

where α is the material thermal diffusivity ($m^2 s^{-1}$).

With multi-layered constructions, the response function method involves an analytical solution of these equations by the use of the Laplace transform (Carslaw and Jaeger 1959, Churchill 1958, Davis 1978). This is essentially a four stage procedure:

- Equations 1 through 3, in the time domain, are transformed into subsidiary equations in an imaginary space.
- These subsidiary equations are then solved.
- Matrix manipluation techniques are used to combine homogeneous elements to obtain a solution for the multi-layered construction.
- And the inverse transformation is then applied to this solution to obtain the solution, in the time domain, of the initial problem. It is at this point that the time and frequency domain formulations take on their own identities.

The interesting feature of the transform method is that ordinary differential equations are transformed into purely algebraic expressions, with partial differential equations becoming ordinary differential equations.

In the early seventies Stephenson and Mitalas (1971) introduced a further transform, known as the z-transform from a method by Jury (1964). This allows the boundary conditions to be represented as a train of samples with linear (or some other form of) interpolation used to convert the samples to a continuous function as with the standard response function method. Results are identical to those from the response function method - for identical interpolation assumptions - but are substantially quicker to obtain.

The mathematical derivation underlying the first two stages are lengthy and so are omitted here. Such derivations can be found in the literature (Carslaw and Jaeger 1959, Stephenson and Mitalas 1967, Clarke 1985). In matrix notation, the solution to emerge from stage two, for a homogeneous element, is given by

$$\begin{bmatrix}\theta(l,p)\\ q(l,p)\end{bmatrix}=\begin{bmatrix}m_{11}(p) & m_{12}(p)\\ m_{21}(p) & m_{22}(p)\end{bmatrix}\times\begin{bmatrix}\theta(0,p)\\ q(0,p)\end{bmatrix} \qquad (4)$$

where, by inspection, the elements of matrix **M**, the transmission matrix, are given by

$$m_{11}(p)=m_{22}(p)=\cosh[(p/\alpha)^{1/2}l]$$
$$m_{12}(p)=-\sinh[(p/\alpha)^{1/2}l]/k(p/\alpha)^{1/2}$$
$$m_{21}(p)=-k(p/\alpha)^{1/2}\sinh[(p/\alpha)^{1/2}l]$$

where l is the element thickness (m), p is the complex Laplace operator, and **M** has unit determinant; that is $m_{11}m_{22}-m_{12}m_{21}=1$.

For composite constructions ($0<x<L$) comprised of a number of layers in intimate contact, the matrix formulation can be directly extended to give

$$\begin{bmatrix}\theta(L,p)\\ q(L,p)\end{bmatrix}=\begin{bmatrix}A(p) & B(p)\\ C(p) & D(p)\end{bmatrix}\times\begin{bmatrix}\theta(0,p)\\ q(0,p)\end{bmatrix} \qquad (5)$$

where the value of the elements $A(p)$, $B(p)$, $C(p)$ and $D(p)$ will depend on the properties of the component elements of the multi-layered construction and the order in which the individual element transmission matrices are combined. Thus for a multi-layered construction with individual elements denoted by 1, 2, 3,..., n specified outside ($x=0$) to inside ($x=L$), the overall transmission matrix is given by

$$\begin{bmatrix}A(p) & B(p)\\ C(p) & D(p)\end{bmatrix}=\mathbf{M}_1\times\mathbf{M}_2\times\mathbf{M}_3\times\cdots\times\mathbf{M}_n$$

where, in general, $m_{11}=m_{22}$ but $A(p)\neq D(p)$.

Equation 5 is the fundamental relationship which underpins the response function method. For solution in the time domain a rearrangement is necessary:

$$\begin{bmatrix}q(0,p)\\ q(L,p)\end{bmatrix}=\begin{bmatrix}D(p)/B(p) & 1/B(p)\\ 1/B(p) & A(p)/B(p)\end{bmatrix}\times\begin{bmatrix}\theta(0,p)\\ \theta(L,p)\end{bmatrix} \qquad (6)$$

The basic strategy of the response function method is to predetermine the response of a multi-layered construction to some unit excitation (time domain) or periodic state (frequency domain). The response of a linear, invariant equation system to this excitation is termed the unit response function (*URF*) with, in the time domain, the time-series representation being the response factors.

Time domain formulation This is a three step procedure:

1. The various excitation functions are resolved into an equivalent time-series; contemporary systems favouring triangular representation.
2. The *URF*'s and their corresponding excitation function are combined to give the system's response to each possible excitation. This is achieved by application of the convolution theorem which states that the response to a linear, invariant system is given by the products of the response of the same system to a unit excitation (the *URF*) and the actual excitation, given that the appriopriate adjustments are made during the multiplication process. That is

$$R(t)=\sum_{m=0}^{\infty}RF(m\Delta)E(t-m\Delta) \qquad (7)$$

where $R(t)$ is the system response at time $t=m\Delta$, $RF(m\Delta)$ is a response factor at time $m\Delta$, $E(t-m\Delta)$ is the excitation at time $(t-m\Delta)$, Δ is the *URF* time-step and m is an integer.

3. The individual responses are superimposed to give the overall response.

Flux unit response functions are the heat fluxes which result when first $\theta(0,p)$ then $\theta(L,p)$, of equation 6, is set to the Laplacian of a unit triangular pulse, whilst the opposite exposed surface temperature, $\theta(L,p)$ or $\theta(0,p)$ respectively, is held at zero. In this case three unit response functions result:

- $X(n\Delta)$; the heat flux *URF* into contruction at $x=0$ for a unit pulse at $x=0$.
- $Y(n\Delta)$; the heat flux *URF* out of construction at $x=0$ or $x=L$ for a unit pulse at $x=L$ or $x=0$ respectively.
- $Z(n\Delta)$; the heat flux URF into construction at $x=L$ for unit pulse at $x=L$.

where $n=0,1,2,3,4,\ldots$..

It follows from equation 7 that at some time $t=n\Delta$:

$$q(0,n\Delta)=\sum_{m=0}^{\infty}\theta[0,(n-m)\Delta]X(m\Delta)-\sum_{m=0}^{\infty}\theta[L,(n-m)\Delta]Y(m\Delta) \qquad (8)$$

$$q(L,n\Delta)=\sum_{m=0}^{\infty}\theta[0,(n-m)\Delta]Y(m\Delta)-\sum_{m=0}^{\infty}\theta[L,(n-m)\Delta]Z(m\Delta) \qquad (9)$$

where $q(0,n\Delta)$ signifies heat flow *into* construction at $x=0$ and $t=n\Delta$; $q(L,n\Delta)$ signifies heat flow *out of* construction at $x=L$ and $t=n\Delta$.

A unit pulse is represented by the superimposition of three ramp functions $r(t)$ defined as

$$r(t)=\begin{cases}0 & t<0\\ t & t\geq 0\end{cases}$$

Thus a unit triangular pulse is given by

$$f(t)=r(t+\Delta)-2r(t)+r(t-\Delta) \qquad (10)$$

so that for $\Delta=1$: $f(-1)=0$; $f(0)=1$; $f(1)=0$.

The Laplacian of the ramp function $f(t)=t$ is $1/p^2$, and so from equation 6 the *URF* due to a ramp function at $x=0$ with $\theta(L,p)=0$ is

$$q(0,p)=D(p)/p^2B(p)$$
$$q(L,p)=1/p^2B(p)$$

and establishing the ramp function at $x=L$ with $\theta(0,p)=0$ gives

$$q(0,p) = -1/p^2 B(p)$$
$$q(L,P) = -A(p)/p^2 B(p).$$

The inverse Laplace transform is now applied to obtain the heat flux time-series in the time domain: these (with appropriate signs) are the *URF*'s $X(m\Delta)$, $Y(m\Delta)$ and $Z(m\Delta)$. This can be achieved by the *method of residues* (Kreyszig 1979). Noticing that the *URF* expressions can each be represented by $\xi(p) = R(p)/p^2 S(p)$ then the residue theorem gives, for $X(m\Delta)$:

$$\xi(t) = \frac{D(0)}{B(0)} t + \frac{d}{dp}\left(\frac{R(p)}{S(p)}\right)\Bigg|_{p=0} + \sum_{j=1}^{\infty} \frac{D(a_j)}{a_j^2 B'(a_j)} e^{a_j t} \qquad (11)$$

where $D(a_j)/B'(a_j)$ is the *residue* of the transfer function D/B at its jth singularity (or pole), a_j. The poles of D/B are the zeros of B and so the next stage is to determine the roots of $B(p)$ as determined from the matrix analysis applied to the construction in question.

The response factor terms are then given by equation 10:

$$X(0) = \xi(\Delta)$$
$$X(\Delta) = \xi(2\Delta) - 2\xi(\Delta)$$
$$X(m\Delta) = \xi[(m+1)\Delta] - 2\xi(m\Delta) + \xi[(m-1)\Delta] \quad m=2,3,4,...$$

with similar expressions emerging for $Y(m\Delta)$ and $Z(m\Delta)$.

For a homogeneous element, equation 4 gives

$$p^2 B(p) = p^2 \sinh[(p/\alpha)^{1/2} l]/k(p/\alpha)^{1/2}$$

and the roots of $\sinh[(p/\alpha)^{1/2} l]$ are given by

$$a_j = -j^2 \phi^2 \alpha / l^2.$$

With multi-layered constructions the elements of the overall transmission matrix are complex hyperbolic functions and determination of the roots of $B(p)$ must be achieved by numerical search procedures which attempt to locate a sign change in $B(p)$ as p is incremented in small steps. Hittle (1981) has proposed an efficient root finding procedure which makes use of the fact that the roots of $B(p)=0$ are bracketed by the roots of $A(p)=0$. The method allows large numerical search increments whilst still ensuring root location. Another root-finding algorithm has been proposed by Gough (1982) in which $B(p)$ is re-expressed as a product expansion in terms of its zeros a_j.

In the z-transform approach, the heat flux at the inside surface of a multi-layered construction is specified as a weighted, *finite* sum of both input and output excitation histories (*cf* equation 9):

$$q(L,t) = \sum_{m=0}^{N} E(m\Delta)\theta(0,t-m\Delta) - \sum_{m=1}^{M} H(m\Delta)q(L,t-m\Delta)$$

where E and H are the z-transfer functions, derived from the ratio of input and output time-series. These are related to the p transfer functions X, Y and Z by

$$E(m\Delta) = \sum_{j=0}^{N} X(j\Delta)H(m\Delta - j\Delta)$$
$$F(m\Delta) = \sum_{j=0}^{N} Y(j\Delta)H(m\Delta - j\Delta)$$
$$G(m\Delta) = \sum_{j=0} Z(j\Delta)H(m\Delta - j\Delta)$$

and $H(m\Delta)$; $m=1,2,3,...$ is obtained as the coefficient of Z^{-m} in the expansion

$$\sum_{m=0}^{\infty} H(m\Delta)Z^{-m} = \prod_{m=1}^{\infty} 1 - Z^{-m} e^{a_j \Delta}$$

where a_j are the poles of $B(p)$ as previously.

In multi-zone building modelling it is next necessary to conduct an energy balance, zone-by-zone, usually centered on the zone air point; this is demonstrated elsewhere (Clarke 1985). This results in a matrix equation containing the wall response formulations just derived. It is the simultaneous solution of this matrix equation which gives the zone response. Alternatively, the matrix equation can be subjected to a number of unit excitations relating to casual gains, climatic stresses and plant interaction, each in turn, with other excitations held at zero. In each case, the matrix equation solution vector defines the individual terms of the zone weighting factors. These are the factors which, when combined with the individual heat gain and loss flowpaths, give the total zone heating or cooling loads.

Frequency domain formulation In this approach real climatic time-series are divided into component sinusoidal variations abount some mean condition by Fourier Series representation. Each harmonic of the time-series is then processed independently and treated as as excitation to produce a *URF*. Recalling equation 5 - the Laplace transforms of the temperature and flux at one exposed surface in terms of the corresponding transforms at the other surface - and expanding the overall transmission matrix to include boundary layers of combined convective/radiative resistance R_L and R_o ($m^2\ C\ W^{-1}$), then

$$\begin{bmatrix}\theta(L,p)\\ q(L,p)\end{bmatrix} = \begin{bmatrix}E(p) & F(p)\\ G(p) & H(p)\end{bmatrix} \times \begin{bmatrix}\theta(0,p)\\ q(0,p)\end{bmatrix} \qquad (12)$$

and by similar reasoning the transforms of the temperature and heat flux at $x=0$ is given by

$$\begin{bmatrix}\theta(0,p)\\ q(0,p)\end{bmatrix} = \begin{bmatrix}I(p) & K(p)\\ K(p) & L(p)\end{bmatrix} \times \begin{bmatrix}\theta(L,p)\\ q(L,p)\end{bmatrix} \qquad (13)$$

where

$$E(p) = A(p) + R_L C(p)$$
$$F(p) = R_o A(p) + R_L R_o C(p) + B(p) + R_L D(p)$$
$$G(p) = C(p)$$
$$H(p) = R_o C(p) + D(p)$$
$$I(p) = A(p) + R_o C(p)$$
$$J(p) = R_L A(p) + R_o R_L C(p) + B(p) + R_o D(p)$$
$$K(p) = C(p)$$
$$L(p) = R_L C(p) + D(p).$$

With equation 12, setting $\theta(0,p)=0$ while the other surface is subjected to a periodic temperature state dictated by $\sin(\omega t)$, where $\omega = 2\pi/L$ is the angular frequency of the variation, gives the frequency response function:

$$\theta(L,p) = F(p)q(0,p)$$
$$q(L,p) = H(p)q(0,p)$$

and for $\theta(L,p)=0$ equation 13 gives

$$\theta(0,p) = J(p)q(L,p)$$
$$q(0,p) = L(p)q(L,p).$$

Note that setting $\theta(?,p)=0$; $?=0,L$ is equivalent to holding the surface temperature constant but not necessarily at zero. Now if $p = j\omega$, where j is the imaginary operator defined by $j^2=-1$, then the Laplace transform becomes the Fourier transform so that

$$\theta_h(L,t) = F(j\omega)q_h(0,t) \qquad (14)$$
$$q_h(L,t) = H(j\omega)q_h(0,t) \qquad (15)$$
$$\theta_h(0,t) = J(j\omega)q_h(L,t) \qquad (16)$$
$$q_h(0,t) = L(j\omega)q_h(L,t) \qquad (17)$$

where θ_h and q_h are the temperature and flux response harmonics.

In the *means and swings technique* (Danter 1960), three separate response factors have been identified: *decrement, surface* and *admittance*. Each factor possesses a corresponding phase angle which gives the response time of the process the factor addresses. When combined with the system excitations, they give the overall zone response.

The *decrement* response factor is defined as the ratio of the cyclic flux transmission to the steady state flux transmission. It is pplied to fluctuations (about the mean) in external temperature or ux harminics impinging on exposed opaque surfaces to give the corresponding cyclic flux at the inside surface. Thus

$$d_{0,L} = q_k(L,t)/U\theta_k(0,t)$$

where $d_{0,L}$ is decrement factor from outside $x=0$ to inside $x=L$ and U is the steady-state overall thermal transmittance ($Wm^{-2}C^{-1}$).

And, from equation 16

$$d_{0,L} = \frac{1}{[R_L A(j\omega) + R_L R_0 C(j\omega) + B(j\omega) + R_0 D(j\omega)]U}$$

This expression will have real and imaginary parts, r_1 and v_1 respectively, so that the decrement magnitude, $\left|d_{0,L}\right|$, and time lag, ϕ_d (radians), is given by

$$\left|d_{0,L}\right| = (r_1^2 + v_1^2)^{1/2} \quad ; \quad \phi_d = \tan^{-1}(v_1/r_1) .$$

The *surface* response factor defines the portion of the heat flux at any internal surface which is readmitted to the internal environmental point when temperatures are held constant. This factor is applied to cyclic energy inputs at a surface to give the corresponding cyclic energy variations at the environmental point. A typical application is the modification of the window transmitted component of solar radiation to give a zone cooling load. For an internal surface

$$s_{L,0} = 1 - \frac{R_L q_k(L,t)}{\theta_k(L,t)}$$

and from equations 14 and 15

$$s_{L,0} = \frac{R_0 A(j\omega) + B(j\omega)}{R_0 A(j\omega) + R_0 R_L C(j\omega) + B(j\omega) + R_L D(j\omega)} .$$

Again the magnitude and time lag is obtained from the real and imaginary parts, r_2 and v_2:

$$\left|s_{L,0}\right| = (r_2^2 + v_2^2)^{1/2} \quad ; \quad \phi_s = \tan^{-1}(v_2/r_2) .$$

The *admittance* response factor is defined as the ratio of the energy entering an internal surface to the corresponding temperature swing:

$$a_{L,0} = q_k(L,t)/\theta_k(L,t)$$

and so from equations 14 and 17

$$a_{L,0} = \frac{R_0 C(j\omega) + D(j\omega)}{R_0 A(j\omega) + R_0 R_L C(j\omega) + B(j\omega) + R_L D(j\omega)} .$$

With the magnitude and *time lead* given by

$$\left|a_{L,0}\right| = (r_3^2 + v_3^2)^{1/2} \quad ; \quad \phi_a = \tan^{-1}(v_3/r_3) .$$

All excitations must conform to cyclic variations and long-wave radiation effects are assumed to act at a single, fictitious environmental point. The use of these factors does allow an assessment of zone loads without the need for weighting factors or zone energy balancing as with the time domain approach.

The numerical approach

As demonstrated, an analytical approach will impose a number of restrictions that will degrade the integrity of the model to emerge. Overcoming the problems to improve model integrity was the challenge taken up by developers in the post response function phase of model evolution.

And so, in recent years, numerical methods have played an increasingly important role in the analysis of heat transfer problems. Whereas the response function approach, mathematically elegant and the outcome of many years of accumulated R&D, is appropriate to differential problems of low order and exhibiting time-invariance, numerical methods can be used to solve complex, time-varying problems of high and low order. The generality of the method provides the key to high integrity modelling; differential equation-sets, representing the dynamic energy and mass balances within *combined* building and plant networks are solved simultaneously and repeatedly at each (perhaps variable) time-step as a simulation is undertaken under the influence of control action. All fundamental properties are assumed to be time dependent and no decoupling is applied as a computational convenience.

The formulation of a numerical method involves several steps. Firstly, the continuous multi-zone building and multi-component plant is made discrete by subdivision into a number of interconnecting, finite volumes. These volumes then possess uniform properties which can vary in the time dimension. Volumes represent homogeneous and mixed material regions associated with air volumes, constructional elements and component heat transfer interfaces; it is not uncommon to have as many as 300 such volumes in a scheme to represent the energy balance within one building zone. Secondly, for each of these finite volumes in turn, and in terms of all surrounding volumes deemed to be in thermal or flow contact, a governing differential equation is developed to conserve the properties of interest - heat energy, mass flow rate, momentum or species balance for example. Thirdly, all differential equations are transformed to give the final state-space representation for subsequent numerical solution. It is normal to transform to *lumped parameter* or *time discrete* formulations, depending on the chosen solution method (as outlined later). Fifthly, control equations are appended to prescribe, limit or impose conditions on system behaviour. And lastly, the entire equation-set is simultaneously solved for some small computational time-step, to obtain the state variables for the current time-row and so allow equation-set re-formulation for the next time-step. Control discontinuities are avoided by time-step variation to ensure that an across-discontinuity integration is not attempted.

If the governing differential equations are known for any finite volume, then a finite difference representation can be directly applied. However this formal approach - differencing by Taylor series expansion - can prove cumbersome and difficult to apply in all but simple cases. Commonly encountered problems include the presence of interacting flowpaths, the temporal and positional dependency of system heat injections, the fact that volumes may not be homogeneous and can be of varying dimensions and shapes, the presence of n-dimensional heat flow, and the interaction of heat and mass transfers. An alternative approach is to rely on formal balancing, which, in the limit, is the technique used to derive the differential equations in the first place. Consider a region I in thermal communication with surrounding regions 1,2,3 & 4 via conduction, convection, radiation and fluid coupling. Internal heat generation can also take place within the region. The heat transfer towards region I from the other regions at some time ξ is given by

$$q(j \rightarrow I,\xi) = \sum_{j=1}^{n} K(j \rightarrow I,\xi)[\theta(j,\xi) - \theta(I,\xi)] \quad n = 1 \rightarrow 4 \ \textit{(in this case)}$$

where $K(j \rightarrow I,\xi)$ is a heat flow conductance which may be some non-linear function of the state variable; temperature in the case of sensible energy balance. Of course θ may represent some other state variable (such as pressure or velocity) with suitable adjustments made to K to represent the fundamental processes. Here K replaces conductivity, k, a convection coefficient, h_c, a long-wave radiation coefficient, h_r, or an air flow conductance, $\rho C\vec{v}$; ($\vec{v}$ *is a volume flowrate vector*), depending on the flowpath.

The heat stored within the region I over some finite time interval δt is given by

$$q_s = \frac{\rho(I,\xi)C(I,\xi)\delta V(I,\xi)}{\delta t}\{\theta(I,t+\delta t) - \theta(I,t)\}$$

where $\rho(I,\xi)$ is the density of region I at time ξ ($kg\ m^{-3}$), $C(I,\xi)$ is the specific heat ($J\ kg^{-1}\ C^{-1}$), and $\delta V(I,\xi)$ is the volume (m^3).

Now, in the limit, the rate at which heat is stored within region I is equal to the net rate of heat flow to the region and so, with $q(I,\xi)$ representing heat generation:

$$\frac{\rho(I,\xi)C(I,\xi)\delta V(I,\xi)}{\delta t}\{\theta(I,t+\delta t) - \theta(I,t)\} \qquad (18)$$

$$= \sum_{j=1}^{n} K(j \rightarrow I,\xi)\{\theta(j,\xi) - \theta(I,\xi)\} + q(I,\xi) + \epsilon.$$

where ϵ is the error resulting from equating the instantaneous flux balance to the change in storage over time. In conventional finite differencing this is equivalent to the truncation error resulting from the approximate representation of the derivatives.

In order to achieve unconditionally stable equations, while retaining maximum accuracy, it is usual to combine explicit $\xi = t$ and implicit $\xi = t+\delta t$ forms of equation 18 giving, after rearrangement:

$$\left[\frac{2\rho(I,t+\delta t)C(I,t+\delta t)\delta V(I,t+\delta t)}{\delta t} + \sum_{j=1}^{n} K(j \rightarrow I,t+\delta t)\right]\theta(I,t+\delta t) - \sum_{j=1}^{n} K(j \rightarrow I,t+\delta t)\,\theta(j,t+\delta t) \qquad (19)$$

$$- q(I,t+\delta t) = \left[\frac{2\rho(I,t)C(I,t)\delta V(I,t)}{\delta t} - \sum_{j=1}^{n} K(j \rightarrow I,t)\right]\theta(I,t)$$

$$+ \sum_{j=1}^{n} K(j \rightarrow I,t)\,\theta(j,t) + q(I,t).$$

In this form equation 19 is *time-discrete* in that both the time and space derivatives have been effectively differenced. It is, of course, possible to retain the time derivative; the scheme will then be *lumped parameter*. In building energy analysis there are three equation types required to describe the active flowpaths. First order ordinary differential equations are used to represent physical regions undergoing multiple heat transfer and possessing averaged thermophysical properties. Second order parabolic partial differential equations are used to describe capacity/insulation regions requiring detailed modelling. And hyperbolic partial differential equations are used to describe fluid flow and convective coupling. Obviously equation 19 is identically equal to the first type as the time increment approaches the limit. It has also been proved (Lambert 1973) that it will become identical to the second and third types if the so-called *semi-discretisation* is applied to the space variables (with K assigned the appropriate meaning).

Time-discrete formulations offer many advantages since they give rise to linear algebraic equations which can be solved by matrix manipulation techniques endowed with knowledge of system behaviour. This is demonstrated later. If the equation-sets are held in lumped parameter differential form, higher level matrix reduction can still be invoked. But, in the author's opinion, modelling accuracy is more linked to the time-dependent assessment of the coefficients of the active equations - representing heat transfer coefficients, heat injections and control conditions - than with a high accuracy solution to a given differential matrix equation. The method of equation 19 has been shown to be consistent, convergent and A-stable, providing the possibility of variable time stepping and well adapted for the solution of *stiff* systems in which time constants may vary by more than an order of magnitude (Mitchell 1969).

After system discretisation has taken place, many interlocking equations will require simultaneous solution. Consider figure 2, which shows a simple multi-zone building incorporating a distributed radiator heating system with central boiler. System discretisation, and the derivation of a simulation equation for each finite volume, is outwith the scope of this paper and only the general form of the resulting matrix equation, and its subsequent solution, is shown here. Clarke (1985) has demonstrated the process for a number of typical building and plant components. In view of the flowpaths shown in figure 1, a discretisation scheme is adopted in which *nodes* (representing the finite volumes) are placed to represent building surface layers, portions of zone air volumes, constructional elements, distribution links, boiler processes and radiator surface exchanges. Nodes are also located in a manner which represents ambient conditions and points of control interaction.

The matrix equation to result is shown in figure 3; note the high level of sparseness and the location of equation clusters in a manner which reflects system connectivities. One method of solution is to proceed as follows. The matrix equation is never formally established. Instead, a series of partitioned matrices are defined, each one representing multi-layered construction heat flow and containing information which links them to the whole system equation-set at the construction boundaries. Zone energy balance matrices, one for each zone, are also extracted at this stage along with partitioned matrix clusters which represent the individual plant components. Again all coupling information is retained and held in these matrices. The solution of the entire system - held in partitioned form - now need only address the *filled* elements of the overall super-matrix. Also, as solution is pursued, it is possible to accomodate any spatial position of control loop sensor and actuator and to incorporate temporal considerations within any control law. The underlying objective is to process each partitioned matrix as far as possible to allow the extraction of one or more *characteristic equations* (or *CE*'s) which embody the dynamics of the related component. These *CE*'s are then gathered together for onward processing to produce a set of whole-system *CE*'s which relate sensor states to control action. The whole-system *CE* set is then solved in terms of user imposed control statements so that back substitution can recommence to give the state variable, simultaneous solution, vector.

Because of the partitioning, different components can be processed at different frequencies; partitioned matrices can even be bypassed if their contents have not changed substantially since the previous time-step. Of course, many complexities accompany the computation of equation coefficients: the time dependent estimation of long- and short-wave radiation processes, surface convection coefficients, inter-zone air flow and casual source injections will require many algorithmic formulations. These, and the overall solution technique, are elaborated in the previous reference.

Features of the Next Generation

Continuing down this evolutionary path, future modelling systems will very likely possess the following features:

- A central database containing a range of dynamic component models. These will not represent component process solution but, instead, will serve to generate the coefficients of the differential and/or algebraic equations which represent the component heat transfer and fluid flow processes. In this way equation formulation is divorced from solution; an effective measure since the latter requires knowledge of the entire system of equations. The database therefore contains *coefficient generators*, one for each building and plant component active in the modelling system at any time.
- A super-matrix builder, acting on information which defines some energy system in terms of components and connections, will receive the coefficients, 'fired' from the generators, for relocation and indexing within the super-matrix partitions. It will be possible to build more than one super-matrix to represent energy balance and multi-phase mass and momenmtum balance. These matrices will then operate in tandem throughout any

simulation to provide the means for combined heat and mass transfer modelling.

- High level, intelligent matrix processing software will exist to achieve simultaneous solution whilst respecting all control characteristics and inter-component connectivities.
- Many support algorithms will be required to assist in coefficient estimation and I/O control. It is therefore essential that a new software approach is adopted in which dedicated software *tools* are combined to form the desired model. What is envisaged is a library of specific function object modules, each serving only one purpose, with an executive 'harness' program existing to allow module concatenation.
- The domain-specific knowledge will be separated out from these software tools to make them generally applicable, and to pave the way for an expert system interface. User interfaces will undoubtedly be expert, although such systems will not emerge until the early 1990's when artificial intelligence techniques proliferate. In the interim it is possible to endow systems with an expert 'feel' by arranging that standard functions - building regulation compliance checks, comfort assessment, equipment sizing and energy consumption computation - are available through a single command syntax.

The paper now concludes with a description of the ESP system. ESP is a modelling system which incorporates many of these desirable features and so offers a radical departure from the modelling techniques traditionally employed.

The ESP System

ESP is a multi-program system developed by the ABACUS CABD unit within the University of Strathclyde with funding from the UK Science and Engineering Research Council. System refinement continues as part of a current SERC grant active until October 1986.

Background

The theoretical basis of ESP was researched between 1974 and 1977 when the various techniques for modelling energy flow - response functions in time and frequency domain and numerical methods - were investigated and tested (Clarke 1977). For a number of reasons numerical methods were favoured: it is essential to preserve the spatial and temporal integrity of real energy systems by arranging that whole system (building + plant) differential equation-sets be solved simultaneously and repeatedly at each computational time-step; numerical methods do not need to assume flowpath superimposition and so can handle complex flowpath interaction; they can also be devised to accomodate non-linear processes as well as complex, time-varying heat transfer coefficients; and the building and plant can be treated in an integrated and first principal manner. Each of these factors will act to improve modelling accuracy and flexibility.

By 1977, then, a prototype version of ESP was in operation. The system utilised a state-space building representation of implicit enumeration in which the governing differential equations are made time-discrete. These equations - held in matrix form and representing the zonal dynamic energy flows - are then processed by a customised numerical method which accomodates variable time-stepping and 'stiff' systems in which time constants can vary by more than an order of magnitude.

Between 1977 and 1982 ESP was extensively re-modelled to allow simultaneous multi-zone processing and to create the framework for a number of simulation support modules covering data input, graphical results display and interrogation, climate and construction database management, and the computation of shading, solar beam tracking, view factors and window spectral behaviour. Two other tasks were undertaken during this period.

- To validate the underlying theory, three test houses were instrumented to allow comparison between measurements and predictions (Clarke 1978). ESP was also entered as a participant code in the International Energy Agency's Annex 1 (IEA 1982) and Annex 4 (IEA 1984) projects, concerned with model testing against hypothetical and real buildings respectively.
- To test ESP's interface, the program was installed in a large multi-disciplinary design practice where its application to a number of live design projects was passively observed (Maver and Ellis 1982).

These two activities lead to further theoretical and user-interface refinements so that, by 1982, a robust and user-friendly system was beginning to emerge. ESP was now equipped to perform comprehensive whole-building dynamic energy simulation. The predictive accuracy, if not proven, was in diminishing doubt and, thanks to a small SERC grant, system documentation was extensive and wide ranging (Clarke 1982).

Nevertheless ESP still had two main defects. Whilst it was entirely possible to represent plant in a conceptual manner - by mimicking plant characteristics in terms of control regime and points of building interaction - it was not yet possible to model the dynamic behaviour of plant components, individually or when connected within a network. Nor was it possible to model building mass balance in terms of the movement of air due to the effects of pressure and buoyancy acting on the building's distributed leakage.

In response to these shortcomings, a research proposal was formulated and submitted to SERC. The proposal had two objectives. To implement an advanced air flow algorithm within the energy simulation 'engine' of ESP, the module ESPsim. And to develop, integrate and test *dynamic* models of a number of plant components: a gas fired water boiler; a water and air solar collector; a heat exchanger; a cooling and heating coil; a humidifier; a fan and pump; pipes and ducts; radiators; electrical storage units; a rock bed storage unit; and a hot water cylinder with heat exchanger.

In 1984 these aims were achieved and, in the case of the plant modelling developments, the foundations were laid for the current work to develop the modelling infrastructure for a system capable of simulating *any* energy system comprised of component parts acting to process or convert mass and energy (Maver and Clarke 1984). In this approach component models act only to generate the coefficients of the representative differential equations, and not to determine component performance (output) for any given input. Thus, each component is replaced by a coefficient generator and, at each simulation time-step, coefficient vectors are 'fired' at some super-matrix builder for subsequent simultaneous solution under control action constraints.

Figure 4 shows the modular structure of the ESP package as it now exists. In this form ESP will allow the simultaneous processing of any problem configuration; from a single zone entity to a full multi-zone structure, from the 'no plant' case to combined building and plant networks, and from an energy only analysis to combined energy and mass balance. The user has full control of the simulation process and is given the software tools to progressively increase the simulation rigour as the design evolves. The system is driven by graphical menu interaction with sophisticated graphical interrogation facilities offerred at output time.

Plant modelling is limited at present by the number of component models presently available within ESP's plant database. This is the restriction being removed in the current work by the introduction of software to control the insertion of user-specified plant components *without the need for coding intervention*.

To demonstrate ESP's plant modelling technique, consider the derivation of a simple state-space representation for a packaged air handling unit. More comprehensive multi-node derivations are presented elsewhere (Clarke 1985) for the components of active solar and wet central heating systems.

The dynamic simulation of air conditioning systems is complicated by the fact that the working fluid (air) is not homogeneous, but comprised of a dry air and vapour mix. Also, a large number of components may be present with complex inter-component connections and control actions. Consider the simple air handling unit of figure 1. Outside air at temperature θ_o, humidity ratio g_o and enthalpy h_o is mixed with zone return air at temperature θ_r, humidity ratio g_r and enthalpy h_r and passed to a chilled water cooler, a humidifier and a re-heater to achieve the required zone supply conditions to offset the zone sensible and latent loads. In the usual way, it is possible to

establish, for each component, an energy balance for any arbitrary time ξ:

For component 1

$$m_0 h_0 + m_r h_r - m_1 h_1 + q_{c1} = \left.\frac{d(\overline{\rho}_1 V_1 h_1)}{dt}\right|_{t=\xi} \quad (20)$$

For component 2

$$m_1 h_1 - m_2 h_2 - m_c h_r + q_{c2} - q_{s2} = \left.\frac{d(\overline{\rho}_2 V_2 h_2)}{dt}\right|_{t=\xi} \quad (21)$$

For component 3

$$m_2 h_2 - m_4 h_4 - m_3 h_3 + q_{c3} = \left.\frac{d(\overline{\rho}_3 V_3 h_3)}{dt}\right|_{t=\xi} \quad (22)$$

For component 4

$$m_3 h_3 - m_4 h_4 + q_{c4} + q_{s4} = \left.\frac{d(\overline{\rho}_4 V_4 h_4)}{dt}\right|_{t=\xi} \quad (23)$$

For component 5

$$m_4 h_4 - m_5 h_5 + q_{c5} = \left.\frac{d(\overline{\rho}_5 V_5 h_5)}{dt}\right|_{t=\xi} \quad (24)$$

where m is the mass flowrate of air/vapour mixture ($kg\ s^{-1}$), h is the mixture specific enthalpy ($J\ kg^{-1}$), q_{ci} the component i heat exchange with surroundings (W), q_{s2} the cooling coil total heat transfer (W) and q_{s4} the re-heater coil total heat transfer (W); $\overline{\rho}_i$ is the volume weighted mean density of component i ($kg\ m^{-3}$), V_i is the total volume of component i (m^3), o & r relate to ambient and zone return air states respectively, c relates to the cooler moisture extraction, and h relates to the humidifier moisture addition.

Since each component is represented by a single point, nodal capacity must be expressed as a function of the average component state. A more refined approach results from the introduction of a multi-node representation since then the capacity of each component element (air, metal casing, coil metal, water, etc.) can be treated separately. Such a refinement is demonstrated later for the cooling coil. For the current one node model the mean density, $\overline{\rho}_i$, is given by a volume weighted average of the densities of the N elements such that:

$$\overline{\rho}_i = \sum_{j=1}^{N} (\rho_j V_j) \Big/ \sum_{j=1}^{N} (V_j) . \quad (25)$$

A mass balance, component-by-component, for the dry air and vapour phases separately, will yield, for some time-row ξ:

For component 1

$$m_0^d + m_r^d - m_1^d = 0 \,|_{t=\xi} \quad (26)$$

$$m_0^d g_0 + m_r^d g_r - m_1^d g_1 = 0 \,|_{t=\xi} \quad (27)$$

For component 2

$$m_1^d - m_2^d = 0 \,|_{t=\xi} \quad (28)$$

$$m_1^d g_1 - m_2^d g_2 - m_c = \left.\frac{d(\rho_L V_c)}{dt}\right|_{t=\xi} \quad (29)$$

For component 3

$$m_2^d - m_3^d = 0 \,|_{t=\xi} \quad (30)$$

$$m_2^d g_2 - m_3^d g_3 + m_h = \left.\frac{d(\rho_L V_h)}{dt}\right|_{t=\xi} \quad (31)$$

For component 4

$$m_3^d - m_4^d = 0 \,|_{t=\xi} \quad (32)$$

$$m_3^d g_3 - m_4^d g_4 = 0 \quad (33)$$

For component 5

$$m_4^d - 2m_5^d = -m_s \quad (34)$$

$$m_4^d g_4 - m_5^d g_5 = 0 \quad (35)$$

where m^d is the mass flowrate of dry air ($kg\ s^{-1}$), g the humidity ratio ($kg\ kg^{-1}$) and ρ_L the density of liquid remaining in cooler ($kg\ m^{-3}$); V_c is the cooler residual liquid volume (m^3) and V_h is the humidifier residual liquid volume (m^3); m_c is the cooler vapour extraction rate ($kg\ s^{-1}$); m_h is the humidifier vapour addition rate ($kg\ s^{-1}$) and m_s is the source of mass used to control the fan.

A time-discrete state space representation can now be obtained for each component by concatenating the explicit and implicit forms of equations 20 through 24:

For component 1

$$\begin{aligned}[2\overline{\rho}_1 (t+\delta t) V_1 + m_1(t+\delta t)\delta t]\, h_1 (t+\delta t) - \delta t q_{c1} (t+\delta t) \quad (36)\\ = [2\overline{\rho}_1 (t) V_1 - m_1 (t)\delta t]\, h_1(t)\\ + m_o (t+\delta t)\, \delta t h_o (t+\delta t) + m_r (t+\delta t)\, \delta t h_r (t+\delta t)\\ + m_o (t)\, \delta t h_o (t) + m_r (t)\, \delta t h_r (t) + \delta t q_{c1}(t).\end{aligned}$$

Now, since a building equation-set is not being considered here (it would of course exist in any ESP run), it is not possible to expand the q_{ci} terms by the introduction of an exchange resistance based on fundamental heat transfer considerations. For the present purpose the assumption is made that the $q_{ci}(t+\delta t)$ can be determined independently and so this term is removed to the right-hand, or known, side of equation 36. In the same way, the $h_o(t+\delta t)$ term is treated as a known quantity here, although it would be represented by an active state variable (and hence matrix equation) if a building equation-set was active.

With reference to figure 5, which shows the overall system energy balance matrix equation, the state-space equation for component 1 becomes

$$a_{11} h_1 (t+\delta t) = b_{11} h_1 (t) + c_1 \quad (37)$$

where

$$\begin{aligned} a_{11} &= 2\overline{\rho}_1 (t+\delta t)\, V_1 + m_1 (t+\delta t)\, \delta t\\ b_{11} &= 2\overline{\rho}_1 (t)\, V_1 - m_1 (t)\delta t\\ c_1 &= m_o (t+\delta t)\, \delta t h_o (t+\delta t) + m_r (t+\delta t)\, \delta t h_r (t+\delta t)\\ &\quad + m_o(t)\delta t h_o(t) + m_r(t)\, \delta t h_r(t) + \delta t [q_{c1}(t+\delta t) + q_{c1}(t)] . \end{aligned}$$

For component 2

$$\begin{aligned}[2\overline{\rho}_2 (t+\delta t)\, V_2 + m_2(t+\delta t)\delta t]\, h_2 (t+\delta t) - m_1 (t+\delta t)\, \delta t h_1 (t+\delta t) \quad (38)\\ + m_c (t+\delta t)\, \delta t h_c (t+\delta t) - \delta t q_{c2} (t+\delta t) + \delta t q_{s2} (t+\delta t)\\ = [2\overline{\rho}_2 (t) V_2 - m_2 (t)\delta t]\, h_2 (t) + m_1 (t)\delta t h_1 (t)\\ - m_c (t)\delta t h_c (t) + \delta t q_{c2} (t) - \delta t q_{s2} (t) .\end{aligned}$$

As with component 1, the future time-row energy exchange with surroundings is removed to the equation right-hand side. Also, in the absence of a more detailed component model, the coil total heat transfer and condensate exit flux must be assumed to be independently calculable from some free standing coil algorithm (perhaps existing in the style of a TRNSYS component). This assumption can be removed by deriving a more detailed, multi-node model.

The component 2 equation becomes

$$a_{21} h_1 (t+\delta t) + a_{22} h_2 (t+\delta t) = b_{21} h_1 (t) + b_{22} h_2 (t) + c_2 \quad (39)$$

where

$$\begin{aligned} a_{21} &= -m_1 (t+\delta t)\, \delta t\\ a_{22} &= 2\overline{\rho}_2 (t+\delta t)\, V_2 + m_2(t+\delta t)\, \delta t\\ b_{21} &= m_1(t)\delta t \end{aligned}$$

$$b_{22} = 2\overline{\rho}_2(t)\,V_2 - m_2(t)\,\delta t$$
$$c_2 = -m_c(t+\delta t)\,\delta th_c(t+\delta t) - m_c(t)\,\delta th_c(t)$$
$$+\delta t\,[q_{c2}(t+\delta t) + q_{c2}(t) - q_{s2}(t+\delta t) - q_{s2}(t)]\,.$$

For component 3

$$[2\overline{\rho}_3(t+\delta t)\,V_3 + m_3(t+\delta t)\delta t]\,h_3(t+\delta t) - m_2(t+\delta t)\,\delta th_2(t+\delta t) \quad (40)$$
$$- m_h(t+\delta t)\,\delta th_h(t+\delta t) - \delta tq_{c3}(t+\delta t)$$
$$= [2\overline{\rho}_3(t)V_3 - m_3(t)\delta t\,]h_3(t) + m_2(t)\delta th_2(t)$$
$$- m_h(t)\delta th_h(t) + \delta tq_{c3}(t)\,.$$

Again, in the absence of a multi-node humidifier model, some free standing algorithm must exist to determine the vapour supply term $m_h h_h(t+\delta t)$ and so this equation becomes

$$a_{32}h_2(t+\delta t)\;a_{33}h_3(t+\delta t) = b_{32}h_2(t) + b_{33}h_3(t) + c_3 \quad (41)$$

where

$$a_{32} = -m_2(t+\delta t)\,\delta t$$
$$a_{33} = 2\overline{\rho}_3(t+\delta t)\,V_3 + m_3(t+\delta t)\,\delta t$$
$$b_{32} = m_2(t)\delta t$$
$$b_{33} = 2\overline{\rho}_3(t)\,V_3 - m_3(t)\,\delta t$$
$$c_3 = -m_h(t+\delta t)\,\delta th_h(t+\delta t) - m_h(t)\,\delta th_h(t)$$
$$+\delta t[q_{c3}(t+\delta t) + q_{c3}(t)].$$

For component 4

$$[2\overline{\rho}_4(t+\delta t)\,V_4 + m_4(t+\delta t)\delta t]\,h_4(t+\delta t) - m_3(t+\delta t)\,\delta th_3(t+\delta t) \quad (42)$$
$$- \delta tq_{c4}(t+\delta t) - \delta tq_{s4}(t+\delta t)$$
$$= [2\overline{\rho}_4(t)V_4 - m_4(t)\delta t\,]h_4(t) + m_3(t)\delta th_3(t)$$
$$+ \delta tq_{c4}(t) + \delta tq_{s4}(t)$$

which gives

$$a_{43}h_3(t+\delta t) + a_{44}h_4(t+\delta t) = b_{43}h_3(t) + b_{44}h_4(t) + c_4 \quad (43)$$

where

$$a_{43} = -m_3(t+\delta t)\,\delta t$$
$$a_{44} = 2\overline{\rho}_4(t+\delta t)\,V_4 + m_4(t+\delta t)\,\delta t$$
$$b_{43} = m_3(t)\delta t$$
$$b_{44} = 2\overline{\rho}_4(t)\,V_4 - m_4(t)\,\delta t$$
$$c_4 = \delta t\,[q_{c4}(t+\delta t) + q_{c4}(t) + q_{s4}(t+\delta t) + q_{s4}(t)].$$

For component 5

$$[2\overline{\rho}_5(t+\delta t)\,V_5 + m_5(t+\delta t)\,\delta t]\,h_5(t+\delta t) \quad (44)$$
$$- m_4(t+\delta t)\,\delta th_4(t+\delta t) - \delta tq_5(t+\delta t)$$
$$= [2\overline{\rho}_5(t)\,V_5 - m_5(t)\,\delta t]\,h_5(t) + m_4(t)\,\delta th_4(t)$$
$$+ \delta tq_{c5}(t)$$

and this becomes

$$a_{54}h_4(t+\delta t) + a_{55}h_5(t+\delta t) = b_{54}h_4(t) + b_{55}h_5(t) + c_5 \quad (45)$$

where

$$a_{54} = -m_4(t+\delta t)\,\delta t$$
$$a_{55} = 2\overline{\rho}_5(t+\delta t)\,V_5 + m_5(t+\delta t)\,\delta t$$
$$b_{54} = m_4(t)\delta t$$
$$b_{55} = 2\overline{\rho}_5(t)\,V_5 - m_5(t)\,\delta t$$
$$c_5 = \delta t\,[q_{c5}(t+\delta t) + q_{c5}(t)].$$

The matrix equation of figure 5 will normally be combined with some multi-zone building matrix to give a super-matrix (figure 3) defining whole system energy balance. In the present treatment the two matrix systems are decoupled so that the time-series of required supply conditions is assumed to be a known boundary condition. Therefore the component 5 equation is written

$$a_{54}h_4(t+\delta t) = b_{54}h_4(t) + b_{55}h_5(t) + c_5' \quad (46)$$

where

$$c_5' = c_5 - [2\overline{\rho}_5(t+\delta t)V_5 + m_5(t+\delta t)\delta t]h_5(t+\delta t).$$

The form of the figure 5 system of equations is simple for the trivial, linear problem considered here. For complex arrangements, involving recirculating loops, more complex matrix structures will result. But, for any system, two possibilities exist for matrix solution:

- The q_{s2} and q_{s4} terms (plus the internal heat transfer processes active in any other participating component) of the **C** matrix can be assessed by independent component algorithms, when given knowledge of current component state variables and future time-row control objectives.
- Alternatively, the q_{s2} and q_{s4} terms can be removed to the future time-row side of the matrix equation - $\mathbf{A}\mathbf{h}(t+\delta t)$ - and replaced by an expanded, multi-node equation-set which represents internal thermodynamic processes directly.

These approaches are pursued later when an algorithm of the former type is developed and the formulation of a detailed component model is demonstrated. But firstly, a mathematical statement on system mass balance must be established.

By the same reasoning that was applied to the formulation of the energy balance matrix, a mass balance equation can be formulated by taking an equal weighting of the explicit and implicit forms of equations 26 through 35.

For component 1

$$m_1^d(t+\delta t) = m_o^d(t+\delta t) + m_r^d(t+\delta t) + m_o^d(t) + m_r^d(t) \quad (47)$$
$$- m_1^d(t)$$

and therefore, with reference to figure 5, this becomes

$$d_{11}m_1^d(t+\delta t) = e_{11}m_1^d(t) + f_1 \quad (48)$$

and

$$m_1^d(t+\delta t)g_1(t+\delta t) = m_o^d(t+\delta t)g_o(t+\delta t) \quad (49)$$
$$+ m_r^d(t+\delta t)g_r(t+\delta t) + m_o^d(t)g_o(t) + m_r^d(t)g_r(t)$$
$$- m_1^d(t)g_1(t)$$

and therefore

$$d_{22}[m_1^d(t+\delta t)g_1(t+\delta t)] = e_{22}[m_1^d(t)g_1(t)] + f_2. \quad (50)$$

For component 2

$$m_1^d(t+\delta t) - m_2^d(t+\delta t) = -m_1^d(t) + m_2^d(t) \quad (51)$$

and therefore

$$d_{31}m_1^d(t+\delta t) + d_{33}^d(t+\delta t) = e_{31}m_1^d(t) + e_{33}m_2^d(t) \quad (52)$$

and

$$[m_1^d(t+\delta t)g_1(t+\delta t)] - [m_2^d(t+\delta t)g_2(t+\delta t)] - m_c(t+\delta t) \quad (53)$$
$$- 2\rho_L(t+\delta t)V_c/\delta t$$
$$= -[m_1^d(t)g_1(t)] + [m_2^d(t)g_2(t)] + m_c(t) - 2\rho_L(t)V_c/\delta t.$$

Again, in the absence of detailed, multi-node component models, some independent algorithm must be formulated for the assessment of cooler vapour extraction rate based on current cooler conditions and future expectations. The component 2 vapour equation becomes

$$d_{42}[m_1^d(t+\delta t)g_1(t+\delta t)] + d_{44}[m_2^d(t+\delta t)g_2(t+\delta t)] \quad (54)$$
$$= e_{42}[m_1 d(t)g_1(t)] + e_{44}[m_2^d(t)g_2(t)] + f_4$$

where

$$f_4 = m_c(t) + m_c(t+\delta t) + 2V_c[\rho_L(t+\delta t) - \rho_L(t)]/\delta t.$$

For component 3

$$m_2^d(t+\delta t) - m_3^d(t+\delta t) = -m_2^d(t) + m_3^d(t) \quad (55)$$

and therefore

$$d_{53}m_2^d(t+\delta t) + d_{55}m_3^d(t+\delta t) = e_{53}m_2^d(t) + e_{55}m_3^d(t) \quad (56)$$

and

$$[m_2^d(t+\delta t)g_2(t+\delta t)] - [m_3^d(t+\delta t)g_3(t+\delta t)] + m_A(t+\delta t) \quad (57)$$
$$- 2\rho_L(t+\delta t)V_A/\delta t$$
$$= -[m_2^d(t)g_2(t)] + [m_3^d(t)g_3(t)] - m_A(t) - 2\rho_L(t)V_A/\delta t$$

and therefore

$$d_{64}[m_2^d(t+\delta t)g_2(t+\delta t)] + d_{66}[m_3^d(t+\delta t)g_3(t+\delta t)] \quad (58)$$
$$= e_{64}[m_2^d(t)g_2(t)] + e_{66}[m_3^d(t)g_3(t)] + f_6$$

where

$$f_6 = -m_A(t) - m_A(t+\delta t) + 2V_A[\rho_L(t+\delta t) - \rho_L(t)]/\delta t.$$

For component 4

$$m_3^d(t+\delta t) - m_4^d(t+\delta t) = -m_3^d(t) + m_4^d(t) \quad (59)$$

and therefore

$$d_{75}m_3^d(t+\delta t) + d_{77}m_4^d(t+\delta t) = e_{75}m_3^d(t) + m_4^d(t) \quad (60)$$

and

$$m_3^d(t+\delta t)g_3(t+\delta t) - m_4^d(t+\delta t)g_4(t+\delta t) = -m_3^d(t)g_3(t) \quad (61)$$
$$+ m_4^d(t)g_4(t)$$

and therefore

$$d_{86}[m_3^d(t+\delta t)g_3(t+\delta t)] + d_{88}[m_4^d(t+\delta t)g_4(t+\delta t)] \quad (62)$$
$$= e_{86}[m_3^d(t)g_3(t)] + e_{88}[m_4^d(t)g_4(t)].$$

For component 5

$$m_4^d(t+\delta t) - 2m_5^d(t+\delta t) + m_s(t+\delta t) = -m_4^d(t) + 2m_5^d(t) \quad (63)$$
$$- m_s(t)$$

and since m_5^d is the known zone mass flowrate (but only in the absence of a building model as assumed here):

$$d_{97}m_4^d(t+\delta t) + d_{99}m_5^d(t+\delta t) = e_{97}m_4^d(t) + e_{99}m_5^d(t) + f_5 \quad (64)$$

and

$$m_4^d(t+\delta t)g_4(t+\delta t) - m_5^d(t+\delta t)g_5(t+\delta t) = -m_4^d(t)g_4(t) \quad (65)$$
$$+ m_5^d(t)g_5(t)$$

and therefore

$$d_{108}[m_4^d(t+\delta t)g_4(t+\delta t)] + d_{1010}[m_5^d(t+\delta t)g_5(t+\delta t)] \quad (66)$$
$$= e_{108}[m_4^d(t)g_4(t)] + e_{1010}[m_5^d(t)g_5(t)].$$

It is important to note that the mass balance matrix is merely a mathematical statement of inter-component linkages. In complex circuits, involving recirculating loops, nodes will be required to represent the branching points. Even then, matrix solution will not provide a unique result in the absence of statements concerning the diversion ratios prevailing at each branching point. Two approaches are possible. A flow simulation model can be established in which individual flowstreams and fittings are expressed in terms of characteristic flow equations. This model, operating in terms of pressure difference and flow resistance, is appended to the mass balance matrix equation to dictate flowstream behaviour at branching points. The derivation of such a model, involving many empirical relationships for pipe, duct and fitting behaviour, is beyond the scope of the current work. A second approach has therefore been adopted in which simple statements are made on the design or desired diversion ratios at each branching point. Thus, in the current example, the component 1 recirculating statement would be (say) $m_s/m_r = 0.15$. With similar statements supplied by a user of ESP for each network branching point. These statements are then located in the plant matrix equation before it is coupled to the building matrix equation. In any event, the system mass balance matrix equation will allow component mass flowrate determination against control action applied at any point in the plant network.

If nodes located downstream from a control valve do not experience a positive flowrate until some time after valve operation due to network inertia, this can be modelled by delaying the introduction of a non-zero mass flowrate to the matrix coefficient entry until some later matrix equation formulation, depending on the node location, fluid velocity and simulation time-step.

The two matrix equations - one for energy balance, the other for mass balance - are now solved in the context of component algorithms which are used to establish the q and m terms as present within the **C** and **F** matrices. Assuming here that the building side matrix has been reduced by the partitioning technique detailed in reference 8, then one method, of many possible methods, is to proceed as follows.

Step 1

At each time-step establish

$$\mathbf{Ah}(t+\delta t) = \mathbf{Bh}(t) + \mathbf{C} \quad (67)$$

$$\mathbf{D}\Phi(t+\delta t) = \mathbf{E}\Phi(t) + \mathbf{F} \quad (68)$$

and initialise $q_{ci}(t+\delta t)$, $q_{s2}(t+\delta t)$, $m_c(t+\delta t)$ and $m_A(t+\delta t)$ to zero.

Step 2

Assume no humidification or dehumidification and determine circuit humidity ratios, $g_i(t+\delta t)$, from

$$\Phi(t+\delta t) = \mathbf{D}^{-1}[\mathbf{E}\Phi(t) + \mathbf{F}]. \quad (69)$$

Set $\delta g = (g_5' - g_5)$ where g_5 is the desired humidity ratio ($kg\ kg^{-1}$).

If $\delta g < 0$ go to step 3
> 0 go to step 4
$= 0$ go to step 5.

Step 3

Humidification required; determine $m_c(t+\delta t)$ to give required $g_5(t+\delta t)$ from the iterative application of equation 69. Then go to step 5.

Step 4

Dehumidification required; determine $m_c(t+\delta t)$ to give required $g_5(t+\delta t)$ from iterative application of equation 69. From cooler algorithm determine minimum $q_{s2}(t+\delta t)$ to give required $g_2(t+\delta t)$.

Step 5

Estimate $q_{ci}(t+\delta t)$ from known component conditions (unless a building matrix is present so that component casing nodes are coupled to containing zone nodes).

Step 6

With $q_{s4}(t+\delta t)$ remaining at zero and $q_{s2}(t+\delta t)$ set at the value determined from step 4 (or zero if step 4 was bypassed), determine circuit enthalpies, $h_r(t+\delta t)$, from

$$\mathbf{h}(t+\delta t) = \mathbf{A}^{-1}[\mathbf{Bh}(t) + \mathbf{C}]. \quad (70)$$

Set $\delta h = (h_5' - h_5)$, where h_5 is the desired supply enthalpy ($J\ kg^{-1}$).

If $\delta h < 0$ go to step 7
> 0 go to step 8

=0 go to step 9.

Step 7

Re-heat required; determine $q_{s4}(t+\delta t)$ from the iterative application of equation 70. Then go to step 9.

Step 8

Further cooling, then humidification required; determine $q_{s2}(t+\delta t)$ from the iterative application of equation 70. From cooler algorithm assess new $m_c(t+\delta t)$ and determine new $m_h(t+\delta t)$ to give required $g_5(t+\delta t)$ from iterative application of equation 69.

Step 9

The desired supply conditions are now achieved, corresponding to

$$\text{minimum } [abs\ q_{s2}(t+\delta t)+q_{s4}(t+\delta t)]$$

that is for minimum cooler and re-heater energy.

Since the duty and entering and leaving air states are now known, active component algorithms can be invoked to determine internal operating conditions - such as coil outlet water temperatures or required chilled water flowrates. Should the component be unable to perform as required then it can be reset to its limiting condition (that is, in the present example, one or more of $q_{s2}(t+\delta t)$, $q_{s4}(t+\delta t)$, $m_c(t+\delta t)$ or $m_h(t+\delta t)$ can be fixed). In the presence of a building matrix equation, any deviation from the required supply condition will be manifest in an environmental penalty for some time after. If control constraints are imposed prior to simulation, then some limit condition may be indicated during steps 4 or 8. In this case, supply conditions will not be met and conditions will deviate from the set point. Note that the procedure of steps 1 through 9 is entirely independent of component location since the matrices represent the linking protocol. In ESP it is also possible to bypass this procedure by establishing a number of control loops which act, throughout simulation, to fix coil and humidifier inputs, at each time-step, on the basis on conditions sensed elsewhere.

We now continue with the formulation of a component model; firstly in a form suitable for use as described above, and secondly in a form which allows the removal of the $q_{s2,4}(t+\delta t)$ terms to the matrix uation future (unknown) time-row. The component selected for this treatment is the cooler of the system of figure 1.

Free-standing component models

It is possible to derive an entirely free-standing component algorithm to represent the performance capabilities of any component. This algorithm then operates in tandem with the matrix reduction to limit component capabilities if system demands are too great. That is, the algorithms represent component operation whilst the matrix scheme represents component inertia and inter-component connections.

Consider a counterflow cooling coil. The following procedure - based on the sensible heat ratio method - is active in ESP to compute coil performance based on inlet conditions.

1. At any time-step, the following quantities are known: inlet water temperature and mass flowrate, θ_{wi} and m_w; inlet air dry bulb temperature, mass flowrate, humidity ratio and enthalpy, θ_{ai}, m_a, g_{ai} and h_{ai}; airside, waterside and metal thermal resistances, R_a, R_w and R_m; coil surface area, A; atmospheric pressure, P_a; and air and water specific heats, C_{pa} and C_{pw}.
2. Calculate the coil bypass factor, β, from
$$\beta = \exp[-A/(C_{pa} m_a R_a)].$$
3. Guess coil effectiveness, E, perhaps the previous time-step value.
4. Guess sensible heat ratio, SHR.
5. Calculate the coil U-value, U, the number of heat transfer units, NTU, and the capacity-rate ratio, CRR, from
$$\phi_1 = (m_a C_{pa})/SHR$$
$$\phi_2 = m_w C_{pw}$$
$$C_{min} = \min(\phi_1,\phi_2)$$
$$C_{max} = \max(\phi_1,\phi_2)$$
$$U = 1/[(R_a SHR) + R_m + R_w]$$
$$NTU = AU/C_{min}$$
$$CRR = C\min/C\max.$$
6. Establish if guessed E and SHR match by applying:
$$E = \{1 - \exp[-NTU(1 - CRR)]\}/\{1 - CRR\exp[-NTU(1 - CRR)]\}.$$
If $CRR \to 1$, $E = NTU/(1 + NTU)$
7. If not matched, return to step 4 and iterate until SHR is established corresponding to guessed E.
8. Evaluate coil heat transfer from
$$Q = C_{min} E(\theta_{ai} - \theta_{wi}).$$
9. From Q calculate outlet air enthalpy, h_{ao}.
10. Calculate the saturation enthalpy, h_s, at coil surface temperature from
$$h_s = (h_{ao} - \beta h_{ai})/(1 - \beta).$$
11. Determine the coil surface temperature, θ_s, and saturation humidity ratio, g_s, from the saturation enthalpy and atmospheric pressure.
12. Calculate outlet air temperature, θ_{ao}, and humidity ratio, g_{ao}, from
$$\theta_{ao} = \beta(\theta_{ai} - \theta_s) + \theta_s$$
$$g_{ao} = \beta(g_{ai} - g_s) + g_s.$$
13. Calculate the corresponding sensible heat ratio, SHR', from
$$SHR' = (\theta_{ai} - \theta_{ao})C_{pa}/(h_{ai} - h_{ao}).$$
14. Compare SHR' with SHR and, if different, return to step 3 and iterate until agreement is obtained.
15. Eventually - perhaps after changing coil parameters to achieve desired coil performance - terminate algorithm, and insert Q in system matrix equation to give final circuit enthalpies and humidity ratios.

Component matrix representation

As an alternative to operating with a free-standing component algorithm, it is possible to introduce a number of equations to the system matrix equation to represent, directly, internal component processes. For example, Holmes (1982) has suggested the following simple coil model, consisting of two first-order, ordinary differential equations:

$$C_w \frac{d\theta_1}{dt} = \frac{\theta_o - \theta_1}{R_1} - \frac{\theta_1 - \theta}{R_{mw}} \quad (71)$$

$$C_m \frac{d\theta}{dt} = \frac{\theta_1 - \theta}{R_{mw}} - \frac{\theta}{R_a + R_4} \quad (72)$$

$$\theta = \frac{\theta_2(R_a + R_4)}{R_4}$$

where $\theta_o = \theta_{wi} - \theta_{ai}$; $\theta_1 = \theta_{wo} - \theta_{ai}$; $\theta_2 = \theta_{ao} - \theta_{ai}$; $R_1 = 1/(m_w C_{pw})$; R_{mw} is the metal + water film resistance; $R_4 = 1/(m_a C_{pa})$; C_w is the water thermal capacity ($J\ K^{-1}$) and C_m is the metal thermal capacity.

Equations 71 and 72 define a 2-node component model. A finite difference approximation applied to these equations gives the final state-space representations:

Air equation

$$-\left[\frac{2C_m(R_a + R_4)}{\delta t R_4} + \frac{R_a + R_4}{R_{mw} R_4}\right]\theta_{ai}(t+\delta t)$$

$$+\left[\frac{2C_m(R_a+R_4)}{\delta t R_4}+\frac{(R_a+R_4)}{R_{mw}R_4}+\frac{1}{R_{mw}}\right]\theta_{ao}(t+\delta t)$$

$$-\frac{1}{R_{mu}}\theta_{uo}(t+\delta t)=-\left[\frac{2C_m(R_a+R_4)}{\delta t R_4}-\frac{(R_a+R_4)}{R_{mw}R_4}\right]\theta_{ai}(t)$$

$$+\left[\frac{2C_m(R_a+R_4)}{\delta t R_4}-\frac{(R_a+R_4)}{R_{mw}R_4}-\frac{1}{R_{mw}}\right]\theta_{ao}(t)+\frac{1}{R_{mw}}\theta_{uo}(t)$$

Water equation

$$\left[\frac{2C_w}{\delta t}-\frac{(R_a+R_4)}{R_{mw}^2}+\frac{1}{R_{mw}}\right]\theta_{ai}(t+\delta t)$$

$$-\frac{(R_a+R_4)}{R_{mw}^2}\theta_{ao}(t+\delta t)+\left[\frac{2C_w}{\delta t}+\frac{1}{R_1}+\frac{1}{R_{mw}}\right]\theta_{wo}(t+\delta t)$$

$$=-\left[\frac{2C_w}{\delta t}-\frac{1}{R_{mw}}+\frac{(R_a+R_4)}{R_{mw}^2}\right]\theta_{ai}(t)-\frac{(R_a+R_4)}{R_{mw}^2}\theta_{ao}(t)$$

$$+\left[\frac{2C_w}{\delta t}-\frac{1}{R_1}-\frac{1}{R_{mw}}\right]\theta_{wo}(t)+\frac{1}{R_1}[\theta_{wi}(t)+\theta_{wi}(t+\delta t)]$$

Figure 5 shows the addition of these representations to the whole system energy balance matrix equation.

Some 25 component models now exist, in state-space form within ESP's plant component database, available for selection for interconnection to define a plant system. This system is then attached to the building description and the entire configuration processed simultaneously. Any number of control loops can be defined to determine flux injections on the basis of sensed nodal conditions. And, the control action can vary from time-step to time-step as a simulation proceeds. To support the new plant modelling features a number of other developments have been necessary. Database management software (ESPpdb) organises the component models in terms of their defining matrix templates and essential manufacturers data. A technical proforma has been designed to allow component models to be clearly specified for transfer to this database. Coding has been added to ESPsim to enable automatic multi-matrix formulation; for energy balance and, optionally, for each of two phase mass balances (representing dry air and water vapour in an A/C system for example). ESPsim's treatment of control has also been radically modified to allow the use of *any* number of user-specified algorithms to dictate component flux injection or flowrate control. And, lastly, ESPout has been modified to enable plant process graphical display.

In this form ESP has been acquired by a number of organisations in the private and public sector.

Conclusions

A new generation of building energy analysis software has begun to emerge. Based on advanced numerical methods, these systems attempt to treat combined building, plant and control systems in a rigorous and fully integrated manner. If the promise of this new generation is to be realised then the research community must find ways to act collaboratively. This process has already begun in a project (Hirsch 1985) which seeks to produce an agreed structure for future systems.

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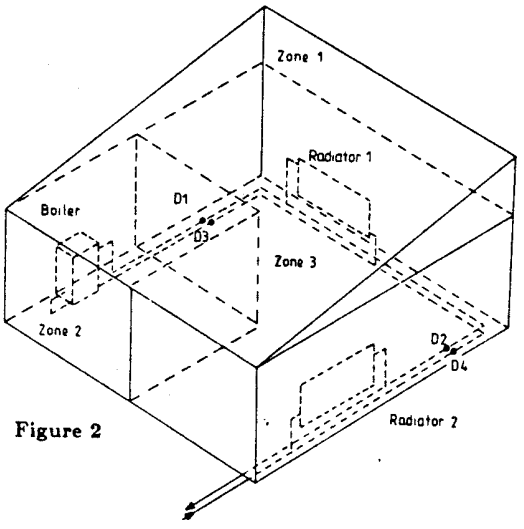


Figure 2

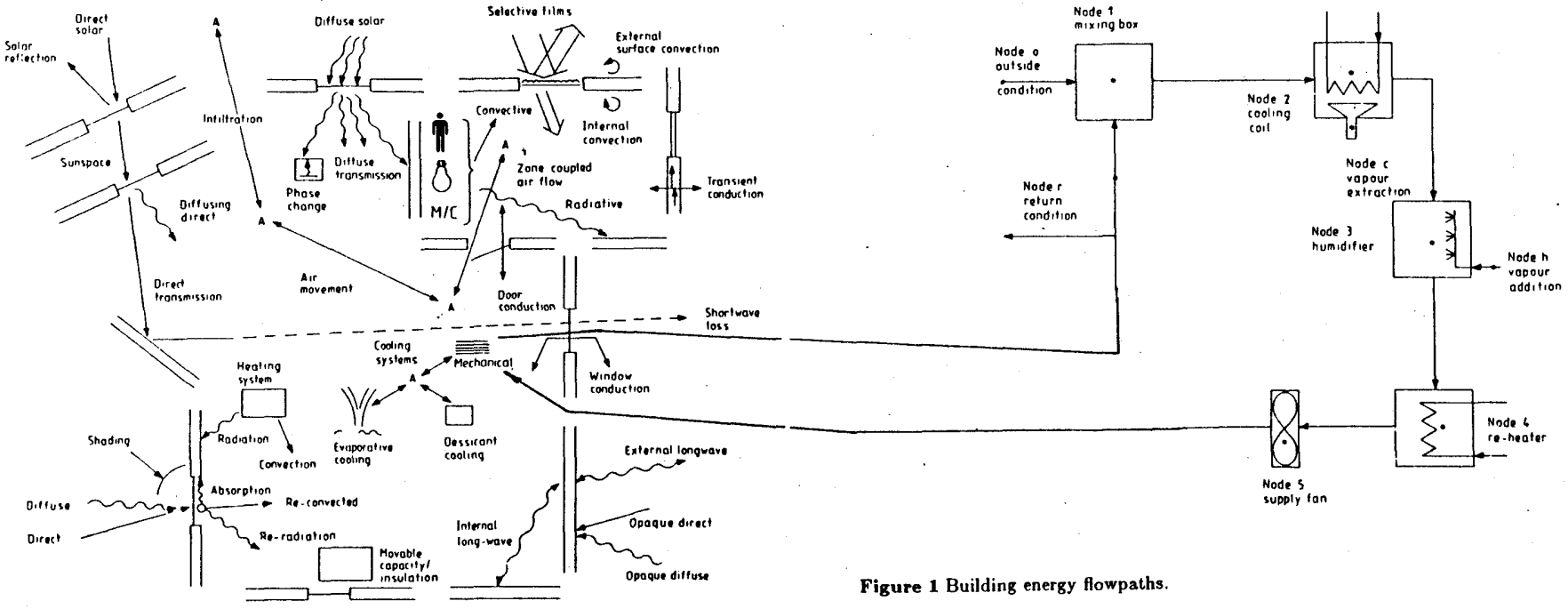
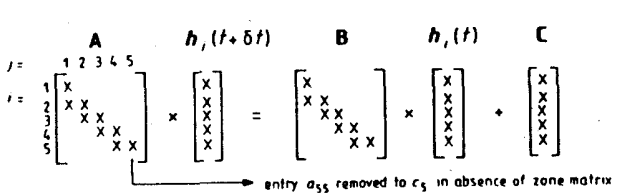
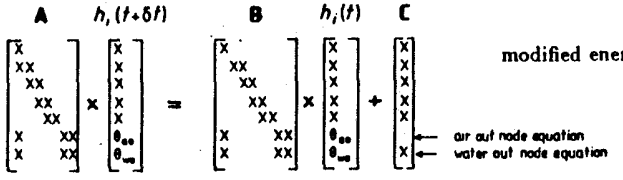


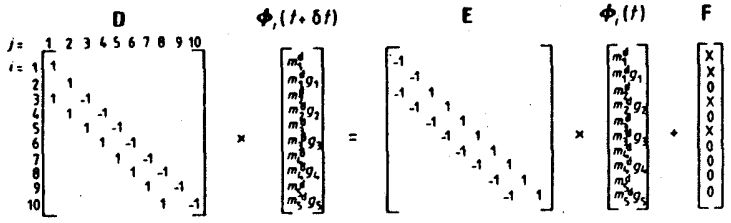
Figure 1 Building energy flowpaths.



energy balance

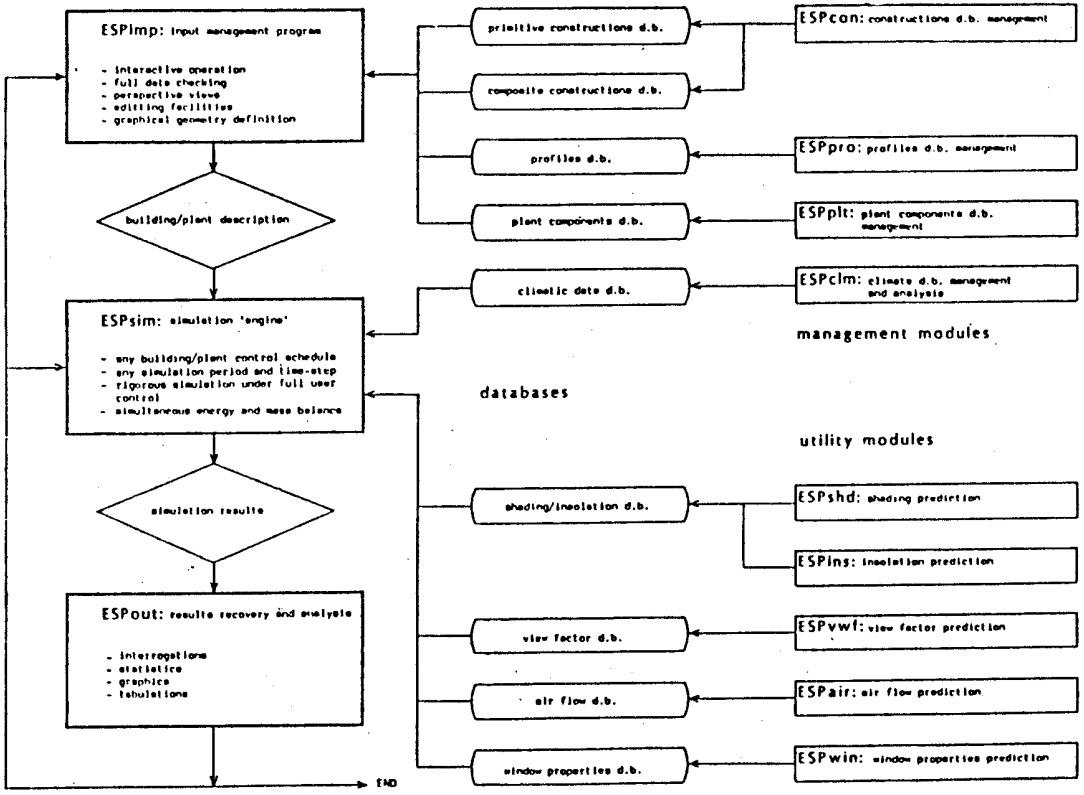


modified energy balance



mass balance; two phase

Figure 5 The energy and mass balance matrix equations for the system of figure 1.



Shown here are the connections between the various program and database modules which comprise ESP. These include:

ESPimp A program which allows the interactive definition of the combined building/plant network to be subjected to some weather influence and simulated over time. All items, as input, are processed through legality and 'within acceptable range' checks before being located in an output file for transfer to the simulation engine ESPSIM. Other facilities include a range of interactive editting commands and automatic access to construction, casual gain profile and plant component databases.

ESPcon A program to manage (create, modify, delete, list) a primitive constructions database containing the thermophysical properties (conductivity, density, specific heat, solar absorptivity and emissivity) of a number of miscellaneous homogeneous elements. This program will also allow the creation and editting of a second project-related database containing whole composite multi-layered constructions formed from elements extracted from the primitives database.

ESPpro A program to manage a casual gain profiles database containing any number of standard or project-specific profiles defining the time varying heat gain from occupants, lights etc.

ESPplt A program to manage the plant components database containing component descriptions, component models (in matrix form) and essential manufacturers data.

ESPshd A program to predict the time-series shading of external opaque and transparent surfaces as caused by surrounding site and facade obstructions.

ESPins A program to predict the time-series insolation of internal opaque and transparent surfaces as caused by solar penetration through windows.

ESPvwf A program to compute view factors between the surfaces bounding any building zone.

ESPair A program which simulates the flow of air throughout any multizone system (where each zone is connected by flow resistances representing windows, doors, cracks, etc) subjected to boundary wind loading.

ESPwin A program to perform a spectral analysis of a multilayered window system to determine overall transmission, reflection and absorption.

ESPclm A program to manage the climatological database allowing creation (by prediction), modification and analysis of the hourly time-series values of the various climatic parameters retained.

ESPsim Is the central simulation 'engine' which simulates energy flow by a heat balance matrix inversion technique of implicit enumeration. The technique is unconditionally stable for all computational time-increments and operates in conjunction with hourly values of various climatological parameters recovered from the climatological database. As the model steps though the user-defined time-increments it continuously predicts and environmental and energy status of the simulated building. This is done by placing 'nodes' at many points of interest, for each node performing a heat balance between all interacting regions (over time and space dimensions), and then simultaneously solving the resulting set of equations at each time-step.

ESPout Is the output program which operates on the simulation results located in the results database by ESPSIM. A variety of output facilities are available: visualisation, result interrogation; graphical display; statistical analysis; and tabulations.

Figure 4 The ESP system.

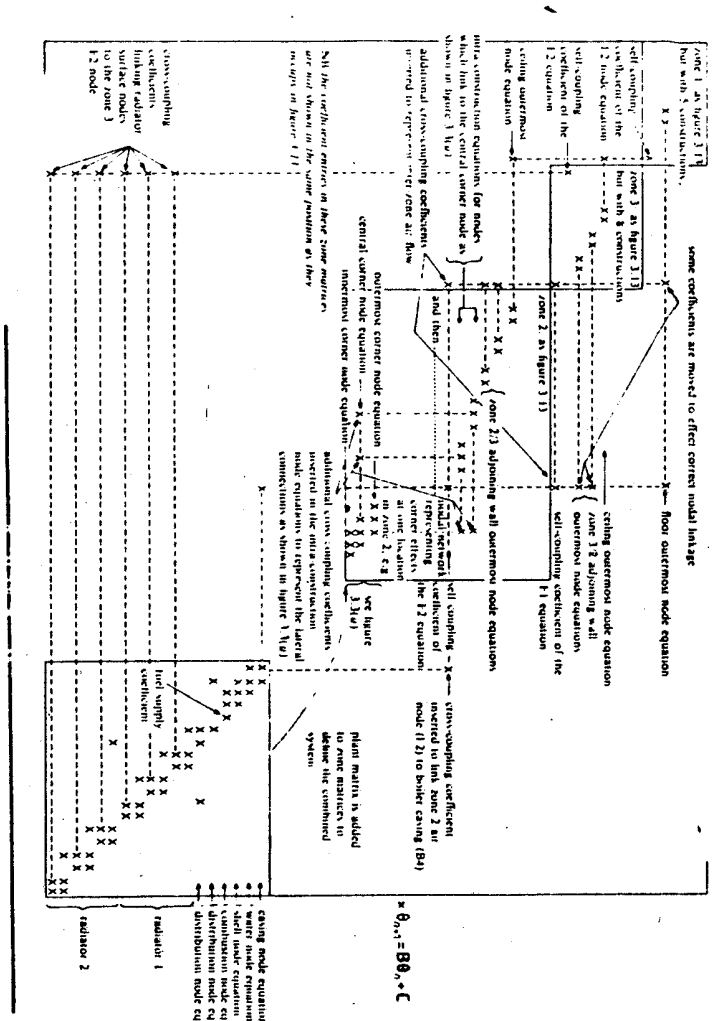


Figure 3 The super-matrix for the system of figure 3. (external references are to Clarke 1984)