SIMULATION OF HEAT FLOW FROM A LINE SOURCE IN SUPPORT OF DEVELOPMENT OF A THERMAL PROBE

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ABSTRACT

The use of a thermal probe allows a range of in-situ conditions to be taken into account that impact upon the properties of building materials (λ, α) as encountered in reality. In this study, the transient thermal simulation of a model representing a line source in an infinite material sample has been used to guide the development of an experimental thermal probe apparatus. Simulation produced a series of datasets that have been used to test the data analysis routines used with the experimental probe. Findings show that by careful application of these routines close agreement (errors of less than 1 percent) with input values can be achieved. This validates the analysis routines and provides a deeper appreciation of the behaviour a theoretical line source model. The model provides valuable insights into the level of error and uncertainty in in-situ measurements.

KEYWORDS

Material properties, thermal probe, transient simulation, experiments

INTRODUCTION

Reliable data concerning the thermal properties of building materials, in particular the thermal conductivity (λ) and thermal diffusivity (α), is needed for the proper simulation of the thermal behavior of buildings, whether in a design stage, refurbishment project, or research context. However, much of the data currently used in building performance simulation is obtained under laboratory conditions, which results in two main problems. Firstly, material properties established using techniques like the guarded hot box method might reflect a dried-out condition, which differs from material properties in actual use. Secondly, actual material properties might be hard to obtain from a handbook due to the multitude of variants of any material, resulting for instance from different manufacturing processes and differences in ingredients.

An alternative to the use of laboratory data is the use of in-situ measurements, allowing the capture of actual thermal properties of materials in buildings in use. One technique for measuring such data on site is the use of a thermal probe apparatus. Thermal probes have been developed and used in other industries, such as geotechnics, food and plastic manufacturing; it is only recently being applied to buildings. The technique is based on the measurement of transient heat flow from a line source. A needle is inserted into the material; a constant power per unit length (*Q*) is then applied to the needle, with the temperature rise ΔT monitored and plotted against the natural logarithm of elapsed time ln(*t*). For the resulting graph of the rise in temperature ΔT versus the natural logarithm of the elapsed heating time *(t)*, where the slope of the straight section is S, the intercept I, the thermal conductivity (λ) and thermal diffusivity (α) can then be calculated by equations (1) and (2):

$$\lambda = Q / (4 \pi S) \qquad (1)$$

$$\alpha = (r^2/4)*\exp((I/S) + \gamma - 2\lambda/rH) \qquad (2)$$

where *r* is the radius from the line source, and γ is Euler's Constant, and H is the probe to sample conductance. Note that if H is infinite, the term 2λ/rH = 0 and the volumetric heat capacity ρC is given by λ/α.

These formulas are derived from equation (3), which describes the temperatire rise in a medium at a distance r from the line source (Blackwell, 1956)

$$\Delta T = A\,[\ln t + B + (1/t)(C \ln t + D)] \qquad (3)$$

With:

$A = Q/4\pi L\lambda$
$B = \ln(4\alpha/r^2) - \lambda + 2\lambda/rH$
$C = (r^2/2\alpha)[1 - \alpha m c_p/\pi r^2 L\lambda]$
$D = (r^2/2\alpha)[\ln(4\alpha/r^2) - \gamma + 1 - B\,\alpha m c_p/\pi r^2 L\lambda]$

in which *L* is the length of the line source (probe), and *m* the mass per unit length of the probe. For a futher discussion of the Blackwell equation see Goodhew and Griffiths (2004), building on previous work by Blackwell (1956), Batty *et al* (1984) and Carslaw and Jaeger (1959).

However, the application in practice is not always straightforward. Several research papers report varying levels of success in measuring λ, but little success in obtaining values for α. While repeatability of results similar conditions (same lab, same type of samples) has been found to be excellent in many cases, application of the thermal probe technology across a broader range of materials with unknown properties remains to be demonstrated. In an attempt to improve the accuracy of the technique two different iterative routines have been developed using the Solver function within Microsoft Excel that allow the simultaneous determination of the three unknowns *α*, λ and *H.* The first routine, named Solver 2.3 is a routine that uses only the first two constants A and B from equation 3, whilst the second routine, named Solver 4.3 employs all four constants.

PROBLEM

The thermal probe equipment, shown in figure 1, together with a measurement procedure has been used in field experiments on real buildings (Pilkington *et al*, 2005; Pilkington *et al*, 2007). With this probe a large set of material samples have been studied. From these results, λ-values can mostly be identified with an accuracy of within ± 15%; this level of accuracy is well within the bounds of an ASTM standard for measuring the properties of soils and soft rock (ASTM, 2000), materials that are similar to those used in construction. However, obtaining values for α proves more challenging.

Figure 1: experimental thermal probe measurement kit (during calibration phase, measuring agar immobilised water)

Typical results obtained by using the thermal probe under laboratory conditions are demonstrated in figure 2. Values for the thermal conductivity and thermal diffusivity can be calculated from this data through using equations (1) and (2), by selecting a straight section of the dataline and then finding the slope of and intercept of this straight section. In practice this is realised by using the Solver 2.3 and 4.3 routines.

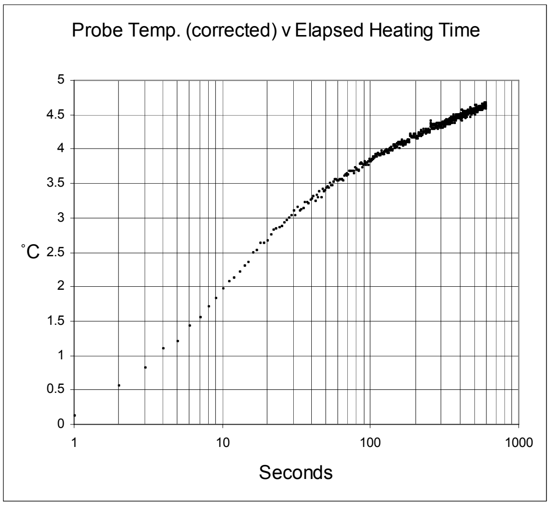

Figure 2: temperature rise ΔT against the natural logarithm of elapsed time ln(t) for a Non Hydraulic Lime sample, measured using the thermal probe under laboratory conditions

However, the measured data can be expected to be influenced by a number of physical factors like the contact resistance between the probe and the needle, heat losses at either end of the probe, heat losses from the material sample, and changeover from heating up to steady state conditions. Findings can also be influenced by the measurement procedure, for instance the amount of heating power provided to the probe or time series used. Furthermore, the data analysis techniques might need further improvement, especially regarding the identification and selection of a suitable section of the slope $\Delta T/\ln(t)$.

The assessment of the impact of each of these factors on the accuracy of measuments undertaken with a thermal probe is essential to guide the ongoing research and development of the measurement kit. Building performance simulation is a useful approach to study the relationship between single, isolated variables and results. This paper addresses the development of a model of a line source as a first step towards modelling of an actual thermal probe in full detail.

OBJECTIVE

In order to guide further development of the measurement kit and data analysis procedures, a thermal simulation model has been employed. Studies with the thermal model provide the opportunity to study the thermal probe from a theoretical point of view, allowing a rigorous testing of the assumptions, mathematics and the data analysis procedures. Results from the model can be used used to gain a better understanding of empirical datasets and the factors influencing the measurement and analysis outcomes.

Findings will guide further development of the thermal probe technique, which might in principle follow three main different paths:

1. improvement of the data analysis techniques;
2. possible re-engineering of the actual probe itself;
3. work on improving the measurement procedure for using thermal probes in the laboratory and on actual buildings.

Results will thereby direct the future efforts needed concerning the above topics, all of which are currently believed to have an impact on experimental and analytical outcomes.

The overall objective of the use of thermal simulation is to provide numerical reference datasets that can be used to test the data analysis procedure, and to provide a series of (numerical) reference sets that can be compared with experimental results obtained with an actual thermal probe apparatus. Simulation will allow a comparison of the effects of individual factors one by one, studying their impact on results obtained, something which is hard to do in physical experiments. Furthermore the use of simulation allows the research to venture into 'extreme' situations, thereby improving the understanding of the theoretical interaction between λ, α and temperature series. This will take advantage of the fact that a model allows the study values of both the thermal conductivity and the diffusivity without being constrained by existence of actual materials with these properties.

METHODOLOGY

The general strategy followed in this research project for investigating the different factors that influence experimental results starts by modeling the ideal, theoretical situation of an infinitely thin and long line heat source in an infinite homogenous block of material. From this starting point a step-by-step process can then be followed that allows the production of a model that is progressively closer in all aspects to the real-life thermal probe. This will evolve to eventually represent a needle with finite dimensions, and a thermal contact resistance between the heater and the sample.

The work described in this paper is limited to the first stage of an infinitely thin and long line heat source in an almost infinite homogenous block of material. Numerical results thereby represent what might be labeled as data from a 'theoretically very small diameter line heat source', and do not show any effects related to presence of a (modeled) probe. For the study, use has been made of the Physibel program Voltra that allows calculation of transient heat transfer using the energy balance technique (Physibel, 2005).

Within Voltra, a model has been built of very large slice of a material sample (outer dimensions of 2400 x 2400 mm, but with a thickness of only 1 mm). The boundary conditions on both faces of this slice of material have been defined as adiabatic, rendering the material infinite in the direction perpendicular to these faces. The line source is modeled at the middle of this slice, again perpendicular to the faces (positioned at 1200 mm from the boundaries of the sample). Due to the fact that Voltra uses a rectangular grid, gridlines have been put closely together closer to the line source (1 mm apart) in order to approximate the radial distribution of heat, gradually widening the grid further away (reaching 20 mm at the outer edges), see figure 3. For this model, various combinations of thermal conductivity (λ) and thermal diffusivity (α) have been entered, with homogenous properties across the whole sample. The modeling of the thermal probe is limited to the application of power at the heart of the line source, thereby modeling an infinitesimal thin probe without any contact resistance issues.

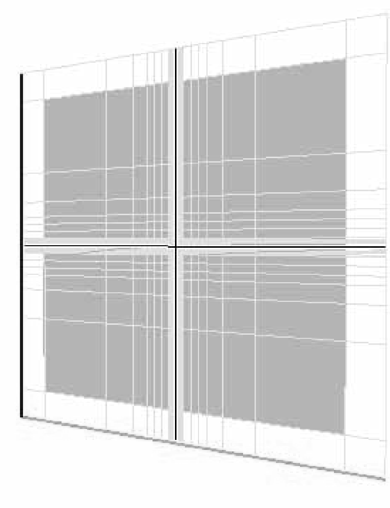

Figure 3: material sample model in Voltra. The line source is simulated at the origin, perpendicular to the (adiabatic) face of the sample

Different combinations of λ and α have been analysed, see table 2. Within the results section of this paper the heating curve, ΔT/ln(t) line and the output data for a material with middle range λ and α properties will be presented first. Then results for a broader range of typical construction materials will be presented. For comparison, material properties for λ, ρ and C have been taken from the ASHRAE Handbook of Fundamentals (2005). Finally, a range of theoretical materials has been studied, varying values of λ and ρC over a wide domain, allowing assessment over the whole range and including very extreme cases, in order to see how these impact on simulation results. The discussion of the results focusses upon the materials with low thermal conductivity, which numerically differs the most from measured data.

Each simulation experiment starts under steady state conditions, where the material sample is at 20.0^{o}C and sits in an environment of the same temperature. After an initial 60 seconds (for acclimatisation) a step function is then used to apply a constant power of 3 W/m to the line source. The temperatures are monitored at distances of 1, 2, 5, 10 and 50 mm away from the line source. Simulation results then have been exported to Microsoft Excel, where the first 60 seconds (steady state situation at 20^{o}C) has been discarded and the temperature rise transformed to data for Δ*T*/ln(*t*). Regression analysis is then carried out, and the thermal conductivity (λ) and thermal diffusivity (α) calculated according to formulas (1) and (2). The values for α are then translated into volumetric heat capacity (VHC) or ρC values using the resulting λ, as VHC is a more commonly used terminology.

RESULTS

The technique has been applied to a range of materials. Initially, when the probe technique was being developed, the chosen materials were selected for their suitability for calibration purposes, materials such as agar and glycerine. When the technique had been proven within the laboratory, construction materials were then investigated and the following results are those from arguably one of the most ubiquitous.

A typical building material: brick

Typical analysis results are shown in figure 4 and 5, for the building material: brick. Input material properties for the Voltra simulation are based on values from the ASHRAE Handbook of Fundamentals (2005): λ = 0.7 W/mK, ρ = 1970 kg/m^3, and C = 800 J/kgK (giving a VHC of 1.576 MJ/m^{3}K). Figure 4 shows the heating curve resulting from the simulation at 1 mm away form the line source, figure 5 shows the Δ*T*/ln(*t*) plot.

The data in figure 4 shows an initially steep increase in temperature when power is applied to the line source in the previously described model, then the temperature trailing of towards a steady state condition. Obviously, the actual steady state condition will only be encountered after a long time. Plotting temperature rise in the simulated brick model against ln(*t*) produces the almost straight line in figure 5.

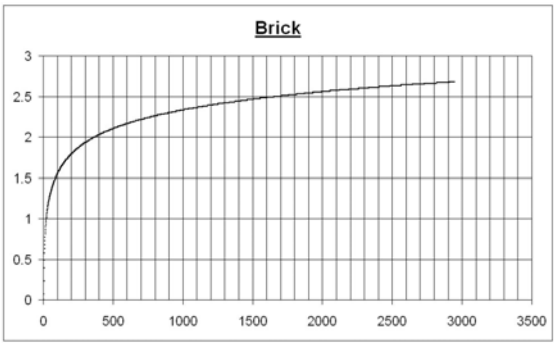

Figure 4: temperature rise ΔT against the elapsed time (t) ('heating curve') for a simulated line source in brick, obtained from Voltra simulations.

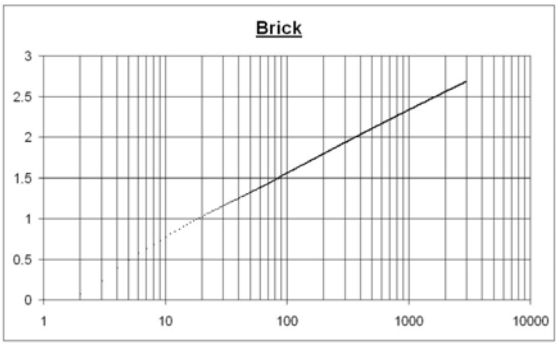

Figure 5: temperature rise ΔT against the natural logarithm of elapsed time ln (t) for a simulated line source in brick, obtained from Voltra simulations

By applying equations (1) and (2) to the data in figure 5, λ and VHC values have been calculated. Comparing the outcomes with the input parameters for the Voltra model shows that the deviaton in the resulting λ is 0%, while the deviation in the resulting ρC is within 5%.

Typical construction materials

Applying the same approach as demonstrated in section 5.1 (figures 4 and 5), further experiments across a wide range of typical construction materials have been carried out; results are presented in table 1. From the Δ*T*/ln(*t*) plot, λ and ρC have been calculated. Table 1 presents the calculated thermal properties of the materials studied, as well as the deviation of the resulting

values related to input data. Similar results have been obtained beyond the 1 mm distance from the line source. Unsurprisingly, deviations increase with the distance to the line source, as the simulations are based on a rectangular grid with increasing distance between gridlines. These therefore are not shown in the paper.
The temperature series underlying the results in table 1 match expectations, showing that steady state temperature will be reached earlier when ρC is lower, or when λ is higher, and that these steady state temperatures will become lower with an increase in λ or distance to the line source.

Interestingly, the deviation found in calculating λ from simulation results for the materials in table 1 shows an error margin in the order of 0% to 10%, while the error in obtaining ρC from the simulation data is much larger, i.e. the order of 5% to 20%. This effect shows the same trend as experimental datasets, where obtaining values for α has been found to be less accurate than finding values for λ.

Table 1. Resulting deviations in λ and ρC values of a range of building materials after simulation and regression analysis

Material	Input λ [W/mK]	Input ρ [kg/m^3]	Input C [J/kgK]	Deviation on resulting λ [%]	Deviation on resulting ρC [%]
Brick	0.7	1970	800	0.00	4.64
Cellular glass	0.05	136	750	-0.45	6.43
Cellulose	0.057	54	1300	-1.59	12.12
Concrete/stone	0.93	2300	653	-1.27	10.21
Fireclay brick	1.0	1790	829	-1.30	10.53
Limestone	0.93	1650	909	-1.28	10.27
Marble	2.6	2600	880	-0.88	9.20
Portland cement	0.029	1920	670	5.28	-16.46
Sand	0.33	1520	800	4.48	-10.58
Wood (fir)	0.12	540	1210	7.70	-19.24
Wood (oak)	0.176	750	2390	7.41	-19.62

Theoretical materials

In order to get a better understanding of the impact of input parameters on computational outcomes, a set of theoretical materials have been studied, varying the inputs of λ and ρC. The following combinations have been reviewed, see table 2.

Table 2. The combination of the pairings of magnitude of λ and ρC

low λ low ρC	average λ low ρC	high λ low ρC
low λ average ρC	average λ average ρC	high λ average ρC
low λ high ρC	average λ high ρC	high λ high ρC

The input for the Voltra simulations as shown in table 2 has been implemented by λ values of 0.01 (low), 0.6 (average) and 2.0 (high). Values for ρC have been varied by changing C only, with ρ taken to be constant at 1000. Values used for C are 100 (low ρC), 2000 (average ρC) and 6000 (high ρC). Note that 'low', 'average' and 'high' values are related to common materials in building construction. Most of the Voltra studies showed the characteristic behaviour previously exhibited when the rise in probe temperature was plotted against the natural logarithm of the elapsed heating time. Values of the calculated thermal conductivity from the Voltra simulations generally agreed with the values inputted to the model. However, with the low thermal conductivity study, with a value of 0.01W/m^{2}K, for the three associated values of the specific heat capacity, (100, 2000 and 6000J/kgK) and with the adopted heating input of 3W/m, the rises in temperature at 1mm from the heater were large compared with the laboratory studies. The Voltra model showed typical temperature rises of 40^{o}C to 100^{o}C in 200s, whereas in the laboratory or field studies 6 ^{o}C to 9 ^{o}C would be expected. To analyse the low thermal conductivity data from the model studies provided an interesting vehicle for exploring the analysis routine of Goodhew and Griffiths (2004).

There are practical reasons for restricting the data analysis to 300 to 400s when analysing data from the laboratory and field. For example, when the increase in the probe rise in temperature becomes too small for practical power inputs, there is considerable data scatter and axial heat losses mar the results. However, the Voltra studies consider the heat flow in an infinite medium with no material probe and therefore the results provide an opportunity to view the thermal probe technique from a simple theoretical stance, or to test the model, the assumptions, the mathematics or theory and the data analysis strategies. As there is no probe in the Voltra simulation the probe to sample conductance, H is infinitely large and the thermal capacity of the probe must be zero. Here is an opportunity to test the analysis routine employing equations 1 and 2 with regression analysis to determine both the thermal conductivity and the diffusivity.

When analysing probe data for each and every power level per m, probe and sample combination the question arises, "What is a long time, or when can the Blackwell (1954) two constant expression be safely applied to the collected data?". Or, in other words, what time must elapse before equations 1 and 2 describe the experimental data? Goodhew and Griffiths (2004) provided a possible solution by suggesting that the percentage error (E%) 2 or 4 constants value of 1% be used as a criterion for selecting the regression analysis time window. A Solver routine employing the Blackwell four constants expression is used to determine approximate values of sample conductivity and diffusivity, mainly as a guide, but more importantly to determine the elapsed heating time required to allow the assumption of long time; that is the time that must elapse so that the error between the Blackwell two constants and the Blackwell four constants is less than 1%.

The chosen set of data for this discussion was the Voltra study with thermal conductivity 0.01W/m^{2}K, thermal capacity 100J/kgK, and density 1000kg/m^3, giving a diffusivity of 10^{-7} m^2/s. The heater power was 3W/m, with the temperature rise in the medium measured at 1mm from the line source. A graph of the rise in temperature versus the natural logarithm of the elapsed heating time is shown in figure 6. Visual inspection of the curve in figure 6 confirms that there is the characteristic "s" shape before the asymptotic approach to the final straight line beyond natural logarithm time of 5.2, or beyond a time of about 180s. The temptation is to apply the regression analysis to this data from 30 to 1500s, or natural logarithm 3 to 7. Here, the resulting conductivity is 0.0103W/m^{2}K and diffusivity 1.12x10^{-7}m^2/s. The conductivity is 3% above the true or input value, while the diffusivity is 12% above the input value. These errors are of similar magnitude to those often found in the experimental laboratory and field work.

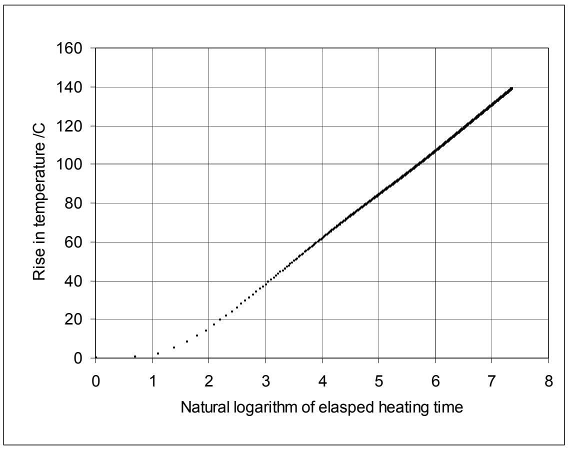

Figure6. Graph of rise in temperature versus natural logarithm of elapsed heating time for Voltra simulation. Temperature measured at 1mm from line heat source.

As described within the introduction, an analysis recipe proposed by Goodhew and Griffiths (2004) was used to simultaneously determine the three unknowns *α*, λ and *H*. A Solver 4.3 routine was applied to this data over the time interval 1300 to 1500s. Solver 4.3 is a Blackwell 4 constant expression with 2 variables, the sample thermal conductivity and diffusivity. The probe conductance was set to a large number (inferring very good thermal contact between the heat source and the medium to be measured) and the probe capacity is set to zero. Figure 7 shows the main graph from the Solver sheet. Here the Solver line describes the data only at times greater than 400 to 500s. The Solver 4.3 routine gave low values of conductivity and diffusivity, but the interesting feature here is that the time that must elapse before the error between the 2 and 4 constant expressions falls to below 1% is 750s, as can be seen in figure 8. Therefore, the conclusion to be drawn is that the equations 1 and 2 cannot be applied to this data set until the heating time exceeds 800s.

Applying the regression analysis to the Voltra data over the time interval 850 to 1500s, results in a thermal conductivity of 0.01002W/m^{2}K (+0.2%), and diffusivity 0.973x10-7m^2/s (-2.7%). These errors are calculated in terms of the known true values for conductivity and diffusivity that were initially inputted into the Voltra package. With the Solver 2.3 routine and the data set in the time window 850 to 1500s, the resulting conductivity is 0.01002W/m^{2}K, (+0.2%), diffusivity 0.975x10-7m2/s (-2.5%), again errors in terms of true values.

Finally, moving the Solver 2.3 time window to longer times, namely 1300 to 1500s, the resulting conductivity is 0.01008W/m^{2}K, (+0.8%), diffusivity 1.01x10-7m2/s (1%), again errors in terms of true value. When the regression analysis was applied to the data set in the time window 1300 to 1500s, the resulting conductivity was found to be 0.01007W/m^{2}K, (+0.7%), and diffusivity 1.006x10-7m^2/s (0.6%). This represents close agreement between model, theory and the Voltra simulation.

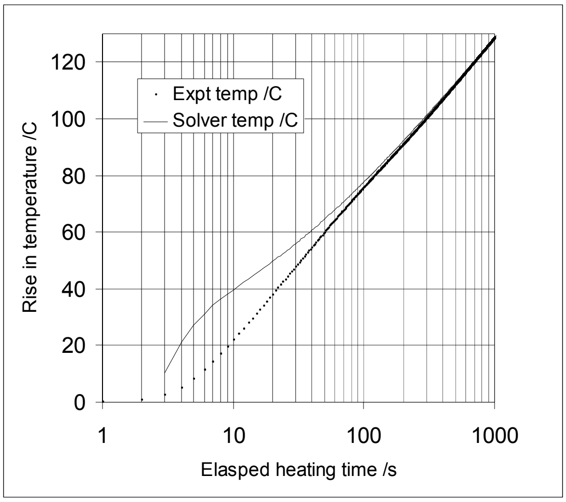

Figure 7. The Solver 4.3 predicted temperature rise plotted with the Voltra simulation results, here denoted as "Expt temp /C", both as functions of the elapsed heating time in seconds.

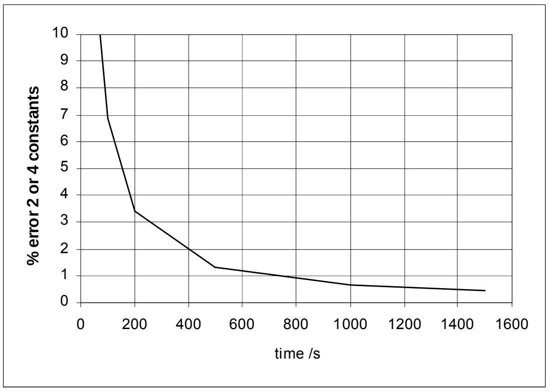

Figure 8. The E% 2 or 4 constants plotted as a function of time for the Voltra simulation. A 1% error or less is only obtainable after a heating time of 750s.

CONCLUSIONS AND REMARKS

This paper describes the use of simulation to guide the development of a thermal probe apparatus. The following conclusions have been drawn from the results:

1. Using the transient heat transfer program Voltra, a line source in an (almost) infinite material sample has been modeled. The simulation model has been used to generate datasets which have been analysed for λ and α by means of the same formula that are currently being used to analyse experimental data from an actual thermal probe apparatus.
2. The Voltra model, the assumptions and the theory provided by Blackwell (1954) appear to be satisfactorily describing the practical arrangement
3. The importance of selecting the appropriate power for the experimental work is underlined, since long times also require measurable temperature rises, i.e. temperature rises that are clearly above the experimental scatter.
4. Applying the regression analysis to data sets at inappropriate time windows, that is short times, often leads to values for the thermal constants that appear of suitable magnitude, but closer and more careful examination of the data can lead to more accurate values for these thermal constants.
5. The Solver 4.3 routine provides the answer to the question of what is a long time, by allowing the error between the Blackwell 2 and 4 constant expressions to be explored.
6. The Solver 2.3 routine applied to the correct time window leads to accurate values of both conductivity and diffusivity.
7. The Voltra study has allowed a demonstration of the probe technique and supports the analysis routine proposed, Goodhew and Griffiths (2004); albeit when applied to a simplified model.

FUTURE WORK

In general, simulation results help to gain a deeper appreciation of the behaviour of results from a theoretical line source model. This may be required to understand and address the level of error and uncertainty caused by the conditions imposed by the constraints of any practical apparatus.

Further work will be carried out to advance the understanding of the behaviour of an actual thermal probe apparatus. This will involve modeling of a probe with finite dimensions and its own material properties, moving towards both a probe and material sample with given dimensions, incorporation of the thermal resistance between probe and sample, and inclusion of boundary effects for both probe and sample.

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