A RESPONSE-FACTOR METHOD FOR CALCULATING COUPLED HEAT AND MOISTURE TRANSFER IN BUILDINGS

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ABSTRACT

Analytical solutions for coupled diffusion of heat and moisture through a material are used to develop a generalisation of the wellknown 2 x 2 matrix method for describing heat flow only. A new 4 x 4 matrix is derived which relates temperatures, humidities, heat flows and moisture flows at one surface of a slab with those at the other. Multilayer slabs and surface boundary conditions, including moisture-impermeable surfaces, are easily handled by multiplication of matrices. The method is well suited for implementation in an existing program, CHEETAH, which was designed to calculate hourly temperatures and sensible air-conditioning loads in small buildings, using real weather data. The new version calculates hourly absolute humidities and latent air-conditioning loads in addition to temperatures and sensible loads, taking into account the effects of outdoor air temperature and humidity, solar radiation, ventilation, internal sources of heat and moisture, and the presence of adsorbing materials. This paper briefly outlines the theoretical basis of the method, and presents some results showing the effects of adsorbing materials on temperatures and humidities in a simple building.

INTRODUCTION

The simultaneous diffusion of heat and water vapour through a porous adsorbing material can have a significant effect on the thermal performance of buildings, where adsorbing materials such as organic fibrous insulations, furnishings, lightweight gypsum plaster, timber or brick may be used. It is therefore of interest to assess the effects of adsorbing materials on indoor air temperatures and indoor humidities (and hence sensible and latent air-conditioning loads if the building is conditioned, or comfort conditions if it is not). Interest in this topic is increasing, and numerous workers have recently investigated the effects of moisture transfer on building thermal performance (e.g. Isetti *et al.* 1988; Kerestecioglu *et al.* 1990; Kerestecioglu and Gu 1990; Shukuya and Saito 1990; Thomas and Burch 1990; Wong 1990).

The equations describing simultaneous heat and moisture transfer are coupled and non-linear, and numerical methods (e.g. finite-difference) are commonly used to solve them. While such methods can deal with problems of great complexity, they yield less insight into the processes involved than analytical treatments, and require significant computing resources. Analytical solutions, known as analogy solutions, can be obtained by exploiting the analogy between the coupled equations and those for heat transfer alone. They were first obtained by Henry (1939) and extended by Banks *et al.* (1970) and Close and Banks (1970). Close (1973) used Henry's and Banks' theory to examine the behaviour of an adsorbent slab, with the outer surface subject to sinusoidal variations in temperature and vapour concentration, but no heat or water vapour transfer at the inner. This was taken as an elementary representation of a wall in a non-air-conditioned building. While such an application yields fairly straightforward results for the response of the slab to sinusoidal variations or step changes in the known fluid state on either side, in a real building (even a very simple one), the situation is much more complicated. First, building components such as walls are usually not homogeneous, but rather can be considered as a succession of homogeneous slabs. Second, the indoor temperature and humidity are determined by several variables: outdoor air temperature, humidity and solar radiation acting on outdoor surfaces, solar heat gains through windows acting on indoor surfaces, infiltration and ventilation, and indoor sources of heat and moisture. This means that even in a non-air-conditioned building, the heat and moisture flow across an indoor surface is not zero in general. Third, a surface may be covered with a thin layer, such as paint, which has a much lower moisture diffusivity than the bulk material.

In this paper, a generalised analogy solution will be used to develop a new matrix formalism to describe coupled heat and moisture transfer in homogeneous adsorbing slabs. It is a generalisation of the well-known 2 x 2 matrix method used for heat transfer alone, and will be used as the basis of a response-factor method for calculating temperatures and humidities in buildings, taking into account the factors mentioned above. This method has been incorporated in an existing computer simulation program, CHEETAH (Delsante 1987), which until now has used the 2 x 2 matrix method to calculate temperatures and sensible air-conditioning loads in multi-zone buildings. Some examples of temperature and humidity response factors will be given, and a simple building constructed largely of a highly adsorbing material will be compared with the same building containing no adsorbing materials.

BASIC THEORY

In the discussion that follows, the adsorbing material will be called the adsorbent, and the air-water vapour mixture in which it is immersed will be called the fluid mixture. The treatment of diffusion within an adsorbent and transfer at the fluid-adsorbent boundary given here is taken from Close and Banks (1970). The following assumptions are made:

(a) The material through which the diffusion process occurs comprises a porous solid adsorbent containing a single-phase mixture of air and water vapour.

(b) For simplicity, a one-dimensional process is considered, so that adsorbent and fluid-mixture properties vary in one direction only.

(c) The fluid mixture and adsorbent are in equilibrium in planes normal to the direction of property variation.

(d) Diffusion of the vapour is governed by its concentration in the fluid mixture, w.

(e) Diffusion processes are governed by Fourier's and Fick's laws, and thermal-diffusion and diffusion-thermo coupling are neglected.

(f) The thermal conductivity and mass diffusivity of the adsorbent material are constant in time and space.

(g) The concentration of the material other than the vapour in the fluid mixture and the adsorbent, ρ and P respectively, are constant in time and space.

(h) The fluid-mixture pressure remains constant.

(i) Sorption is reversible; that is, it is without hysteresis.

(j) To allow w to be used as the potential for mass transfer, the state of the adsorbent is characterised by the state of air that would be in equilibrium with it.

Assuming that the fluid-phase vapour enthalpy $(\partial h/\partial w)_t$ is constant, an energy and mass balance yields, after some manipulation (see Close and Banks 1970)

$$\frac{\partial t}{\partial \theta} = D_t \frac{\partial^2 t}{\partial y^2} + D_w \frac{\lambda \alpha_h}{\mu_d \sigma} \frac{\partial^2 w}{\partial y^2}$$

$$\frac{\partial w}{\partial \theta} = -\frac{D_t}{\alpha_W} \frac{\partial^2 t}{\partial y^2} - \frac{D_w(\lambda \alpha_h + \sigma v)}{\mu_d \sigma \alpha_W} \frac{\partial^2 w}{\partial y^2}$$

where θ is time, y is distance along which diffusion occurs, t and w are temperature and vapour concentration of the fluid, D_t and D_w are thermal and vapour diffusivities;

$$\mu_d = P/\rho,$$

$$\alpha_W = -(\partial t/\partial w)_W,$$

$$\alpha_h = -(\partial t/\partial w)_h$$

$$\lambda = 1 - (\partial H/\partial W)_t/(\partial h/\partial w)_t,$$

$$\sigma = (\partial H/\partial t)_W/(\partial h/\partial t)_w,$$

$$v = -(\partial t/\partial W)_w,$$

W is the water content of the adsorbent, mass ratio to dry material, h the fluid enthalpy per unit mass of dry air, and H the adsorbent enthalpy, including adsorbed water, per unit mass of dry material.

These equations can be expressed as

$$\frac{\partial F_i^*}{\partial \theta} = D_i \frac{\partial^2 F_i^*}{\partial y^2}, \; i = 1,2 \qquad (1)$$

where

$$F_i^* = t + \bar{\alpha}_i^* w, \qquad (2)$$

and $\bar{\alpha}_i^*$ is an appropriate constant value of

$$\alpha_i^* = \frac{1}{2}\left[\frac{\lambda\alpha_h + \sigma v}{N} + \alpha_W - (-1)^i\left(\left(\frac{\lambda\alpha_h + \sigma v}{N} + \alpha_W\right)^2 - \frac{4\alpha_W \lambda\alpha_h}{N}\right)^{1/2}\right]$$

N, the diffusivity ratio, and D_i, the effective diffusivity, are given by

$$N = \mu_d \sigma D_t / D_w,$$

$$D_i = D_t \sigma/\bar{\gamma}_i^* = (D_w/\mu_d\sigma)\,(N\sigma/\bar{\gamma}_i^*)$$

where

$$\frac{\gamma_i^*}{\sigma} = 1 - \frac{N\alpha_i^* - \lambda\alpha_h}{\sigma v} = \frac{\alpha_W}{\alpha_W - \alpha_i^*}, \; i = 1,2.$$

If we assume a sinusoidal time dependence for all variables of the form $exp(j\Omega\theta)$, then equations (1) and (2) can be solved formally for t and w at any point within the material, giving

$$t(y) = \left[\bar{\alpha}_2^* A_1 \exp(j\beta_1 y) + \bar{\alpha}_2^* B_1 \exp(-j\beta_1 y) - \bar{\alpha}_1^* A_2 \exp(j\beta_2 y) - \bar{\alpha}_1^* B_2 \exp(-j\beta_2 y)\right] / \left(\bar{\alpha}_2^* - \bar{\alpha}_1^*\right) \qquad (3)$$

and

$$w(y) = \left[A_1 \exp(j\beta_1 y) + B_1 \exp(-j\beta_1 y) - A_2 \exp(j\beta_2 y) - B_2 \exp(-j\beta_2 y)\right] / \left(\bar{\alpha}_1^* - \bar{\alpha}_2^*\right) \qquad (4)$$

where $\beta_i = (1-j)(\Omega/(2D_i))^{1/2}$, $j = (-1)^{1/2}$, and where the A_i and B_i, $i = 1,2$, are to be found from the boundary conditions. Equations (3) and (4) are analytical solutions based on the assumption of constant properties. In reality of course the properties depend on the state of the material, but for a sinusoidal temperature variation with a period of 24 hours, and an amplitude appropriate for building applications, the analytical solution agrees very well with a finite-difference solution of the non-linear equations. For a step change in temperature at one surface the agreement is good if the constant properties are chosen appropriately.

DEVELOPMENT OF MATRIX FORMALISM

In the usual matrix formalism for heat transfer alone, the temperatures and heat flows at one surface of the slab are related to those at the other surface by a 2 x 2 matrix. Writing t_l, t_o for $t(l)$ and $t(0)$ respectively, and similarly Q_l and Q_o for the heat flows, we have (e.g. Carslaw and Jaeger, Section 3.7, 1959)

$$\begin{bmatrix} t_l \\ Q_l \end{bmatrix} = \begin{bmatrix} \cos(H) & -R\sin(H) \\ H\sin(H)/R & \cos(H) \end{bmatrix} \begin{bmatrix} t_o \\ Q_o \end{bmatrix} \qquad (5)$$

where $H = (-j\Omega CR)^{1/2}$, $R = l/k_t$ is the thermal resistance, $C = lk_t/D_t$ is the thermal capacitance, and k_t and D_t are the thermal conductivity and diffusivity respectively. Heat flow is taken to be positive in the direction of increasing y. The generalisation of (5) to include moisture transfer effects is obtained by first using (3) and (4) to calculate

$$Q(y) = -k_t\, \partial t(y)/\partial y,$$

$$M(y) = -k_m\, \partial w(y)/\partial y$$

at $y = 0$ and $y = l$, where $k_m = PD_w$ is the vapour conductivity, and $M(y)$ is the mass flow rate. For example, we find

$$M_o = (k_m/\alpha_{12})[t_l(\beta_2 cosec(\beta_2 l) - \beta_1 cosec(\beta_1 l)) + t_0(\beta_1 cot(\beta_1 l) - \beta_2 cot(\beta_2 l)) + w_l(\bar{\alpha}_2^* \beta_2 cosec(\beta_2 l) - \bar{\alpha}_1^* \beta_1 cosec(\beta_1 l)) + w_0(\bar{\alpha}_1^* \beta_1 cot(\beta_1 l) - \bar{\alpha}_2^* \beta_2 cot(\beta_2 l))\,], \qquad (6)$$

where $\alpha_{12} = \bar{\alpha}_1^* - \bar{\alpha}_2^*$. Similar expressions are found for Q_o, Q_l, and M_l. After some manipulation, they can be expressed as

$$\begin{bmatrix} t_l \\ w_l \\ Q_l \\ M_l \end{bmatrix} = [G] \begin{bmatrix} t_o \\ w_o \\ Q_o \\ M_o \end{bmatrix} \quad (7)$$

where *[G]* is a 4 x 4 matrix that generalises (5). The elements of *[G]* are rather complicated; they are available from the author, and will be published elsewhere. Again, heat and mass flow are taken to be positive in the direction of increasing y. Clearly, if we have a building element consisting of a series of homogeneous slabs, then (7) still applies except that *[G]* is now the product of the individual slab matrices, and the subscripts l and o refer to the outer surfaces of the multilayer slab.

Boundary conditions can be treated by including in *[G]* a matrix, say *[S]*, representing heat and mass transfer from a surface to the air. The elements of this matrix are $S_{ii} = 1$, $S_{12} = -1/h_t$, $S34 = -1/h_m$, and all other elements zero, where h_t is the surface heat transfer coefficient and hm the surface mass transfer coefficient. For unrestricted mass transfer, h_m can be obtained from the Lewis relation, $h_m = h_t / c_p$, where c_p is the specific heat of air. However, if a relatively impermeable film is present, it can be taken into account by decreasing h_m appropriately.

CALCULATION OF TRANSIENT RESPONSE

The computer simulation program CHEETAH (Delsante 1987) runs on small computers and calculates hourly temperatures or sensible air-conditioning loads in up to ten zones of a building over any period up to one year, using real hourly weather data. The theoretical basis of the program is given in Walsh and Delsante (1983). This section describes how this theory can be extended to take into account coupled heat and mass diffusion in building materials.

The solutions obtained in the previous section are for a particular frequency. In the standard version of CHEETAH, a heat balance on the building is done in the frequency domain. In the new version, a simultaneous heat and mass balance is done. The heat balance takes the form

$$\sum_i A_i Q_{oi} - Q + \frac{\rho a V c_p (t_o - t_e)}{3600} + j\Omega c_p \rho V t_o = 0 \quad (8)$$

where A_i is the area of surface i, Q_{oi} is the heat flux entering surface i, Q represents internal heat sources or sinks (e.g. lights, people, air-conditioning plant), ρ is the density of air, a the air change rate in air changes per hour, V the volume of the space, c_p the specific heat of air, t_o and t_e the indoor and outdoor air temperatures, and Ω the frequency. The effects of solar radiation transmitted through windows can be accommodated in Q, but the details are too complex to include here. Similarly, the mass balance takes the form

$$\sum_i A_i M_{oi} - M + \frac{\rho a V (w_o - w_e)}{3600} + j\Omega \rho V w_o = 0 \quad (9)$$

where M_{oi} is the mass flow rate of vapour entering surface i, M represents internal sources or sinks, and w_o and w_e are the indoor and outdoor absolute humidities. Note that all variables are amplitudes (i.e. their time dependence is $exp(j\Omega\theta)$). The matrix *[G]* defined by (7) can be used to obtain expressions for Q_{oi} and M_{oi} in terms of t_e, t_o, w_e, and w_o, for each building element. Equations (8) and (9) can then be solved for the unknowns t_o and w_o. This brief description has omitted some details that need to be taken into account in practice (e.g. the treatment of solar radiation and the extension to multi-zone buildings), but includes all the important steps. The complete analysis will be reported elsewhere.

The solutions obtained for t_o and w_o give the frequency response of the room air temperature and humidity to the combined effects of outdoor environmental drivers (outdoor air temperature and humidity, various sol-air temperatures, and solar radiation), and indoor sources of heat and moisture. Each term in the solution gives the frequency response to a particular driver. Standard methods of linear system theory (e.g. Papoulis 1962) are used to convert these responses to transient responses (Walsh and Delsante 1983), usually responses to a unit triangular, ramp or square pulse. It should be noted that the transient responses so calculated are whole-building (or whole-zone) responses, that is, they include the effects of all elements comprising the building.

Outdoor environmental variables are represented by a time series of ramp or triangular pulses, while indoor sources of heat or moisture are represented by a series of square pulses. The response of the indoor air temperature or humidity is obtained by summing the responses to each pulse.

If the building is to be air-conditioned, the procedure for calculating sensible and latent loads is as follows. At each time, the indoor temperature and humidity that would be attained at the end of the time step, in the absence of heat or moisture flow from the plant, are calculated. If the results are within the deadbands, no further calculation is needed. If not, the (constant) heat flow and moisture flow rates required to bring the temperature and humidity to their set-points at the end of the hour are calculated from the known response of the building to a unit pulse of heat or moisture, and the difference between the set-points and the floating values first calculated. The sensible and latent loads so calculated are room loads; CHEETAH does not attempt to simulate the actual loads on an air-conditioning plant.

SOME EXAMPLES

Consider a simple rectangular single-zone building with a 10 m^2 window facing the equator, protected by a 1 m roof overhang. The building (of dimensions 8 x 10 x 3 m) consists of a carpeted suspended timber floor over a well ventilated subfloor space, with the walls and (flat) roof constructed from lightweight panels containing either 50 mm of a highly adsorbing organic insulating material (such as wool) or 50 mm of a non-adsorbing insulating material of the same thermal resistance. The infiltration rate is 0.5 air changes per hour. Two extreme conditions for moisture transfer at the indoor surface will be considered: free transfer, or obstructed transfer (the latter corresponding to an impermeable film or thin panel). Moisture transfer at the outdoor surface is assumed to occur freely. For this building, 10 temperature response factors and 10 humidity response factors would be calculated, these being the response of indoor air temperature or humidity to outdoor air temperature, outdoor humidity, north, east, south, west sol-air temperatures, roof sol-air temperature, solar heat gain, indoor heat flow, and indoor moisture flow. The response factors are calculated at each hour for the first six hours, then at longer intervals up to 128 days.

In the discussion that follows, 'humidity' means absolute humidity. Figure 1 gives the response of indoor air temperature to a ramp increase (over one hour) of 1.0 g/kg in the outdoor air humidity, for the building containing the adsorbing material, and for the two conditions of indoor moisture transfer (free or obstructed). As would be expected, the temperature response is positive because an increase in outdoor humidity causes moisture to be adsorbed in the structure, which releases heat. The differences in response for the two indoor surface boundary conditions are not very great, because the change in the driver is on the outdoor side, where moisture transfer is taken to be free in both cases. Note that the temperature response takes a considerable time to reduce to zero, reflecting the fact that most of the building contains an adsorbing material. This emphasises that the response factors are specific to the building.

Figure 2 gives the response of indoor air humidity to an indoor moisture source of 0.1 g/s applied for one hour, again for the two conditions of indoor surface moisture transfer. Here the response for the obstructed moisture transfer case is larger and takes longer to reach zero than the response for the free transfer case. This is because with an impermeable film at the indoor surface, the only way the additional moisture can be removed is via infiltration, whereas for the free transfer case, the additional moisture is also readily adsorbed into the structure.

Figure 3 compares indoor temperatures for the adsorbing building (with free moisture transfer at the indoor surface) with the non-adsorbing building over three hot days in Alice Springs, Australia, which has a hot-dry inland climate (note that the indoor temperatures are higher than outdoors because of the intense solar radiation). The temperatures for the case of obstructed moisture transfer are not shown because they are very similar to those for free transfer, as would be expected from Fig. 1. There is a substantial difference between the two buildings, amounting to a reduction in maximum temperature of over 5 K on the third day; on the other hand, the minimum temperature for the adsorbing building is higher. It is interesting to note that the adsorbing insulation is similar to that of a dense material in terms of its effect on the indoor temperature amplitude, but quite different in terms of its effect on the phase difference between indoor and outdoor temperature.

Figure 4 compares indoor humidities. The humidity for the non-adsorbing insulaion is very close to the relatively constant outdoor humidity, the differences being attributable to the low infiltration rate. In contrast, the humidity for the adsorbing insulation varies considerably, more or less in phase with the outdoor temperature, but out of phase with the outdoor humidity.

While the examples chosen may be somewhat extreme, they highlight the ability of the method to handle the variety of situations that are encountered in buildings.

CONCLUSION

The use of analytical solutions to the coupled heat and mass transfer equations has enabled an existing building thermal performance simulation program to be modified to calculate humidities as well as temperatures in buildings, and sensible and latent air-conditioning loads. This has resulted in considerable gains in execution speed compared to numerical solutions, since hourly time steps can be used (compared to time steps of the order of 10 seconds required for accurate numerical results), and there are no lumping errors involved. The penalty that must be paid is that constant properties must be used, whereas with a numerical solution the properties can be recalculated at every time step (but with much slower execution speed).

Apart from the calculation of indoor air temperatures and humidities, the method has other applications. One is to calculate the water content at various points within a building element. This may be of particular interest in tropical areas, from the viewpoint of material degradation. If, for example, the water content at the mid-point of one of the materials comprising a wall is required, the matrix representing the material can be separated into a product of two matrices, each representing a slab of half the thickness. The plane at the mid-point of the material can be treated as a zone in cheetah, and the temperature and humidity calculated there.

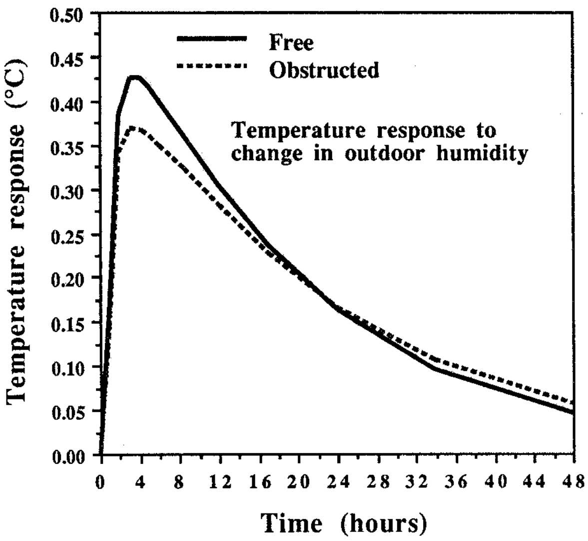

Fig. 1. Indoor air temperature response for a building containing absorbing insulation, for free or obstructed moisture transfer (impermeable film) at the indoor surface. The response is to a linear increase in outdoor humidity of 1.0 g/kg over one hour.

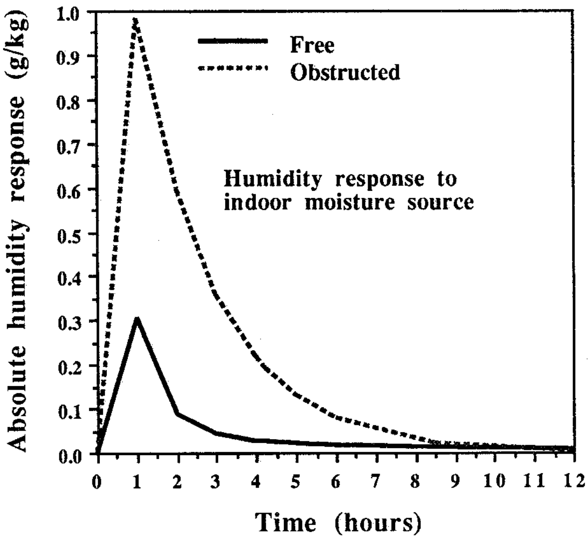

Fig. 2. Indoor air humidity response for a building containing absorbing insulation, for free or obstructed moisture transfer (impermeable film) at the indoor surface. The response is to a constant indoor moisture source of 0.1 g/kg over one hour.

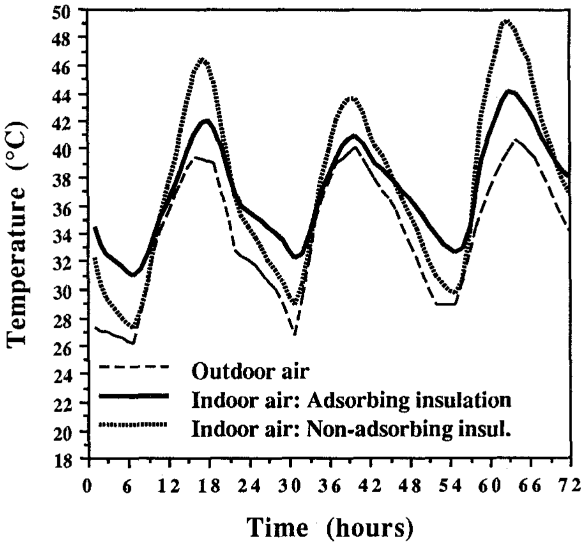

Fig. 3. Comparison of temperatures inside a simple building in Alice Springs, Australia, over three hot days (6–8 January) for a building containing absorbing or non-absorbing insulation.

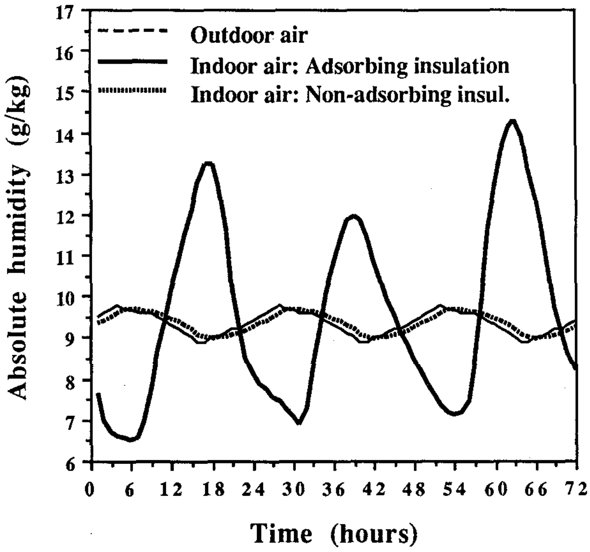

Fig. 4. As for Fig. 3, but for absolute humidities.

Another application is to explicitly calculate indoor surface temperatures and compare them with the dewpoint of the indoor air. This would then enable an hourly assessment of condensation risk to be obtained.

The development and implementation of the method into a viable computer package is still in its early stages. While some simple validation work has been done on the theory using isolated slabs, further work using test cells or buildings is required.

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