



The Determination of Vapour and Liquid Transport Coefficients as Input to Combined Heat and Mass Transfer Models

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The prediction of the effects of moisture in buildings is now a well established part of the building design process. A significant international effort is taking place to develop combined heat and mass transfer models which will enable the thermal and hygric performance of building envelopes to be more accurately assessed. However, the universal solution of the defining equations requires the separation of the total moisture flow into its liquid and vapour components for any specified set of humidity conditions. Up till now this has not been possible and partial model solutions have required the premise that liquid water flow in porous building materials only becomes significant at humidities approaching 100%. Such an assumption is simplistic and clearly at variance with the body of experimental evidence. This paper presents an analytical and experimental procedure which allows the transport parameters defining liquid and vapour flow to be described for any given material as functions of relative humidity. Such information allows for a complete model solution without recourse to questionable assumptions regarding the initiation of capillary condensation.

1 Introduction

An understanding of the phenomenon of moisture transfer through porous materials is of great importance if more accurate combined heat and mass transfer models are to be developed. Models of this nature are an essential requirement if the thermal and moisture behaviour of building structures is to be predicted accurately. An ability to estimate such effects is particularly relevant at a time when the prevention of interstitial condensation and the accumulation of moisture within the fabric of buildings is being recognised as

a crucial consideration during the design process. The importance of this subject area is recognised world-wide as demonstrated by the formation of Annex 24 of the International Energy Agency which aims to co-ordinate and consolidate activities in this field.

Glaser in 1959 (Glaser 1959) produced one of the earliest procedures for predicting such effects, a method which is still used widely today by designers and building technologists in the UK and other European countries. Its popularity is primarily due to its simplicity, being a steady-state method based on the easy-to-understand Fick's Law theory. Such models were produced originally for use as hand calculation techniques when computers were not widely available. As a result of this they were based upon a simplified representation of the actual processes taking place.

Steady-state models of the type developed by Glaser assume that moisture transfer occurs in vapour form only and neglect the importance of liquid water transfer. Condensation is supposed to be exclusively associated with a structural temperature at or below the dew-point temperature,

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and in most cases only one driving potential, the vapour pressure, is used in the calculation of the transfer rate. The fact that such models are steady-state further undermines their reliability, particularly since the time constants associated with moisture movement are known to be large. Because of these many deficiencies considerable attention is now being devoted to constructing more sophisticated transient models.

While many such models have been developed over recent years, they do not generally evaluate the interactions between vapour, liquid water and the solid matrix using a rigorous theoretical approach. Instead these interactions are described using empirical material parameters determined by experiment. However, even when such an empirical approach is used, the formulation of the model equations is difficult, in that the solution of the energy conservation equation requires information on the individual vapour and liquid water flow rates through the material. This is required to enable the convection energy transfer rates and condensation/evaporation rates to be obtained. The experimental separation of vapour and liquid flow is not readily achieved as these processes are coupled and conventional techniques allow only the total moisture transfer to be measured. If these coefficients cannot be measured or determined theoretically, then the transient models become indeterminate unless appropriate assumptions are made.

Luikov (Luikov 1966) in his work included a phase conversion factor to describe the ratio of the vapour flow rate to the total moisture diffusion flow, without providing a method for its determination. This concept of a phase conversion factor is applied by many other researchers (Kohonen 1984), while others directly use the concepts of vapour and liquid water transfer coefficients (Pedersen 1990). In all cases no method of measurement is correctly defined. In fact, most models either directly or indirectly assume that moisture transfer takes place as vapour alone for relative humidities below about 98% relative humidity, equivalent to assuming that Luikov's phase conversion factor is effectively unity. This rather simplistic approach undermines the whole conceptual basis of such models and makes them unreliable.

In this paper a combined mathematical and experimental approach is presented which can be used to predict the individual vapour and liquid transfer rates which make up the total moisture diffusion flow. This approach involves the use of straightforward test procedures which could be performed in any laboratory which currently carries out standard vapour permeability measurements.

2 Theoretical Analysis

The moisture flow rate through a sample of material, j , can be divided into two components:

$$j = j_v + j_l \quad (1)$$

where j_v : vapour flow rate, kg/m²s
 j_l : liquid water flow rate, kg/m²s

If the total pressure field is uniform, and the effect of thermal diffusion is neglected (Galbraith 1992), then j_v and j_l can be expressed as:

$$j_v = -D_v \nabla \rho_v \quad (2)$$

$$j_l = -D_l \nabla p_l \quad (3)$$

In the above equation it has been assumed that the driving potential for vapour diffusion can be taken as the density or concentration gradient, $\nabla \rho_v$, while liquid transfer is assumed to be functionally dependant on the gradient of pore liquid pressure, ∇p_l . The coefficients D_v and D_l are experimental coefficients describing the liquid and vapour transfer rates.

If the vapour is assumed to behave as a perfect gas, then the vapour transfer rate j_v of equation (1) becomes:

$$j_v = -D_v \nabla \rho_v = -\frac{D_v}{R_v T} \nabla p_v + \frac{p_v D_v}{R_v T^2} \nabla T \quad (4)$$

where R_v : gas constant for water vapour, J/kgK
 T : temperature, K
 ΔT : temperature gradient, K/m

The relationship between the pore liquid pressure, p_l , and the vapour pressure, p_v , can be given using the following form of the 'Kelvin' equation:

$$p_l = P + R_v T \rho_l \ln \left[\frac{p_v}{p_s} \right] \quad (5)$$

where P : total pressure, N/m²
 p_s : saturation vapour pressure, N/m²

Equation (5) above can be differentiated to give the gradient of p_l as :

$$\nabla p_l = R_v \rho_l \left(\frac{T}{p_v} \nabla p_v - \frac{T}{p_s} \nabla p_s + [\ln \phi] \nabla T \right) \quad (6)$$

where ϕ : relative humidity, p_v/p_s

From the Clausius Clapeyron equation :

$$\frac{dp_s}{dT} = \frac{L}{T(\xi_v - \xi_l)} \quad (7)$$

where ξ_v : specific volume of vapour, m³/kg
 ξ_l : specific volume of liquid, m³/kg
 L : latent heat of vaporization, J/kg

the following expression is obtained for the saturation vapour pressure term in equation (6):

$$\frac{T}{p_s} \nabla p_s = \frac{L}{p_s(\xi_v - \xi_l)} \nabla T \quad (8)$$

Neglecting ξ_l as small compared to ξ_v and applying the perfect gas law, this becomes:

$$\frac{T}{p_s} \nabla p_s = \frac{L}{R_v T} \nabla T \quad (9)$$

Substituting this expression back into equation (6) gives the gradient of p_l as:

$$\nabla p_l = \frac{R_v T p_l}{p_v} \nabla p_v + R_v \rho_l \left[\ln \phi - \frac{L}{R_v T} \right] \nabla T \quad (10)$$

Thus equation (1) can now be expressed as :

$$j = j_v + j_l$$

$$= - \left(\frac{D_v}{R_v T} + \frac{D_l R_v T p_l}{p_v} \right) \nabla p_v - \left(D_l R_v \rho_l \left[\ln \phi - \frac{L}{R_v T} \right] - \frac{p_v D_v}{R_v T^2} \right) \nabla T$$

The above equation simplifies under isothermal conditions to:

$$j = - \left(\frac{D_v}{R_v T} + \frac{D_l R_v T p_l}{p_v} \right) \nabla p_v \quad (11)$$

The moisture flux can also be expressed in terms of permeability as:

$$j = -\mu \nabla p_v \quad (12)$$

where μ : the 'differential' permeability, s

Equations (11) and (12) give:

$$\mu = \frac{D_v}{R_v T} + \frac{D_l R_v T p_l}{p_v}$$

$$= \frac{D_v}{R_v T} + \frac{D_l R_v T p_l}{\phi p_s} \quad (13)$$

The coefficient, D_v , differs from that for vapour transfer in bulk air, D , due to the space confinement for transfer and the complex structure of the capillaries. D is a function of temperature and total pressure and can therefore be considered constant for isothermal conditions under a constant total pressure (Sherwood and Pigford 1958). D_v and D can be related to each other under dry conditions by the tortuosity and the material porosity as (Philip and De Vries 1957):

$$D_v = D \nu \epsilon \quad (14)$$

where ν : correction factor depending on the tortuosity of the material
 ϵ : the material porosity, m³ void space/m³ of material

For a certain volume of material, as the relative humidity increases from the dry condition, so condensation will take place within the pores. This will reduce the void space available for vapour transfer in a complex manner related to the capillary structure and air relative humidity.

If a material volume of 1m³ is considered, then there will exist a volume V_v within the void space which is free of liquid water. It is reasonable to assume that this volume V_v has a tortuosity which will be unchanged having the value ν as given in equation (14). Outwith V_v , the vapour transfer is effectively zero due to the blockage caused by the presence of liquid water. If an effective vapour transfer area factor, τ , is introduced, equation (14) can be rewritten for conditions other than the dry condition as:

$$D_v = \tau D \nu \epsilon \quad (15)$$

where $\tau = V_v / (V_v + V_l)$
and V_l : the volume content of liquid
 $(V_v + V_l)$: the porosity ϵ

$D \nu \epsilon$ can be considered constant for a given material. Letting $D'_v = D \nu \epsilon$, then:

$$D_v = D'_v \tau \quad (16)$$

Given that the material porosity is ϵ and that the sorption isotherm moisture content for any given condition is U (kg moisture/kg dry material), then the effective vapour transfer area factor can be evaluated approximately as (Galbraith 1992):

$$\tau \cong 1 - \frac{\rho_o U}{\rho_l \epsilon} \quad (17)$$

Based on previous experimental work in this field (Galbraith and McLean 1990), the following functional relationship between the liquid transfer coefficient and the relative humidity is proposed by the authors:

$$D_l = D'_l \phi^n$$

The second term of equation (13) then becomes:

$$\frac{D_l R_v T \rho_l}{\phi p_s} = \frac{D'_l R_v T \rho_l}{p_s} \phi^{n-1} = \frac{D'_l R_v T \rho_l}{p_s} \phi^m \quad (18)$$

where the n or m coefficients are determined experimentally

Equation (13) combined with (16) and (18) can then be written as:

$$\mu = \frac{D'_v}{R_v T} \tau + \frac{D'_l R_v T \rho_l}{p_s} \phi^m \quad (19)$$

$$\text{Letting } D'_v = \frac{D_v}{R_v T} \text{ and } D'_l = \frac{D'_l R_v T \rho_l}{p_s},$$

equations (12) and (19) become:

$$j = -\left(D'_v \tau + D'_l \phi^m\right) \nabla p_v \quad (20)$$

where

$$\mu = D'_v \tau + D'_l \phi^m = D'_v \left(1 - \frac{\rho_o U}{\rho_l \epsilon}\right) + D'_l \phi^m \quad (21)$$

In the above equations the first transfer coefficient, $D'_v \tau$, relates to the vapour transfer while the second coefficient, $D'_l \phi^m$, relates to the liquid water transfer. The total moisture flux is thus separated

into its two component parts and the values of D'_v , D'_l and m can be determined experimentally from standard permeability cup tests carried out over a range of relative humidity conditions.

4 Experimental Investigation

The applicability of this approach has been demonstrated by carrying out measurements on several common building materials, namely plywood, expanded polystyrene and plasterboard. Details of these materials are to be found in Table 1 below.

Test Material	Density of material matrix ρ_o (kg/m ³)	Measured Porosity ϵ (m ³ /m ³)	Sorption Isotherm Curve Fit Parameters {equation (25)}		
			U_h	A	B
Plywood (12mm)	56	0.79	0.398	0.309	1.66
Plasterboard (9mm)	881	0.72	0.26	0.000232	4.02
Polystyrene (25mm)	25	0.93	0.051	0.00131	2.1

Table 1 Details of Test Materials

The experimental procedure used followed the recommendations of British Standard 4370, Part 2, 1973, in which the material sample to be tested is sealed into the mouth of an impermeable cup containing a vapour pressure regulator.

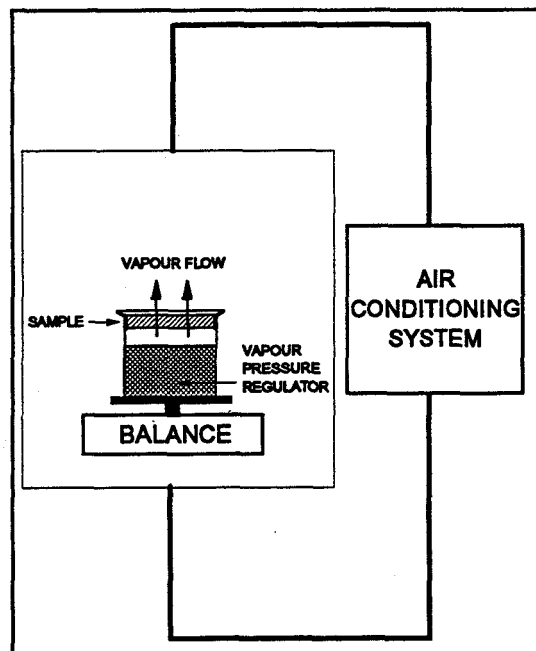


Figure 1 Sketch of Test Apparatus

The cup is then positioned in a test chamber in which specific environmental conditions are maintained. In this way a constant vapour pressure difference is generated across the sample, and after equilibrium is attained, the vapour flow can be determined from the steady change in cup weight (see Figure 1).

The cup used was a 65mm diameter glass beaker. The test specimens were cut to fit tightly into its mouth and were then carefully sealed using melted wax.

The vapour pressure regulators used within the cups are given in Table 2 shown below.

Vapour Pressure Regulator	Relative Humidity Generated at 20°C
CaCl ₂	0%
ZnSO ₄	90%
NH ₄ H ₂ PO ₄	93%
H ₂ O	100%

Table 2 Vapour Pressure Regulators & their Relative Humidities

Conditions within the test chamber were maintained at a temperature of 20°C and a relative humidity of either 60 or 80 % as indicated in Table 3. Control fluctuations in temperature and relative humidity were in all cases limited to ±0.4°C and ±2% RH respectively.

The tests were carried out using four relative humidity ranges, with six specimens of each material being used for each range. The weight change with time was measured daily for each cup until equilibrium was attained and regression analysis could be performed on the data. The average permeability $\bar{\mu}$ for the experiment was then calculated from:

$$j = \frac{\bar{\mu} (p_{vi} - p_{vo})}{l} \quad (22)$$

where l : sample thickness
 p_{vi} : vapour pressure inside cup
 p_{vo} : vapour pressure outside cup

This average permeability is related to the differential permeability by :

$$\bar{\mu} = \frac{1}{\Delta\phi} \int_{\phi_o}^{\phi_i} \mu d\phi \quad (23)$$

where $\phi = \frac{p_v}{p_s}$; $\mu = \mu(\phi)$ and $\Delta\phi = \phi_i - \phi_o$

It has been shown by the authors elsewhere (Galbraith and Mclean, 1990) that by carrying out at least four experiments covering a range of relative humidities, an accurate relationship between differential permeability and relative humidity can be determined. Equation (21) shows that the differential permeability can also be expressed in terms of the vapour and liquid transfer coefficients such that :

$$\bar{\mu} = \frac{1}{\Delta\phi} \int_{\phi_o}^{\phi_i} D_v^* \left(1 - \frac{\rho_o U}{\rho_i \epsilon} \right) d\phi + \frac{1}{\Delta\phi} \int_{\phi_o}^{\phi_i} D_l^* \phi^m d\phi$$

giving

$$\bar{\mu} = D_v^* \left(1 - \frac{\rho_o \bar{U}}{\rho_i \epsilon} \right) + \frac{D_l^*}{\Delta\phi [m+1]} \{ \phi_i^{m+1} - \phi_o^{m+1} \} \quad (24)$$

where $\bar{U} = \frac{U(\phi_o) + U(\phi_i)}{2}$

This equation can be used to determine the values D_v^* and D_l^* and exponent m for any material if its porosity and sorption isotherm are known.

Porosity data for the materials investigated was obtained using a Helium Gas Porosimeter and is given in Table 1. Material sorption isotherms were measured by carrying out absorption and desorption measurements on small material samples, and fitting the results with a single mean correlation. The relationship applied for this correlation was identical to that suggested by Hansen (Hansen 1986), namely:

$$U = U_h \left(1 - \frac{\ln \phi}{A} \right)^{\frac{1}{B}} \quad (25)$$

where U : the moisture content, kg/kg
 U_h , A and B are experimental constants

The values of the constants U_h , A and B obtained for each material are given in Table 1.

4 Experimental Results

Mean values of permeability were calculated from the results of the six sample tests carried out for each of the test conditions and are given for each material in Table 3.

The measured permeability data was used along with the sorption and porosity data to calculate the

vapour and liquid transfer coefficients using equation (24).

Material	% RH Range	Measured (seconds) $\bar{\mu} \times 10^{12}$	Predicted	
			(seconds) $\bar{\mu} \times 10^{12}$	% Difference
Plywood	0-60	1.04	1.04	0.0
	80-60	2.97	2.97	0.0
	93-60	5.34	5.34	0.0
	100-60	8.82	8.82	0.0
Plaster Board	0-60	21.3	21.1	-0.9
	90-60	23.3	23.3	0.0
	100-60	26.3	26.3	-0.4
	100-80	30.4	30.4	0.0
Expanded Polystyrene	0-60	6.4	6.44	0.6
	93-60	6.7	6.51	-2.8
	100-60	6.5	6.52	0.3
	100-80	6.4	6.53	2.0

Table 3 Measured and Predicted Values of Permeability

The determination of these coefficients was carried out using a 'best-fit' regression analysis. The coefficients obtained in this way are given in Table 4 below.

Test Material	Transfer Coefficients		
	$D_v^* \times 10^{12}$	$D_l^* \times 10^{12}$	m
Plywood	1.15	17.8	8.96
Plasterboard	22.2	23.3	9.49
Expanded Polystyrene	6.39	0.16	1

Table 4 Vapour and Liquid Coefficients for the Test Materials

It is clear from this table that the Expanded Polystyrene behaves in a different manner to the other two materials. The value of 1.0 obtained for the coefficient m indicates that this material could be defined as essentially non-hygroscopic in that the permeability does not increase by a measurable amount with relative humidity.

In order to ascertain the suitability of this approach, the parameters in Table 4 have been used to predict the average permeability for conditions corresponding to those of the original tests. The resulting predicted values are given for comparison in Table 3. It is clear from these results that the model equation chosen is in close agreement with the experimental results obtained, the maximum difference being 2.8% for the expanded polystyrene.

The relative merits of this approach can be further explored by investigating in more detail the results for the hygroscopic plywood.

The data in Table 4 enables the vapour and liquid water transfer coefficients to be evaluated as:

VAPOUR TRANSFER :
 $D_v^* \tau = 1.15 \times 10^{-12} (1 - 0.07103U)$

LIQUID TRANSFER :
 $D_l^* = 1.78 \times 10^{-11} \phi^{8.96}$

The above equations are presented in graphical form as Figure 2.

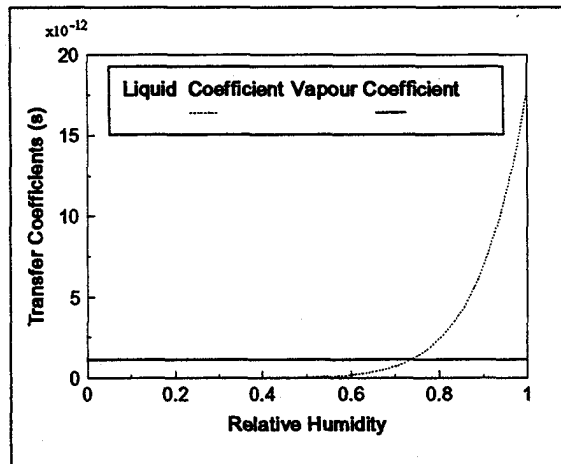


Figure 2 Plotted Vapour and Liquid Coefficients for plywood

From this it can be seen that the liquid transfer coefficient begins at zero as no liquid is present under dry conditions and all of the void space is occupied by the gaseous phase. In the low relative humidity range, below 60-70%, the liquid water transfer increases rather slowly. In the high relative humidity zone the water transfer rate increases sharply as a result of the occurrence of capillary condensation. For this material it is clear that the vapour coefficient can be considered, with little error, to be constant over the complete range of relative humidity. This is due to the fact that even at 100% relative humidity less than 3% of the total void volume is filled with liquid water.

The vapour and liquid transfer coefficients can be combined to give the total differential permeability curve for the plywood.

In 1990 Galbraith and McLean (Galbraith and McLean 1990) proposed the following exponential

form of curve fit for permeability values over the whole relative humidity range :

$$\bar{\mu} = A + B \exp(C\phi) \quad (26)$$

In this equation A , B and C are experimental coefficients which were determined for the same plywood as $A = 1.04 \times 10^{-12}$, $B = 5.505 \times 10^{-16}$ and $C = 0.105$.

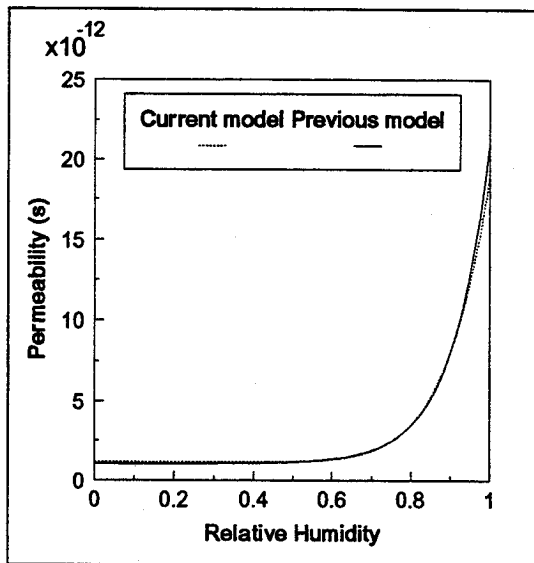


Figure 3 Comparison of Data

The differential permeability curve for the plywood has been plotted in Figure 3 for comparison using both the curve fit proposed in this work along with the curve obtained in 1990 using equation (26). It is clear from this that the model developed in this paper gives a permeability curve in close agreement with that obtained from previously established theory.

5 Conclusion

One of the main difficulties in formulating a complete model for predicting the process of combined heat and mass transfer through building structures has been the determination of vapour and liquid transfer coefficients. Assumptions are made by many researchers to overcome this difficulty, the most common one being that liquid transfer does not occur until relative humidities above 98% are reached. The results of previous research by the authors and this current investigation clearly demonstrate the importance of liquid flow at relative humidities as low as 60%.

The results presented here provide a means of separating the data from standard permeability tests into expressions for the vapour and liquid transfer coefficients. These coefficients are an essential prerequisite to enable complete model solution without recourse to unsubstantiated assumptions. This procedure is easy to carry out in any moisture investigation laboratory and provides information which up till now has not been available.

Reference List

- British Standard 4270, Part 2, 'Methods of Test for Rigid Cellular Materials', 1973.
- Galbraith G H, McLean R C, 'Interstitial Condensation and the Vapour Permeability of Building Materials', *Energy in Buildings*, 14, 1990.
- Galbraith G H, 'Heat and Mass Transfer Within Porous Building Materials', PhD Thesis, *Strathclyde University*, 1992.
- Glaser H, 'Graphical Method for Investigation of Diffusional Process', *Kaltetechnik*, 11, 1959
- Hansen K K, 'Sorption Isotherms, A Catalogue', Technical Report 162/86, Building Materials Laboratory, *The Technical University of Denmark*, 1986.
- Kohonen R, 'A Method to Analyze the Transient Behaviour of Building Materials and Components', PhD Thesis, *Helsinki University of Technology*, Finland, 1984
- Luikov A V, 'Heat and Mass Transfer in Capillary-porous Bodies', *Pergamon Press*, 1966.
- Pederson C R, 'Combined Heat and Mass Transfer in Building Construction', Report No. 214, *Technical University of Denmark*, 1990.
- Philip J R, De Vries D A, 'Moisture Movement in Porous Materials under Temperature Gradients', *Transactions of the American Geophysical Union*, Vol. 38, No. 2, 1957.
- Sherwood T K, Pigford R L, 'Absorption and Extraction', *McGraw-Hill Book Company*, New York, 1958.