SIMULATION IN LIGHTING COST ANALYSIS

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ABSTRACT

Since methods incorporating the time value of money are the only ones that give an accurate picture of life costs of a system, they are the only methods appropriate for the analysis of building lighting systems. However, these procedures (such as the present worth method) and the algorithms for employing them are not without shortcomings. One problem with traditional methods is that they employ single point estimates as input values and result in a single value that is supposed to represent the present worth of the system. There is a strong temptation to adhere tenaciously to this number as the definitive answer without considering sensitivity of the output of the model to variations in input.

One well-known technique for simultaneous analysis of the uncertainties of the various inputs to a present worth analysis involves Monte-Carlo simulation. First described in the finance and capital budgeting literature of the late 60s, this sophisticated approach is based on user input concerning the "odds" of a given cost taking on various values. The system being analyzed is repeatedly simulated. In each trial, samples are taken from the theoretical probability distributions for the cost inputs, and the present worth is calculated. The output from the model is then a distribution of how frequently present worth values were computed on each of many intervals. Such distributions contain significant information about variability and hence risk of the proposed investment.

For maximum accessibility by lighting design professionals, Monte-Carlo software is needed that is specifically tailored to lighting system analysis and is designed for the PC environment. such characteristics are embodied in the MCSEALS* program developed by the author at the University of Kansas. This paper discusses the advantages of Monte-Carlo simulation for the economic analysis of lighting in buildings and describes the probability theory on which the MCSEALS software is based.

NEED FOR A NEW METHOD

The primary objective of this paper is to introduce the concept of Monte-Carlo simulation as an aid in the economic assessment of lighting systems. A public-domain computer program which embodies this method is also presented.

The author has previously demonstrated that some of the methods that have been proposed for evaluating lighting system costs are inappropriate and can lead to selection of inferior alternatives.[1] Among these are the "cost of light" concept which ignores fixture and room geometry, and a simplistic annualized cost method which doesn't consider the time value of money.

The simple models being disqualified, one must move up in sophistication to those models that explicitly account for interest rates and time. The basic present worth technique is a widely used and useful example of this category of methods. Present worth analysis acknowledges that the risk averse individual (i.e. everyone) prefers a dollar today over a dollar tomorrow. Examples of how this method yields superior results as compared with the simplistic methods can be found in any basic finance or engineering economics text. See also reference 1.

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SENSITIVITY ANALYSIS

One problem incurred with even a good method (such as present worth) is that, once selected as being the most appropriate, there is a temptation to adhere tenaciously to the results without considering if those results are reasonable. Constant reminders are necessary that output from any model can be no better than the input. This leads directly to the suggestion that sensitivity analysis should be performed to determine the "stability" of the model's output for a given comparison.

Sensitivity analysis provides a format for examining what effect the input assumptions have on the output of the model. Consider the cost of a maintenance event at some time during the life of the system. This will obviously affect the present worth of the project. However, there is much uncertainty surrounding such an event. For ex-

* MCSEALS is a public domain program available

ample, the timing of a ballast failure/replacement may traditionally be estimated as occurring in the tenth year of system life, but may actually occur at any time. Rather than looking at only the "best guess" point estimate of this event, in sensitivity analysis one investigates an entire range of possible values which the variable can assume. In this manner it can be determined if there are possible values which would make a previously unacceptable alternative emerge as the most attractive.

The opportunity rate for capital is one input having a range of possible values. Figure 1 illustrates the effect of considering different possible opportunity rates for two alternative lighting systems. System "A" involves expenditures of $5000 today plus $2000 ten years from now. In "B", all expenditures are delayed until year ten in which $12,000 must be paid. This could represent, for example, an existing system which could either be A) upgraded today at a cost of $5000 and overhauled ten years from now at a cost of $2000 or, B) completely replaced in ten years at a cost of $12,000.

Over part of the range of interest rates, "A" is preferred (below about 7%). Over the rest of the range, "B" is preferred. A slightly wrong point estimate of the opportunity rate could cause a completely wrong decision. Sensitivity analysis is crucial.

The opportunity rate of capital is just one of the inputs to which the result of the calculation may be sensitive. Similar analyses should be performed to determine if the decision is sensitive to any of the other input variables for which the value

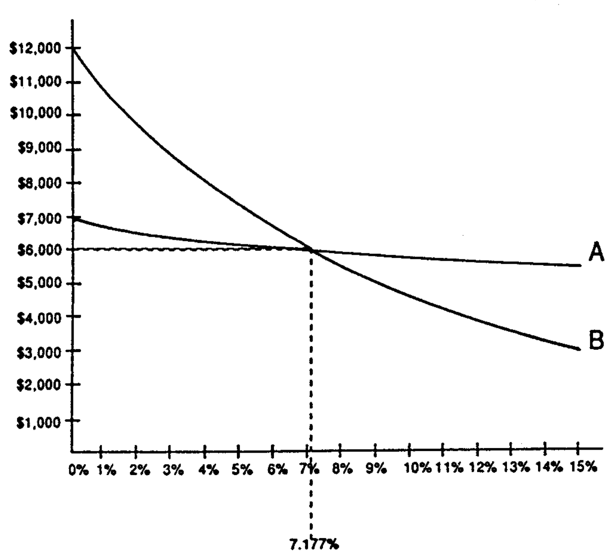

FIGURE 1
Opportunity rate sensitivity analysis

or timing is uncertain. However, researchers in accounting herald the discomforting news that <u>all</u> costs and their timing, even those that have already occurred, are approximations! This indicates that for an economic analysis to be rigorous requires <u>simultaneously</u> taking into account all of the uncertainties in all of the inputs[6]. The most elegant solution to the problem of simultaneous analysis of sensitivity to all variables would be an analytical one. Recalling the simple relationship between two random variables:

$$E(X+Y) = E(X) + E(Y) \quad \text{and,} \qquad (1)$$
$$V(X+Y) = V(X) + V(Y) + 2\,\text{cov}(X,Y),$$

it is tempting to assume that the distribution resulting from an arbitrary combination of well-defined random variables results in a probability distribution whose moments can be analytically determined. There exist, however, formidable obstacles to finding an analytical solution. These include the presence of exponents in the present worth formulae, the fact that some of the distributions may be discrete, and the complex interdependencies of the variables. The analytical approach precluded then, it is necessary to turn to numerical methods.

<u>MONTE-CARLO SIMULATION</u>

One well-known numerical technique that has been applied to sensitivity analysis is Monte-Carlo simulation. Though he didn't apply the name "Monte-Carlo", Hertz first described the procedure in the mid- to late-60s.[6,7] Traditionally, point estimates of costs serve as input to a financial analysis. In this sophisticated approach however, ranges are considered for each variable, and probabilities are assigned indicating the likelihood of each point along that range. Samples are then taken from these theoretical probability distributions and these samples are combined according to the rules of the present worth model. The output is then, itself, a probability distribution containing significantly more information than a single point[11] (as will be discussed with figure 3).

The finance and capital budgeting literature of the 70s and 80s is replete with examples of specific applications for Monte-Carlo simulations of this type. No modern basic finance text would be complete without reference to the method (references 2, 3, and 5 for example). The result is that MBAs and financial analysts are very familiar with such techniques. They are well equipped to use these methods due to mainframe software packages such as the Interactive Financial Planning System (copyright 1982, EXECUCOM Systems Corporation) which has a Monte-Carlo module. The target user of these packages is the financial expert who is a generalist; input format and requirements are not unique to any specific industry. They could theoretically be used equally well to analyze investment in a new assembly line, to compile an appropriate stock portfolio, or to determine the economically superior lighting system to install.

However, much of the work done in the lighting industry is conducted by freelance consultants and small to medium-sized firms. This makes these general purpose packages unsuitable for at least two reasons. 1) They are main-frame based, and 2) they require extensive knowledge of financial theory. Most lighting professionals do not have the luxury of easy access to mainframe computing. Personal computers, though, have proliferated to the point where even the smallest of firms owns at least one. Memory and speed have also expanded to the point where Monte-Carlo analysis is feasible on a microcomputer. Further, the lighting designer, though forced to consider the impact of lighting costs on the client, has neither the time nor the desire to learn complicated investment theory. Neither is the resource of an in-house financial expert often available to these small firms.

For these reasons, using a main-frame to analyze lighting economics is not a reasonable option. Therefore, PC-based software is needed that is capable of addressing the complex issues of economic uncertainty while remaining accessible to lighting professionals having only a basic understanding of economic theory. To demonstrate how these characteristics could be incorporated into software for the lighting professional, the author has developed a program known as MCSEALS.

MCSEALS

MCSEALS stands for: Monte-Carlo Simulation for the Economic Analysis of Lighting Systems. The IBM PC compatible environment was chosen for this program due to its widespread usage among engineers. The program is written in BASIC to provide maximum access to the source code (provided with the program) by the typical user who is not a professional computer programmer. This transparency should be welcome to users who may have ideas for improving programming of the algorithms or who may have specific needs not foreseen by the author that they would like to incorporate in the program. Microsoft's QuickBasic is used. It is a compilable BASIC that runs with good speed and affords more structure than standard interactive BASIC.

INPUT

The user is asked to enter data concerning the system to be analyzed such as equipment and installation costs, energy and maintenance costs, insurance and tax effects, etc. The user may select either spot relamping or group relamping on a predetermined, user defined schedule. For spot relamping, lamp mortality distributions are integrated into the program so as to permit modeling of the stochastic process of individual replacement.

Ballast failure is modeled as a Weibull process as is indicated from the failure statistics literature[8]. Minimum time to first failure is considered zero. This may appear to have limited face validity since industry practice would have the manufacturer replace at its expense a ballast that failed at "time zero". However, the assumption allows failure data to be fitted to a two-parameter Weibull distribution instead of the excessively complex three-parameter Weibull. As modeled, short times to first failure are extremely improbable, which seems to be compatible with the notion that early failures are not of concern in an economic analysis.

The tax benefit of depreciation is, of course, included in the program. The user may define a depreciation schedule or use the default straight-line depreciation model.

Probability distributions are constructed from information provided by the user. The user is asked to input the most likely value of the variable under consideration; this is used as a measure of central tendency of the distribution. Then the user is asked to assess the value which it is highly unlikely for the variable to exceed, and the value which it is highly unlikely that the variable will be less than. These values are combined to determine the dispersion of the distribution. Elements of the analysis handled in this way include: timing of events, energy consumption (operating hours), tax rate, and salvage value. Eventually, the program may also allow initial costs to be treated stocastically. If the user feels initial costs have been precisely determined, then this distribution collapses to a point at the most likely value.

The interest rate at which these cash flows are discounted to their present value is a fixed-point input to the model. This is appropriately taken as the so-called "risk-free" interest rate. MCSEALS follows the standard business practice of using the current rate on 90 day U.S. Treasury Bills as the risk-free rate. At first glance it may seem preferable to use a probability distribution of the opportunity cost of capital as the discount rate. Using the cost of capital, however, causes a double consideration of risk - once in the cost of capital distribution and once in the uncertainties of the individual cash flows. The risk-free rate is used to avoid prejudging risk and thus allows the dispersion of the output to be an appropriate indicator of risk.[9]

The user is asked to input the current inflation rate and the current rate of escalation of energy prices. This information is combined with certain aspects of the historical activity of these measures to create a simplistic econometric model. Elements of that model are:

1) the historic percentage occurrence of annual increases vs. annual decreases in price indices

2) the relative frequency distributions of the magnitudes of the changes in 1).

The assumption on which the inflation models are based is that the underlying structure of the economy over the economic life of the project being considered will be the same as for the 40 years preceding 1988 (since the model is built on data from that period). This assumption is obviously open to discussion. The built-in model is provided simply as a default, however. The user possessing superior information or insight into future price escalation is allowed the option of user-defined inflation models.

Currently (version 1.1), MCSEALS assumes that all future costs except energy escalate at the same rate as general inflation. Since a present worth calculation is a current dollar method, values for all future costs except energy are input in today's dollars and no inflation adjustment is required. Since energy costs are assumed to inflate at a different (though correlated) rate from the cpi, a current dollar adjustment is made using the ratio of general price inflation to energy price inflation. Future versions of MCSEALS may include the possibility to define other inflation rates such as wage inflation.

Based on the above inputs, the computer makes hundreds of random selections of these variables and computes the resulting present worth in each case. For example, on the first pass it might select a labor cost of $20/hr., ballast replacement in year 11, an energy rate of $0.065/kwh, etc. and from these compute a present worth of the alternative. The second time through it might randomly pick a $19/hr. labor rate, ballast replacement in the tenth year, $0.072/kwh, etc., and again calculate the present worth. After hundreds of such computations, a very clear picture emerges of the probability distribution of the present value of the alternative under consideration. This allows construction of confidence intervals and statements such as the following can be made: one can be 95% sure that the present worth of system "A" is between $X and $Y.

A statistician would, of course, shudder at such language. This imprecision is justified, however, due to the fact that each lighting system subjected to analysis by this method will be unique. To be totally precise, the only correct statistical statements that can be made are of this sort: "95% of the systems installed that are correctly represented by the model will have present worths between $X and $Y". Since the system is one-of-a-kind, however, comments like that are meaningless. Therefore, as long as one remembers that it is a sort of shorthand, there is little harm in expressing frequency distributions as confidence levels (e.g. one is 95% sure that the present value lies between $X and $Y).

GENERATING THE VARIATES

Standard formulae for generation of stochastic variates are found in the literature. The following are the formulae used in MCSEALS. They are either taken directly from, or derived from Bulgren[4]. In each case, R denotes a uniform random variate on the appropriate range [usually (0,1)] as obtained from the pseudo-random number generating capability of the computing environment.

Normal Distribution

The default inflation models in MCSEALS utilize the Normal distribution. Two random normal variates are generated that are independent and N(0,1) using:

$$x_1 = (-2 \ln R_1)^{\frac{1}{2}} \cos(2\pi R_2) \quad \text{and,}$$
$$x_2 = (-2 \ln R_1)^{\frac{1}{2}} \sin(2\pi R_2) \qquad (2)$$

Weibull Distribution

Failure of lamps and ballasts are modeled using the Weibull distribution. With s as the scale parameter and p as the shape parameter, the random variates of the two-parameter Weibull distribution are found from:

$$x = s(-\ln R)^{1/p} \qquad (3)$$

Beta Distribution

The beta distribution is discussed extensively in the PERT literature and is shown to be applicable to the question of the probability of the timing of an event. In MCSEALS, the Beta distribution is used to model timing of maintenance events, salvage values, and tax rates.

Consider three estimates concerning an event thought to follow a beta distribution. If the most optimistic estimate of time, cost, etc. is "a", the most likely estimate is "m", and the most pessimistic estimate is "b", then the mean and variance are given by expressions (4) and (5).

$$E(x) = \frac{a + 4m + b}{6} \quad \text{and} \tag{4}$$

$$V(x) = \left[\frac{b - a}{6}\right]^2 \tag{5}$$

A transformation to the range (0,1) is achieved with the expression:

$$E(x') = [E(x) - a] / [b - a] \tag{6}$$

Random variates from the beta distribution having the mean and variance given by (4) and (5) are then generated using:

$$x' = \frac{x_1}{x_1 + x_2} \tag{7}$$

where x_1 and x_2 are the random variates:

$$x_1 = -\ln \left[\prod_{i=1}^{k_1} R_i \right] \tag{8a}$$

and,

$$x_2 = -\ln \left[\prod_{i=1}^{k_2} R_i \right] \tag{8b}$$

where, again, the R_i's are random variates of the (0,1) uniform distribution and:

$$k_1 = \left[\frac{1 - E(x')}{1/36} - \frac{1}{E(x')}\right] \cdot \left[E(x')\right]^2 \tag{9}$$

$$k_2 = K_1 \cdot \left[\frac{1}{E(x')}\right] - 1 \tag{10}$$

The inverse transformation to the original range is accomplished by:

$$x = a + x' \cdot (b - a) \tag{11}$$

STOPPING RULE

How many present worth calculations must be performed in order to achieve a given level of precision? This question is addressed in the program through use of the simplifying assumption that the distribution of the sampled present worth calculations will not differ radically from the normal. It can be shown[10] that under normality:

$$q = \frac{z^2 \sigma^2}{d^2} \tag{12}$$

where q is the number of trials required, z is the normal deviate associated with the desired reliability level, σ is the sample standard deviation, and d is the desired maximum deviation of the sample mean from the true mean. The units of d are the same as for the population being sampled, in this case dollars.

The program assumes a 95% reliability level (corresponding normal deviate = 1.96) and a desired maximum deviation of one percent. Estimates of σ and d are made after each trial and are the n^{th}-trial sample standard deviation and one percent of the n^{th}-trial sample mean, respectively. The estimate of q made after each trial is displayed on the screen as **STOP = (q)**. The number of the current trial is displayed in real-time as ROUND = (n). Each 50^{th} round, MCSEALS graphs the data calculated to that point and checks to see if $n > q$. If not, calculation continues. If so, the desired precision has been achieved and the final graph is presented. Thus the user always has an indication as to how much longer the program will run.

OUTPUT

After the final cost distribution graph is presented on screen, the user is given the option of saving the screen. A utility program called REPLAY is used to retrieve and display any previously saved screen. Figure 2 is an example of such a screen.

The ordinate of the graph represents the frequency of occurrence of a given range of costs in the calculation process. The abscissa is the present worth of the system.

The top line of the screen should be self explanatory. It just echoes the system name and economic life which the user has input to the program.

On the second line are indicated two fractiles and the mode of the distribution. In this example, 5% of the frequency distribution falls below $17,163.79. This corresponds to the portion of the distribution to the left of the left-most vertical dashed line. The highest point of the distribution (the value most frequently calculated) is $17,564.02. The highest 5% of the values calculated are greater than $20,556.72, they appear to the right of the right-most vertical line.

More summary information about the distribution is given on the bottom line. $16,833.16 is the lowest value calculated. $18,401.47 is the arithmetic mean of all values calculated. $22,744.92 is the greatest value calculated.

In addition to providing a more accurate estimate of the most likely value of the present worth of the project, this display

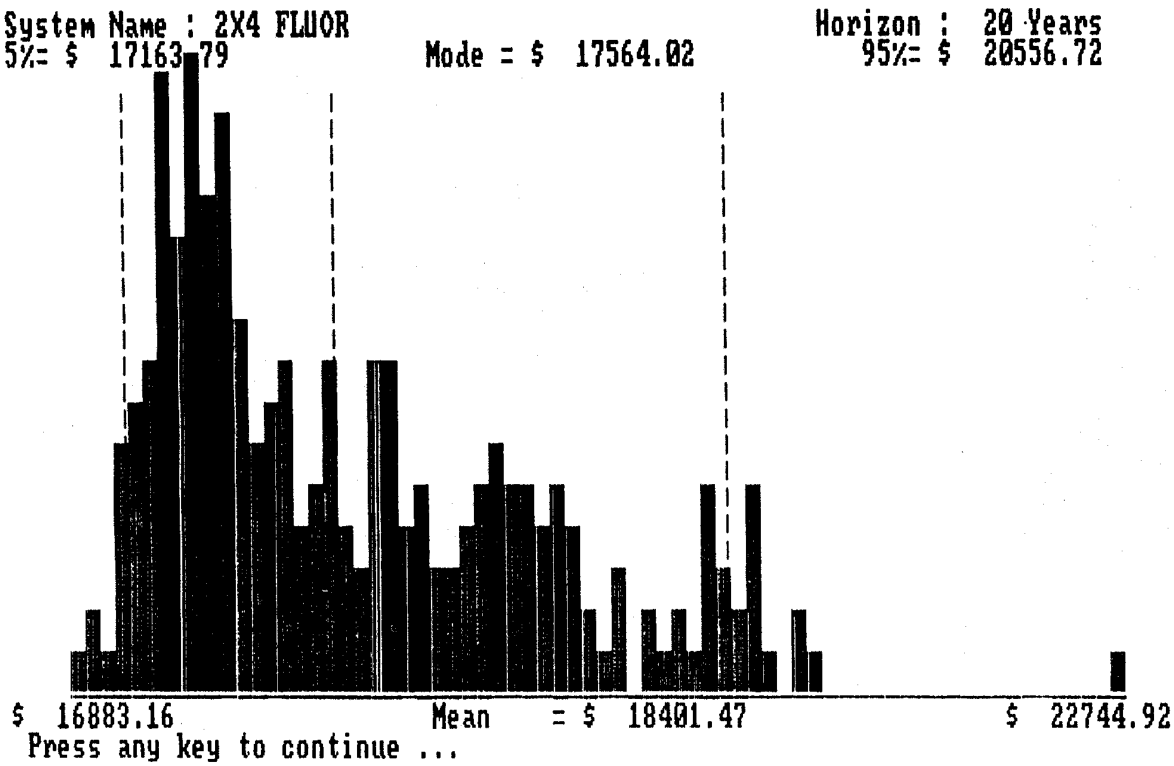

FIGURE 2
Example of output screen from MCSEALS program

conveys information concerning its riskiness. Figure 3 shows this graphically. A very broad distribution (such as A) indicates that the cost of the system is highly uncertain (and therefore risky). A narrow distribution (B), on the other hand, indicates that the cost of the system is relatively predictable. In deciding among alternatives, this dispersion information may lead to the selection of an alternative such as B which has a higher median present worth than the competing system A. This

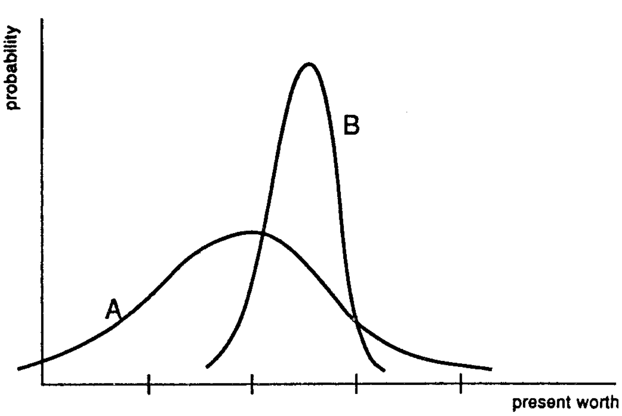

FIGURE 3
Comparisons reveal risk information

preference is due to a lower risk level in B. While it is true that the most likely value of A is lower than the most likely value of B, choosing A exposes one to the slight chance that the cost will be double the anticipated cost. This risk may be intolerable. Such is not the case with alternative B, the cost of which is fairly precisely determined.

FUTURE DIRECTIONS

Variability and uncertainty surrounding the inputs to the classical present worth model of lighting system analysis point out the limited usefulness of the point estimate which that model provides. Monte-Carlo simulation begins to identify some of the areas of uncertainty and incorporate what information is available about those uncertainties into the analysis. Future development of this important concept should proceed on two fronts: software development and data compilation.

SOFTWARE DEVELOPMENT

Since MCSEALS was developed as a simple teaching aid to demonstrate the Monte-Carlo concept, it is less than perfect as a professional tool. Software (which could evolve from MCSEALS or be independent) should be developed that will more adequately meet the Monte-Carlo needs of the lighting professional. The first suggested improvement involves the computer code. Since the author is not trained as a programmer, it is presumed that the existing code is not very sophisticated. More efficient coding should enable it to run more quickly.

The improved program would contain a better inflation model. The user should be allowed options other than the current two: use the crude default model or input the yearly inflation for each year of the economic life of the program. A more sophisticated model would allow the user to input beliefs about the shape of the distribution of future inflation - its skewness and volatility. As mentioned, the program could also be improved by allowing the user to treat more of the input variables as stochastic (such as initial costs). In addition to summary statistics about the output distribution, regression analysis could be employed to tell which of the input variables are causing the most variability in the output. This would tell the user where efforts should be directed to improve information if a less uncertain output is necessary. It could also help determine where the irreducible risks lie.

Whereas this discussion has centered around <u>cost</u> analysis, an economic evaluation is really only complete if <u>benefits</u> are also considered. These could theoretically be included in a MCSEALS analysis as the program now stands. For example, if we know that aesthetic appreciation of a given system results in a three percent increase in productivity due to enhanced worker well-being, this savings can be translated into dollars (or a probability distribution) and be input into the analysis. The barrier at this point is, of course, the dearth of empirical evidence that would enable translating a subjective benefit into something compatible with a dollars and cents analysis. The IES Economics Committee (on which the author serves) recognizes the need for such a system and is moving in the direction of its definition. When that occurs, the framework of MCSEALS will be readily adaptable to that enhancement.

DATA COMPILATION

The perennial question asked by lighting professionals and students concerning economic analysis is: "where does the input data come from?" They usually find little consolation in this author's answer: "Some of it comes from here, some of it comes from there. Some of it is based on your judgment, some of it comes from more experienced colleagues. Some of it you get from a phone call to a manufacturer or a utility company. And some of it simply doesn't exist."

Many papers end by spelling out what is unknown and calling for research on that topic. This one is different in that the research necessary does not, by and large, involve unknown information, but rather information that is unusable due to its dispersion. To make a MCSEALS-type analysis accessible to lighting professionals requires that the existing data necessary for input to the program (location specific fuel costs, impact of installed watts on first cost of various HVAC systems, lighting equipment costs, relevant tax information, etc.), be compiled into one readily available source. It is hoped that progress on this front will proceed at least as rapidly as software evolution.

ACKNOWLEDGMENTS

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