THE NUMERICAL MODELLING OF THE DIFFUSIVE-CONVECTIVE WATER VAPOUR TRANSPORT THROUGH THE BUILDING CONSTRUCTIONS

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ABSTRACT

The following paper is focused on the effects of the combined water vapour transport caused by diffusion and convection. The governing equation of this transport mechanism is presented together with the possibilities of its numerical solution by means of the finite element method.

In the second part of the paper, a typical slope roof construction with thermal insulation between the rafters and with small leakages in the water vapour barrier is analysed using 2D calculation model. General conclusion discussing the significance of the combined diffusive-convective heat and water vapour transport is included in the final part of the paper.

INTRODUCTION

The air infiltration through the permeable building constructions or through the cracks in them is important factor, which can considerably affect their hygrothermal performance. The thermal effects of this transport mechanism include deformation of the temperature field in the construction and significantly increased heat loss. The moisture effects are even more important because the air flow is able to transport high amounts of water vapour into the construction and subsequently cause interstitial condensation with very high condensation rate. This can even lead to serious damage of such permeable and/or leaky constructions.

The analysis of the combined diffusive-convective water vapour transport can be based on the partial differential equation for the two-dimensional steady-state water vapour transport in a porous medium. This equation can be expressed as

$$\frac{D}{\mu}\nabla^2\rho_v - \vec{v}\cdot\nabla\rho_v = 0 \qquad (1).$$

The first term on the left-hand side of equation (1) represents the water vapour transport due to diffusion; the second term represents the water vapour transport due to convection. The Newton boundary condition for the equation (1) can be defined as

$$-\frac{D}{\mu}\frac{\partial\rho_v}{\partial n} + v_n\rho_v = \beta\left(\rho_v - \overline{\rho}_v\right) \qquad (2).$$

The numerical solution of the equation (1) with the boundary condition (2) was in the presented paper derived using the following assumptions:

- water vapour transport is steady-state and two-dimensional
- convection of air through the building construction is caused only by pressure difference
- air is incompressible
- flow of air is linear according to Darcy´s Law

$$\vec{v} = -\frac{k}{\eta}\vec{\nabla}P \qquad (3)$$

- pressure distribution is governed by Laplace equation

$$k\cdot\nabla^2 P = 0 \qquad (4).$$

ANALYSIS

The equation (1) belongs to the family of convective-diffusion equations. It is always more complicated to find the numerical solution of such equation than to find the solution of the related usual diffusion equation. The main cause is the convective transport term, which can - under certain conditions - produce instabilities in the numerical solution.

The finite element method (FEM) was used in the presented paper in order to obtain the numerical solution of the equation (1). The general finite element formulation was derived by means of the Petrov-Galerkin process, which is one of the weighted residuals methods. Therefore the derivation of the finite element formulation starts with the condition

$$\int_{\Omega^{(e)}}\left[\frac{D}{\mu}\nabla^2\rho_v - \vec{v}\cdot\nabla\rho_v\right]\cdot W\,\mathrm{d}\Omega = 0 \qquad (5)$$

which is also the mathematical expression of the requirement that the residual of the numerical solution of the equation (1) must be orthogonal to the weighting functions *W*.

The unknown function ρ_v in equation (5) is taken as approximation

$$\rho_v = N_i^T \cdot \rho_{v,i} \qquad (6).$$

While interpolation functions N_i are known functions depending on the type of the chosen finite elements, weighting functions W_i must be derived from the following equation recommended by Zienkiewicz

$$W_i = N_i + \varepsilon \frac{b}{2} \frac{v_x \frac{\partial N_i}{\partial x} + v_y \frac{\partial N_i}{\partial y}}{|v|} \qquad (7).$$

Now, if the value of ε is chosen as

$$\varepsilon = \coth Pe - \frac{1}{Pe} \qquad (8)$$

and Peclet number as

$$Pe = \frac{\mu \cdot |v| \cdot b}{2 \cdot D} \qquad (9)$$

then according to Zienkiewicz numerical oscillations do not arise for any possible rate between convective and diffusive water vapour transport.

The general finite element formulation can be finally derived from the equation (5) by means of integration by parts and by means of introduction of the boundary condition (2). The final general FEM formulation is

$$\left(K_D + K_V + K_\beta\right) \cdot \rho_{v,i} = q_\beta \qquad (10).$$

The diffusion matrix K_D is defined as

$$K_D = \int_{\Omega^{(e)}} \frac{D}{\mu} \left(\frac{\partial W_i}{\partial x} \frac{\partial N_i^T}{\partial x} + \frac{\partial W_i}{\partial y} \frac{\partial N_i^T}{\partial y} \right) \mathrm{d}\Omega,$$

the convective transport matrix K_v as

$$K_v = \int_{\Omega^{(e)}} \left(v_x \cdot W_i \frac{\partial N_i^T}{\partial x} + v_y \cdot W_i \frac{\partial N_i^T}{\partial y} \right) \mathrm{d}\Omega,$$

the boundary conditions matrix K_β as

$$K_\beta = \int_{\Gamma^{(e)}} (\beta - v_n) W_i N_i^T \, \mathrm{d}\Gamma$$

and the boundary conditions vector q_β as

$$q_\beta = \int_{\Gamma^{(e)}} \beta \cdot W_i \cdot \overline{\rho}_v \, \mathrm{d}\Gamma \cdot$$

Note that the convective transport matrix K_v is asymmetrical, which leads to the asymmetrical matrix of the linear equations system for unknown nodal values $\rho_{v,i}$.

The field of partial water vapour densities $\rho_{v,i}$ calculated from the equation (10) is correct only in the case with no interstitial condensation – in other words, when the following condition is met in any part of the solved area

$$\rho_v < \rho_{v,sat} \qquad (11).$$

Otherwise, the water vapour condensation occurs in the building construction and the calculated field of partial water vapour densities is deformed according to this fact. In the presented paper, the simplified iteration process was used to obtain the area of water vapour condensation and the final field of partial water vapour densities. This iteration process is based on the condition

$$\rho_v \le \rho_{v,sat} \qquad (12)$$

which can be expressed also as requirement that the partial water vapour density cannot in any part of the construction exceed the saturated partial water vapour density. The iteration process itself can be programmed in the following steps:

- at first, the field of partial water vapour densities and the field of saturated partial water vapour densities are calculated separately and compared – the result of comparison shows if the water vapour condensation occurs in the construction
- if there is condensation, the maximum difference between partial water vapour density and the saturated partial water vapour density is found
- the value of saturated partial water vapour density is then introduced as Dirichlet type of boundary condition in the mesh node with this maximum difference and the whole calculation of partial water vapour density field is repeated
- the newly calculated field of partial water vapour densities is afterwards compared with known field of saturated partial water vapour densities in order to find out if there are still mesh nodes where the partial water vapour density exceeds the maximum limit
- if yes, the maximum difference is found again and the calculation of partial water vapour density field is repeated with all the existing and new Dirichlet conditions.

The iteration process continues until the condition (12) is fulfilled with chosen accuracy in all mesh nodes. Convergence of this iteration is reliable and quite rapid. The result of the iteration process is the field of partial water vapour densities in the construction with interstitial water vapour condensation. More common field of relative humidities can be calculated from

$$\varphi = \frac{\rho_v}{\rho_{v,sat}} \cdot 100 \qquad (13).$$

It can be seen from the already described methods and equations that the temperature field must be determined at first because the saturated partial water vapour density is calculated using equation

$$\rho_{v,sat} = \frac{p_{sat}}{461{,}9 \cdot (273{,}15 + \theta)} \quad (14).$$

The temperature field must be of course calculated in the same conditions of combined convective-conductive transfer as the water vapour density field. The governing equation for this heat transport mechanism can be expressed as

$$\lambda \nabla^2 \theta - \vec{v} \cdot \rho_a c_a \cdot \nabla \theta = 0 \quad (15).$$

The numerical solution of the equation (15) by means of FEM is similar to already described numerical solution of the equation (1) and can be found in already published papers (e.g. Svoboda, 1999, 2000).

The developed computer model „WIND2D“ is based on the FEM solution of the equations (1) and (15). It calculates the steady-state air pressure field within two-dimensional porous building components, the air flow velocity field, the temperature field and the relative humidity field.

The analysis of the numerical stability of the solution obtained by „WIND2D“ was realised using the comparison of the numerical solution with the exact analytical solution of the equation

$$\frac{D}{\mu} \frac{d^2 \rho_v}{dx^2} - v_x \frac{d\rho_v}{dx} = 0 \quad (16)$$

with boundary conditions

$$\rho_v(0) = 0{,}002; \ \rho_v(1) = 0 \quad (17).$$

The analytical solution of the equation (16) is

$$\rho_v(x) = \frac{0{,}002 \cdot \left(e^{Bx} - e^{B}\right)}{1 - e^{B}} \quad (18)$$

with the value of *B* defined as

$$B = \frac{\mu \cdot v_x}{D} \quad (19).$$

The numerical solution of the equation (16) for the value of *B=5* and for various number of the finite elements in comparison with the exact solution can be seen in Table 1. The analysis shows clearly that the results of the numerical solution converge to the exact solution with increasing number of the finite elements covering the solved area.

SIMULATION

A typical slope roof construction with thermal insulation between the rafters has been chosen to be the object of simulation. The layers of analysed construction are described in Table 2.

Table 1
Results of the exact and the numerical solution

DISTANCE	PARTIAL VAPOUR DENSITY			
	EXACT	NUMERICAL SOLUTION		
x [m]	ρ_v [kg/m^3]	$\rho_{v,1}$ [kg/m^3]	$\rho_{v,2}$ [kg/m^3]	$\rho_{v,3}$ [kg/m^3]
0.0	0.00200	0.00200	0.00200	0.00200
0.2	0.00198	0.00198	0.00198	0.00198
0.4	0.00191	0.00191	0.00191	0.00191
0.6	0.00174	0.00172	0.00173	0.00174
0.8	0.00127	0.00124	0.00125	0.00126
1.0	0.00000	0.00001	0.00001	0.00000
Number of elements	---	40	80	160

Table 2
Layers of the analysed construction

LAYER	THICKNESS [mm]	THERMAL CONDUCTIVITY [W/(m.K)]	EQUIVALENT DIFFUSION THICKNESS [m]	PERMEABILITY [m^2]
Roof tiles	Not considered in calculation			
Ventilated air layer				
Waterproof barrier	0.1	0.21	0.02	10^{-12}
Mineral wool	160	0.04	0.18	10^{-9}
Vapour barrier	0.1	0.21	1.44	10^{-17}
Mineral wool	40	0.04	0.04	10^{-9}
Plasterboard	12.5	0.22	0.11	10^{-12}

There is no major risk of huge interstitial condensation in this model construction if the water vapour barrier is tight. On the other hand, if the water vapour barrier is perforated, the situation can be quite different. Let us suppose that in the plasterboard and in the water vapour barrier are relatively small openings 1 mm wide – and to be more realistic – these openings are located in the distance of 500 mm. The safety waterproof barrier is considered tight (with loosely overlapped partial layers).

The calculated relative humidity fields in this model construction for the air exfiltration caused by various pressure gradients are shown on Fig. 1. The results are valid for the external air with temperature -15 ˚C and relative humidity 84% and for internal air with temperature 21 ˚C and relative humidity 50%.

The water vapour condensation zone for the pressure difference of 40 Pa is very interesting. The apparent reduction of this zone in comparison with the zones for the lower pressure gradients is caused by higher heat transport through the leaky construction (Fig. 2). However, this higher heat transport does not lead to analogous reduction of water vapour condensation

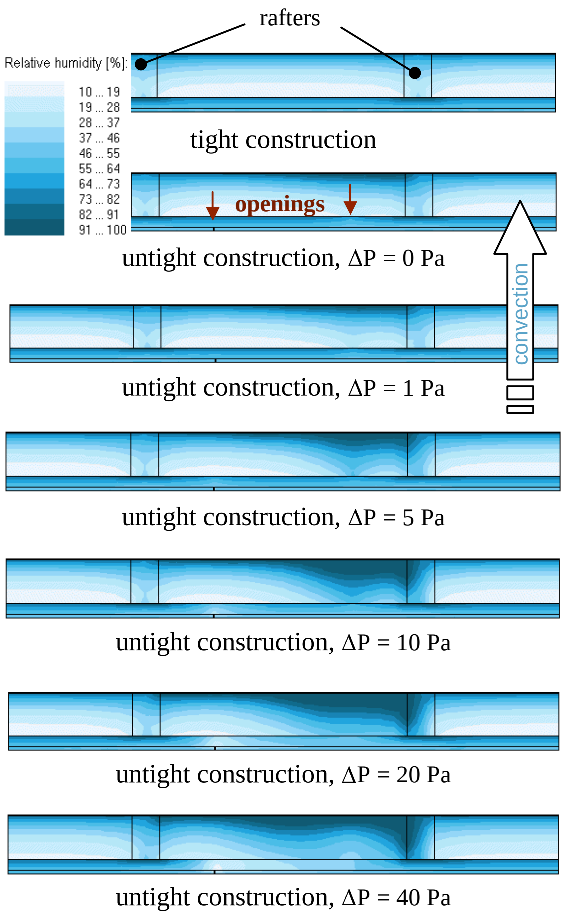

Figure 1 Relative humidity field in the slope roof construction – the case of air exfiltration

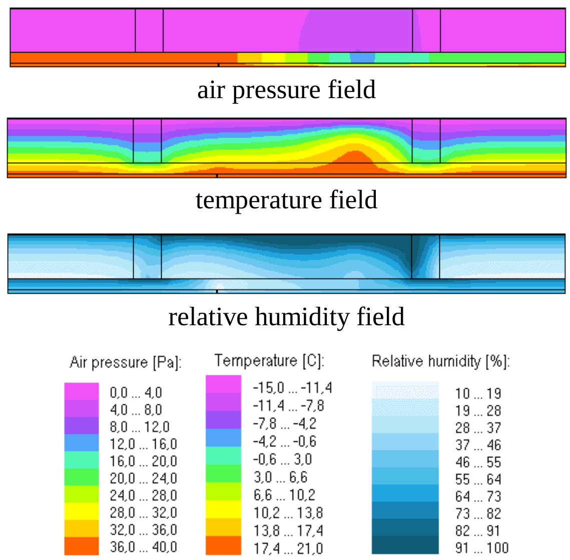

Figure 2 Various fields for exfiltration through the slope roof caused by pressure gradient of 40 Pa

rate as can be seen in Table 3 – the only effect is the displacement of the condensation zone to colder parts of the construction.

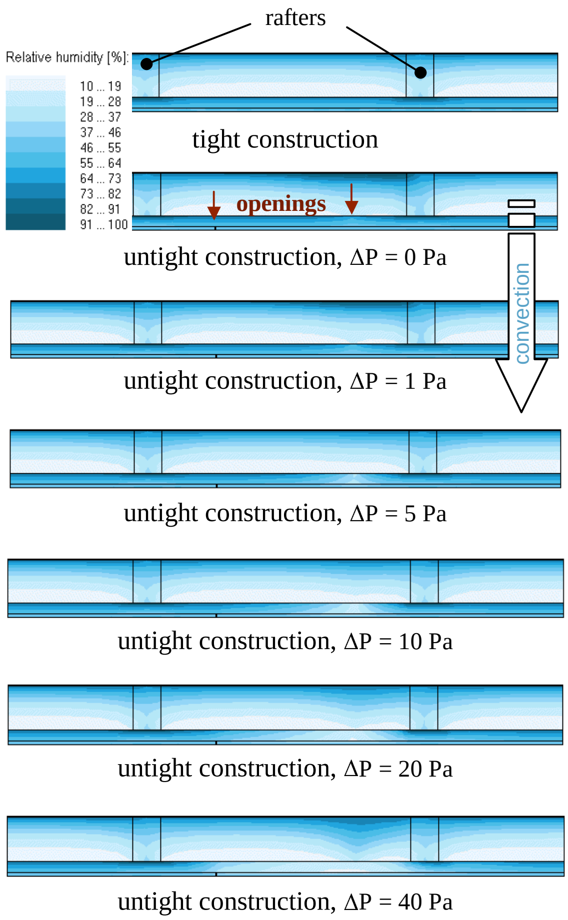

Figure 3 Relative humidity field in the slope roof construction – the case of air infiltration

While the air exfiltration (from interior to exterior) causes obvious enlargement of the water vapour condensation zone, the air infiltration (from exterior to interior) leads to more subtle changes in the extent of the condensation zone (Fig. 3). Nevertheless, the water vapour condensation rate rises up in both cases as the pressure difference gradually moves to higher levels (Table 3 and Fig. 4).

Table 3
Water vapour condensation rate

CONSTRUCTION	**PRESSURE DIFFERENCE**	**WATER VAPOUR CONDENSATION RATE [kg/(m^2.s)]**	
		EXFILTRA-TION	**INFILTRA-TION**
Tight slope roof	0 Pa	8,15.10^{-8}	
Untight slope roof	0 Pa	1,08.10^{-7}	
	1 Pa	1,47.10^{-7}	1,23.10^{-7}
	5 Pa	2,97.10^{-7}	2,02.10^{-7}
	10 Pa	4,71.10^{-7}	2,93.10^{-7}
	20 Pa	7,60.10^{-7}	4,50.10^{-7}
	40 Pa	1,23.10^{-6}	7,20.10^{-7}

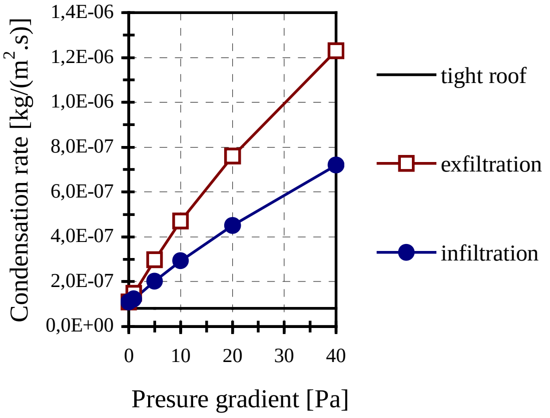

Figure 4 Water vapour condensation rate

Although the increase of the water vapour condensation rate is substantial mainly in the case of air exfiltration (e.g. for the pressure difference of 40 Pa, the condensation rate is about 15-times higher than for the case of tight construction), it is not negligible also in the case of air infiltration. These effects are caused almost exclusively by convection because the increased diffusion through the openings leads itself to the increase of condensation rate only around 33% (Table 3).

CONCLUSIONS

The newly developed computer model „WIND2D“ (version 2004) can be used for the calculation of the temperature and relative humidity distribution in various types of two-dimensional building components in the conditions of steady-state combined heat and water vapour transport caused by conduction/diffusion and convection.

This paper presents its application as a tool for the analysis of water vapour transport in a typical slope roof construction.

The results of the numerical modelling show these major conclusions:

- modern constructions with permeable thermal insulation from mineral wool are very sensitive to the convective heat and water vapour transfer
- any opening in water vapour barrier and/or safety waterproof layer can cause the air infiltration or exfiltration through the thermal insulation and subsequently the essential modifications in the temperature and relative humidity fields
- the results of such deformations in the temperature and water vapour distribution include:
  - considerable increase of the heat loss through the construction
  - substantial increase of the water vapour condensation rate and – mainly in the case of air exfiltration – also the enlargement of the condensation zone in the construction
- higher moisture transport through the leaky construction caused mainly by the convection can easily lead in specific conditions (e.g. in buildings with moist internal microclimate – swimming pools etc.) to severe and surprisingly rapid damage of such construction (beyond all design assumptions).

The requirement for the maximum tightness of water vapour barrier and other air-tight layers is therefore in the case of modern lightweight constructions really essential.

ACKNOWLEDGMENT

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NOMENCLATURE

b size of an element in the velocity direction [m]

c_a thermal capacity of the air [J/(kg.K)]

D water vapour diffusion coefficient in air [m^2/s]

k permeability of the porous medium [m^2]

K_D diffusion matrix

K_v convective transport matrix

K_β boundary conditions matrix

N_i interpolation functions vector

P air pressure [Pa]

p_{sat} saturated partial water vapour pressure [Pa]

q_β boundary conditions vector

$|v|$ magnitude of vector of air flow velocity [m/s]

$\vec{v}$ vector of air flow velocity [m/s]

v_x velocity component in the x axis direction

v_y velocity component in the y axis direction

v_n velocity component normal to the boundary

W_i weighting functions vector

β water vapour boundary transfer coefficient [m/s]

η dynamic viscosity of the air [Pa.s]

φ relative humidity of the air [%]

λ thermal conductivity [W/(m.K)]

μ water vapour diffusion factor [-]

θ temperature [°C]

ρ_a density of the air [kg/m^3]

ρ_v partial water vapour density [kg/m^3]

$\rho_{v,sat}$ saturated partial water vapour density [kg/m^3]

$\rho_{v,i}$ vector of unknown partial water vapour density values [kg/m^3]

$\overline{\rho}_v$ known partial water vapour density at element boundary [kg/m^3]

Γ_e boundary of a finite element

Ω_e area of a finite element.