A SIMPLE METHOD TO INCORPORATE THERMAL BRIDGE EFFECTS INTO DYNAMIC HEAT LOAD CALCULATION PROGRAMS

Akihiro Nagata

Faculty of Urban Environmental Sciences
Tokyo Metropolitan University
Tokyo 192-0397, Japan

ABSTRACT

The aim of this study is to develop a simple method to predict heat flows and minimum surface temperatures in dynamic heat load calculation programs. A composition of two linear flow components is assumed instead of an element including a thermal bridge. A fractional area of the thermal bridge part is determined by the maximum and the average thermal transmittances. We also discuss a method to constitute a one-dimensional imaginary component of the thermal bridge.

INTRODUCTION

There are many studies to predict the steady-state performance of thermal bridges, while few studies addressed the unsteady-state performance (Burch, D.M., Seem, J.E., Walton, G.N., and Licitra, B.A., 1992). Recently, however, thermal bridge effects have become incorporated into dynamic heat load calculation programs (for example, EnergyPlus, 2004). Although they address only whole heat flow, the minimum surface temperature is also important issue in considering thermal bridge effects. In this paper, we propose a simple method to consider both heat flows and minimum surface temperatures.

Thermal bridge gives rise to complex two or three dimensional heat flows, which is calculated by a two or three dimensional finite difference method (FDM) in the Laplace transform domain. A composition of two linear flow components is assumed instead of an element including a thermal bridge. One is a thermal bridge part of the element and the other is a general part. The maximum and minimum of the distribution of the thermal transmittance correspond to the thermal transmittance of the thermal bridge component and that of the general component, respectively. The fractional area of the thermal bridge component is determined to preserve the whole thermal transmittance. This is the fundamental idea of the proposing method to consider both heat flows and minimum surface temperatures.

The argument in the above steady state is extended to an unsteady state. As for the transmittance and admittance transfer functions, the similar relations are supposed. If a heat load calculation program is based on the response factor method, what is necessary is only the Laplace transform inversion numerically. If, however, a program is based on FDM etc., it is necessary to constitute a one dimensional imaginary component. Therefore, a method to constitute the 3-layer model is also proposed. This method is simple and deterministic unlike the iterative method adopted by EnergyPlus. It is applied to a flat-surface thermal bridge and gives a reasonably good approximation.

THERMAL TRANSMITTANCE

In ISO 14683, the calculation of the transmission heat transfer coefficient L includes the contribution due to thermal bridges, according to Equation (1).

$$L = \sum_i A_i U_i + \sum_k \Psi_k l_k + \sum_j \chi_j \qquad (1)$$

where,

U_i : the thermal transmittance of element i [W/(m²K)]

A_i : the area of element i [m²]

Ψ_k : the linear thermal transmittance of linear thermal bridge k [W/(mK)]

l_k : the length of linear thermal bridge k [m]

χ_j : the point thermal transmittance of the point thermal bridge j [W/K]

Default values of linear thermal transmittance Ψ for various types of thermal bridges are also presented in ISO 14683.

In Japan, the thermal transmittance of a component including thermal bridges is first estimated by parallel-path method and the effect of the thermal bridge is corrected by multiplying the average thermal transmittance $\overline{U}$ by the "thermal bridge factor" β to meet the actual thermal transmittance U. Thus,

$$U = \beta \overline{U} \qquad (2)$$

and default values of thermal bridge factor β are given in "Exposition of the Energy Consevation Standard for Residential Buildings".

Although these two approaches treat only whole heat flow rate, the minimum surface teperature is also important issue in considering the thermal bridge effect. In this paper, we propose a simple method to consider both heat flows and minimum surface temperatures.

A composition of two linear flow components is assumed instead of an element including a thermal bridge. One component is a thermal bridge part of the element with a fractional area f and the other component is a general part with a fractional area $1-f$. The maximum and minimum of the distribution of the thermal transmittance correspond to the thermal transmittance of the thermal bridge component U_b and that of the general component U_g, respectively. The thermal transmittance U is then obtained by the following equation.

$$U = (1-f)U_g + fU_b \qquad (3)$$

Actually, U, U_b and U_g are calculated numerically by the finite difference method or the finite element method, and these will determine the fractional area f. Thus,

$$f = \frac{U - U_g}{U_b - U_g} \qquad (4)$$

The thermal transmittance disributions are appeared on both interior and exterior surfaces. The fractional area should be determined by the distribution of the interior surface. If both interior and exterior maximum thermal transmittances are conserved, each surface generally has a different fractional area and it is impossible to replace a thermal bridge with a simple linear flow component.

This is the fundamental idea of the proposing method. Figure 1 shows the concepts of above three methods.

STATE-SPACE REPRESENTATION

If we discritize the space using the finite difference method or the finite element method, the linear time invariant multidimensional heat transfer system is generally represented as the following descriptor form.

$$C_\Omega \dot{T}_\Omega(t) = K_\Omega T_\Omega(t) + K_{\Omega\Gamma} T_\Gamma(t) \qquad (5)$$

$$q_\Gamma(t) = K_{\Omega\Gamma}^T T_\Omega(t) + K_\Gamma T_\Gamma(t) - C_\Gamma \dot{T}_\Gamma(t) \qquad (6)$$

where,

$T_\Omega(t)$: internal temperature vector at time t

$T_\Gamma(t)$:boundary temperature vector at time t

$q_\Gamma(t)$:boundary heat flux vector at time t
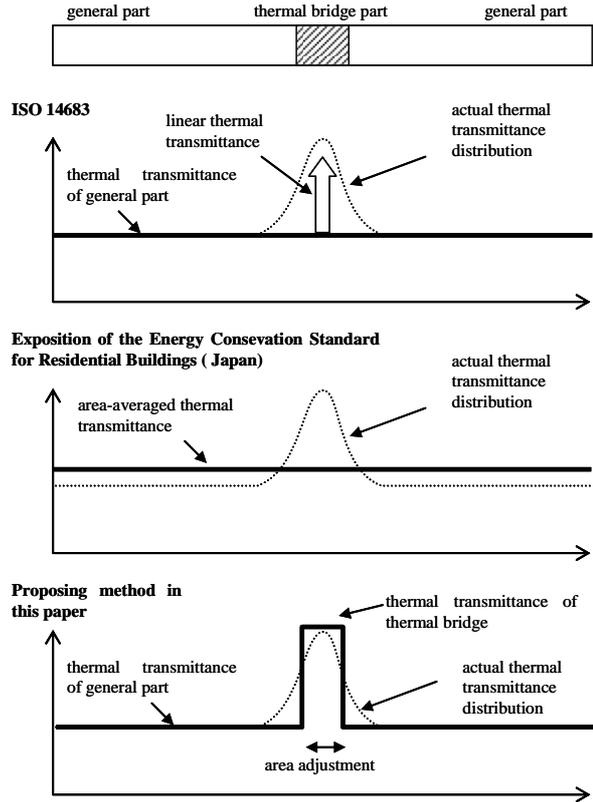
C_Ω :internal heat capacity matrix

Figure 1 Concepts of the models

C_Γ :boundary heat capacity matrix

K_Ω , K_Γ , $K_{\Omega\Gamma}$: heat transfer matrix

and a dot means time derivative. In case of $C_\Gamma \neq 0$, the above representation is not a formal descritor form but we leave C_Γ in the representation. Since there may be a node without a heat capacity, it is not obvious that C_Ω is regular (the inverse matix C_Ω^{-1} exists). Equations (5) and (6) are usually solved by using a finite time difference approach, but sometimes solved by using an analytical time integration approach.

The Laplace transforms of equations (5) and (6) are

$$sC_\Omega T_\Omega(s) = K_\Omega T_\Omega(s) + K_{\Omega\Gamma} T_\Gamma(s) \qquad (7)$$

$$q_\Gamma(s) = K_{\Omega\Gamma}^T T_\Omega(s) + K_\Gamma T_\Gamma(s) - sC_\Gamma T_\Gamma(s) \qquad (8)$$

where s is the Laplace parameter. Input-output form is

$$q_\Gamma(s) = G(s)T_\Gamma(s) \qquad (9)$$

where

$$G(s) = K_{\Omega\Gamma}^T(sC_\Omega - K_\Omega)^{-1}K_{\Omega\Gamma} + K_\Gamma - sC_\Gamma \qquad (10)$$

$G(s)$ is called the transfer function matrix and

$$G(0) = -K_{\Omega\Gamma}^T K_\Omega^{-1} K_{\Omega\Gamma} + K_\Gamma \qquad (11)$$

$$G'(0) = -K_{\Omega\Gamma}^T K_\Omega^{-1} C_\Omega K_\Omega^{-1} K_{\Omega\Gamma} - C_\Gamma \qquad (12)$$

since

$$\left(sC_\Omega - K_\Omega\right)^{-1} = -K_\Omega^{-1} - K_\Omega^{-1} C_\Omega K_\Omega^{-1} s + \cdots \qquad (13)$$

The Fourier transforms can be formally obtained by substituting s into $i\omega$, where i is the imaginary unit and ω is the angular frequency.

If there is only two boundaries, one is interior and the other is exterior, $T_\Gamma(s)$ and $q_\Gamma(s)$ in equation (9) are

$$T_\Gamma(s) = \begin{bmatrix} T_i(s) & T_e(s) \end{bmatrix}^T \qquad (14)$$

$$q_\Gamma(s) = \begin{bmatrix} q_i(s) & q_e(s) \end{bmatrix}^T \qquad (15)$$

where T_i and q_i are the internal environmental temperature and the interior surface heat flux, T_e and q_e are the external environmental temperature and the exterior surface heat flux, respectively. When the sign is positive for the outward heat flux, the tranfer function matrix is specifically

$$G(s) = \begin{bmatrix} -G_{Ai}(s) & G_T(s) \\ G_T(s) & -G_{Ae}(s) \end{bmatrix} \qquad (16)$$

where $G_{Ai}(s)$, $G_{Ae}(s)$ and $G_T(s)$ are called the interior admittance transfer function, the exterior admittance transfer function and the transmittance transfer function, respectively.

SYSTEM REDUCTION AND RESPONSE APPROXIMATION

In the field of system theory, model reduction methods of a linear time invariant system have been almost established (Obinata, G., Anderson, B.D.O. 2000). Among the various methods, modal truncation and balanced realization are major methods. Modal truncation is a method to preserve eigenvalues of $C_\Omega^{-1} K_\Omega$ up to some order and to cut off the higher orders. It is classic and widely used especially in the vibration analysis. Balanced realization is a method to build a system that the observability gramian and controllability gramian may become the same diagonal matrix. It is used in SISLEY software (Déqué, F., Noel, J. and Roux, J.J., 2001) which calculates dynamic properties of a thermal bridge. Approximate accuracy of both models by simple truncation is good for high frequency but not good for low frequency, and steady-state responses are usually unpreserved. It is, however, possible to preserve steady-state responses with a singular perturbation method.

As a method based on a response approximation, Padé approximation is well known. Padé approximation is a way to approximate a transfer function by a rational polynomial, which few terms of Taylor's series coincide with those of the original function.

The collocation method (Schapery, R.A., 1962), sometimes called the least-squares method, is a major technique of the Laplace transform inversion. According to this technique, it is necessary to calculate only few numerical values of a transfer function for real positive values of the Laplace parameters. A rational polynomial satisfied these collocated values can be obtained by solving a system of linear equations. The collocation method was applied to approximate responses of planar walls (Matsuo, Y., 1971), and it was also applied to earth contact walls (Nagata, A. and Matsuo, Y., 1990) and linear heat and moisture transfer problems (Nagata, A. and Matsuo, Y., 1991). A number of the Laplace parameters determines the order of reduced system. The set of the Laplace parameters in hour^{-1} is

$$s = 0 \text{ and } s = 10^{-5+0.5k} \qquad , k = 0,1,\cdots,13 \qquad (17)$$

which is usually enough for building heat transfer problems.

In this study, we use the collocation method for obtaining arroximate responses of thermal bridges.

DYNAMIC PROPERTY OF THERMAL BRIDGE

The argument in the steady state is extended to an unsteady state. The fractional area of a thermal bridge is fixed at the value of the steady state ($s = 0$). This means that the approximate accurary of the dynamic property of the thermal bridge part becomes worth but it is possible to preserve the whole heat flux.

As for the transmittance transfer function $G_T(s)$, a similar relation to equation (3) is supposed. Thus

$$G_T(s) = (1-f)G_{Tg}(s) + f G_{Tb}(s) \qquad (18)$$

where $G_{Tg}(s)$ and $G_{Tb}(s)$ are the transmittance tranfer function of the general component and that of the thermal bridge component, respectively. When $G_T(s)$ and $G_{Tg}(s)$ are obtained from the FDM calculation, then the transmittance transfer function of the thermal bridge component, $G_{Tb}(s)$, is derived from

$$G_{Tb}(s) = G_{Tg}(s) + \frac{G_T(s) - G_{Tg}(s)}{f} \qquad (19)$$

The admittance transfer function of the thermal bridge component is also derived in a similar way.

$$G_{Ab}(s) = G_{Ag}(s) + \frac{G_A(s) - G_{Ag}(s)}{f} \qquad (20)$$

where $G_A(s)$, $G_{Ag}(s)$ and $G_{Ab}(s)$ are the admittance tranfer functions of the whole component, the general component and the thermal bridge component, respectively. Althogh there are interior and exterior admittance transfer functions, we need interior one for a heat load calculation.

If a heat load calculation program is based on the response factor method, what is necessary is only the Laplace transform inversion numerically by the collocation method. If, however, a program is based on FDM etc., it is necessary to constitute a one dimensional imaginary component.

IMAGINARY COMPONENT

In this study, simple and deterministic methods to constitute the 2-layer model and the 3-layer model are also proposed.

Here we have already obtained the transmittance and admittance transfer functions. Both interior and exterior admittance transfer functions are needed to constitute a one dimensional imaginary component.

The imaginary component model is satisfied the following conditions.

a) the thermal transmittance (total resistance) is conserved.

b) the total capacity is conserved.

c) each layer has no negative resistance and capacity.

d) the boundary layers of both surfaces are separated from the body (bound nodes are put on the surfaces).

Careful attention should be given to the surface coefficients of heat transfer (surface resistances). If they vary, the fractional area of the thermal bridge component also varies. As the interior surface coefficient of heat transfer becomes large, the fractional area of the thermal bridge becomes large and the thermal transmittance of the thermal bridge component becomes small. Therefore, the interior surface coefficient of heat transfer should be small for considering risk factor of minimum temepratures and shold be large for considering that of whole heat flow rates.

The transfer function matrix without surface boundary layers is

$$
\begin{bmatrix} -G_{mAi}(s) & G_{mT}(s) \\ G_{mT}(s) & -G_{mAe}(s) \end{bmatrix}
= \left(\begin{bmatrix} -G_{Ai}(s) & G_T(s) \\ G_T(s) & -G_{Ae}(s) \end{bmatrix}^{-1} + \begin{bmatrix} h_i & 0 \\ 0 & h_e \end{bmatrix}^{-1} \right)^{-1} \tag{21}
$$

where

h_i : interior surface coefficient of heat transfer

h_e : exterior surface coefficient of heat transfer

$G_{mAi}(s)$: interior admittance transfer funtion matrix without surface boundary layers

$G_{mAe}(s)$: exterior admittance transfer funtion matrix without surface boundary layers

$G_{mT}(s)$: transmittance transfer funtion matrix without surface boundary layers

We expand the transfer function matrix wothout surface boudary layers in a Taylor's series

$$
\begin{bmatrix} -G_{mAi}(s) & G_{mT}(s) \\ G_{mT}(s) & -G_{mAe}(s) \end{bmatrix}
= \frac{1}{R_m} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -G'_{mAi}(0) & G'_{mT}(0) \\ G'_{mT}(0) & -G'_{mAe}(0) \end{bmatrix} s + \cdots \tag{22}
$$

where R_m is the total resistance without surface boundary layers and

$$
1/R_m = G_{mAi}(0) = G_{mAe}(0) = G_{mT}(0) \tag{23}
$$

The quantity of storage heat in the component when interior surface temperature equals 1 and external surface temperature equals 0, is

$$
C_{mi} = G'_{mAi}(0) - G'_{mT}(0) \tag{24}
$$

and the storage heat in the reverse case is

$$
C_{me} = G'_{mAe}(0) - G'_{mT}(0) \tag{25}
$$

The sum of C_{mi} and C_{me} is the heat capacity of the component.

$$
\begin{aligned} C_m &= C_{mi} + C_{me} \\ &= G'_{mAi}(0) + G'_{mAe}(0) - 2G'_{mT}(0) \end{aligned} \tag{26}
$$

First we define the following two parameters.

$$
r_S = 1 + \frac{4G'_{mT}(0)}{C_m} \qquad (0 \le r_S \le 1) \tag{27}
$$

$$
c_B = \frac{G'_{mAi}(0) - G'_{mAe}(0)}{C_m + 4G'_{mT}(0)} \qquad (-1 \le c_B \le 1) \tag{28}
$$

For constituting a 2-layer model (Figure-2), R_m is devided in the ratio $r_1 : 1 - r_1$, and C_m is devided in the ratio $c_1 : 1 - c_1$. r_1 and c_1 are determined by the first order Padé approximation. The transfer functions of the 2-layer model can be derived analytically. Thus

$$
G'_{mAi}(0) = \frac{C_m}{3} \left[c_1(2 - r_1) + (1 - r_1)^2 \right] \tag{29}
$$

$$G'_{mAe}(0) = \frac{C_m}{3}\left[(1-c_1)(1+r_1)+r_1^2\right] \qquad (30)$$

$$G'_{mT}(0) = -\frac{C_m}{6}\left[c_1(2r_1-1)+(1-r_1)(1+2r_1)\right] \quad (31)$$

then

$$r_1 = \frac{1}{2} + \frac{1-3r_S}{4c_B r_S} \qquad (32)$$

$$c_1 = \frac{1}{2} + \frac{1-3r_S}{4c_B r_S} + c_B r_S \qquad (33)$$

These solutions, however, reach infinity at $c_B = 0$. Since the component is symmetrical at $c_B = 0$, it is preferable that r_1 and c_1 are 1/2. They are, therfore,

$$r_1 = \frac{1}{2} + \frac{1-3r_S}{4} c_B r_S \qquad (34)$$

$$c_1 = \max\left[0, \min\left[\frac{1}{2} + \frac{5-3r_S}{4} c_B r_S\right]\right] \qquad (35)$$

For constituting a 3-layer model (Figure-2), R_m is devided in the ratio $r_1 : 1-r_1-r_3 : r_3$, and C_m is devided in the ratio $c_1 : 1-c_1-c_3 : c_3$. From the result that the transfer functions of 3-layer model are derived analytically and expanded in Taylor's series, we obtain the following relations.

$$r_S = \frac{1}{3} - \frac{2}{3}(1-c_3)(1+r_1-2r_1^2)$$
$$- \frac{2}{3}(1-c_1)(1+r_3-2r_3^2) \qquad (36)$$
$$+ \frac{4}{3}(1-c_1 r_1 - c_3 r_3) - \frac{4}{3}(1-c_1-c_3)r_1 r_3$$

$$c_B r_S = (1-c_3)(1-r_1) - (1-c_1)(1-r_3) \qquad (37)$$

which give

$$c_1 = r_1 + c_B r_S - \frac{1-3r_S + 2c_B r_S(1-2r_1)}{4(1-r_3)} \qquad (38)$$

$$c_3 = r_3 - c_B r_S - \frac{1-3r_S - 2c_B r_S(1-2r_3)}{4(1-r_1)} \qquad (39)$$

As for the thermal resistance allocation, we adopt

$$r_1 = (1-c_B r_S)\frac{1-r_S}{2} \qquad (40)$$

$$r_3 = (1+c_B r_S)\frac{1-r_S}{2} \qquad (41)$$

which satisfy $c_1 = c_3 = r_1 = r_3 = 1/3$ at $r_S = 1/3$ and $c_B = 0$ (a single layer wall).

NUMERICAL EXAMPLE

A simple example of a flat-surface thermal bridge (Figure-4) is shown in this section. Figure-5 shows
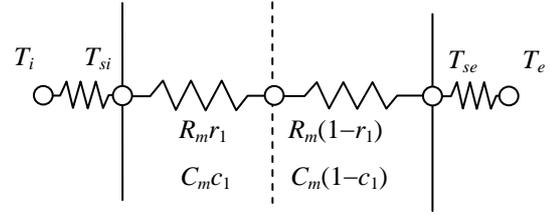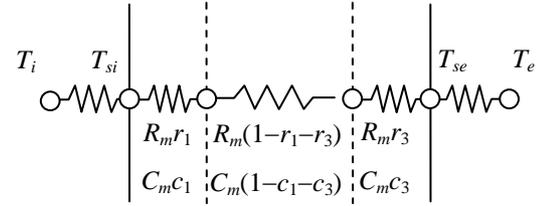

Figure-2   2-layer model
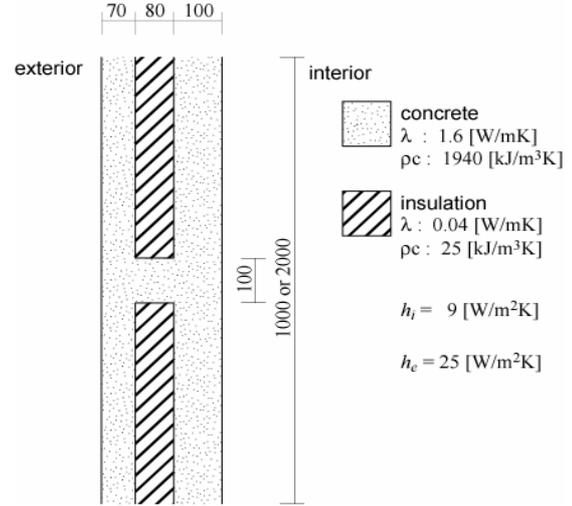

Figure-3   3-layer model
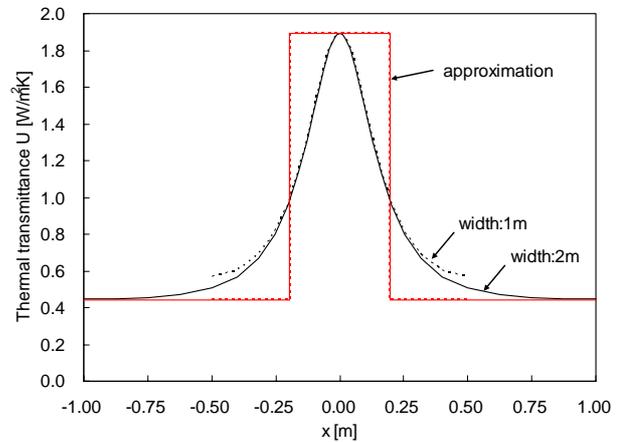

Figure-4 Example of a thermal bridge

Figure-5 Thermal transmittance distribution and its approximation

thermal transmittance distributions along the interior surface which are calculated by FDM program. The pitches of a thermal bridge are set 1m and 2m. Approximations of both case are almost same and the thermal bridge component is symply separated from the general component. Figure-6 shows transfer functions of the thermal bridge component and at the center of the thermal bridge in the Laplace tranform domain. This result means that dynamic properties of the thermal bridge component derived from the proposed method in this paper approximate those at the center of the thermal bridge. Three-layer model gives a reasonably good approximation in this example (Figure-7). The resistances of the layers are 0.065, 0.231, 0.082 $[m^2K/W]$ from interior to exterior, respectively and the heat capacities of the layers are 1.87×10^5, 3.77×10^4, 1.46×10^5 $[J/m^2K]$, respectively.

CONCLUSION

This paper propose a simple method to predict both heat flows and minimum surface temperatures.

REFERENCES

Burch, D.M., Seem, J.E., Walton, G.N., and Licitra, B.A., 1992. Dynamic evaluation of thermal bridged in a typical office building, ASHRAE Trans. 98(1), 291-304.

EnergyPlus, 2004. "Engineering Documentation: The Reference to EnergyPlus Calculations". Building Simulation Group. Lawrence Berkeley National Laboratory. Berkeley, California.

Obinata, G., Anderson, B.D.O. 2000. "Model Reduction for Control System Design", Springer-Verlag.

Qu, Z., 2004. "Model Order Reduction Techniques", Springer-Verlag.

Institute for Building Environment and Energy Conservation. 2002. "Exposition of the Energy Consevation Standard for Residential Buildings" (in Japanese)

ISO 14683: 1999. "Thermal bridges in building construction - Linear thermal transmittance - Simplified methods and default values"

Schapery, R.A., 1962. Approximate Methods of Transform Inversion for Viscoelastic Stress Analysis, Proc. of the Fourth U.S. National Congress on Applied Mechanics, Vol.2, 1075-1085.

Déqué, F., Noel, J. and Roux, J.J., 2001. SISLEY : An open tool for transient-state two-dimensional heat transfer, Proc. of Building Simulation '01, 915-920.
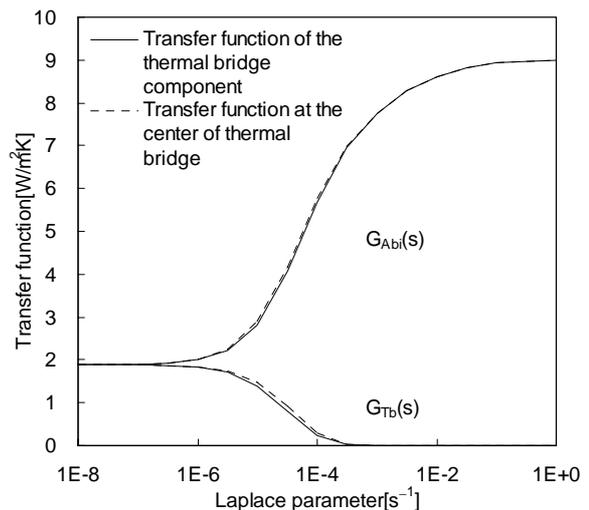
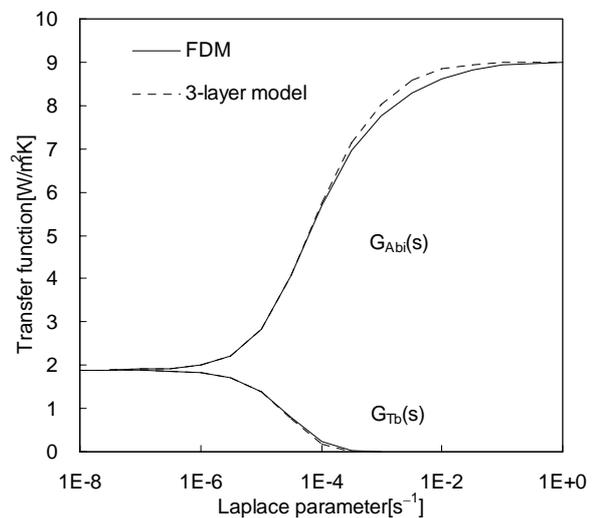*Figure-6 Transfer functions of the thermal bridge (FDM results)*

Figure-7 Transfer functions of the thermal bridge (FDM vs. 3-layer model)

Matsuo, Y., 1971. Approximation method and its application to heat transfer of buildings, Summaries of Technical Papers of Annual Meeting, A.I.J., 133-134 (in Japanese).

Nagata, A. and Matsuo, Y., 1990. Approximation method for heat transfer analysis of semi-infinite solid, Summaries of Technical Papers of Annual Meeting, A.I.J., D, 735-736 (in Japanese).

Nagata, A. and Matsuo, Y., 1991. Response analysis of walls considering simultaneous heat and moisture transfer (part 1) Approximation by collocation method, Summaries of Technical Papers of Annual Meeting, A.I.J., D, 765-766 (in Japanese).