

## A NEW SIMULATION SYSTEM FOR RADIATION CALCULATION WHEN OPAQUE AND HALF TRANSPARENT OBJECTS EXIST TOGETHER

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### ABSTRACT

Here a generalized 3-D radiation simulation system is developed with Monte-Carlo and Gebhart method. There are two kinds of basic units in the system, which are surface unit for opaque objects and body unit for transparent objects respectively. When the bundles were emitted from certain point or surface with Monte-Carlo method, it would enter into an opaque surface or a transparent body. For the former, it would be reflected and absorbed. However, when the second situation happened, there is the third process named penetration beside above two processes. Those three processes would be continued until the bundles reach an opaque surface. With calculating the amount of bundles and that absorbed by another surface or body, the direct area exchange index could be obtained. With Gebhart method and the calculation of net angle coefficient, the radiation energy exchange with short wave and long-wave radiation could be calculated easily.

### KEY WORDS

Radiation calculation, Monte-Carlo and Gebhart method, half-transparent object, opaque object, thermal environment

### INTRODUCTION

In the study of indoor and outdoor thermal environments, the radiation simulation and calculation are very difficult when the opaque and transparent objects coexist, especially when studying the effects of greening to outdoor thermal environment, atrium's thermal environment, and indoor thermal environment with transparent

enclosure structure. The opaque objects mainly include glass, curtains, louvers and the plant canopies. The traditional method is hard to deal with these issues since it needed to suppose a radiation distribution or give a reliable boundary condition and the accuracy is hard to control. In addition, the traditional method is difficult to simulate the air temperature and radiation temperature in continuous hours. For example, when there are some plants canopies exist in outdoor environment, it is difficult to give a boundary heat flux.

In order to solve problems mentioned above, a generalized 3-D radiation simulation system is developed with Monte-Carlo and Gebhart methods. There are two kinds of basic units in the system, which are surface unit for opaque objects and body unit for transparent objects respectively.

### BASIC ASSUMPTION

The radiation principle for transparent objects is similar to the gas with high temperature. According to the Bouguer Law, the radiation intensity decreases exponentially as the distance increases when the heat radiation transmits the absorption and diffuse layers, and the exponential index equates with the integral of local decreasing coefficient with the distance.

$$I_{\lambda}(l) = I_{\lambda}(0) \exp[-kl] \quad (1)$$

It accords with the long-wave and short-wave radiations in transparent objects (especially plant canopies). Therefore, compared with the methods listed in literatures<sup>[2,3]</sup>, a general radiation calculation system is proposed and established when opaque and transparent objects coexist. The other assumptions are as follows:

1) All surfaces are simplified as diffuse grey objects

for radiation calculation;

2) The upwards and downwards reflection rates for the transparent objects are the same;

3) The second reflection is dealt as diffuse reflection, namely the mirror reflections among transparent objects are not considered in this system.

Considering the calculation space as a close cavity, the inside radiation and heat exchange processes could be described as follows.

In a closed blackbody, radiation energy transmits between any two surfaces that could be seen by each other. The radiation energy from surface  $i$  to  $j$  could be described by Formula (2), where  $k_iLAD_iV_i$  is the equivalent area of one plant canopy unit  $i$ .

$$Q_{i \rightarrow j} = D_{ij} E_i = D_{ij} \sigma T_i^4$$

$$= \begin{cases} A_i F_{ij} \sigma T_i^4 & : \text{opaque object} \\ 4k_i LAD_i V_i F_{ij} \sigma T_i^4 & : \text{transparent object} \end{cases} \quad (2)$$

### RADIATION SYSTEM DESCRIPTION FOR THE WHEN OPAQUE AND TRANSPARENT OBJECTS EXIT TOGETHER

There are two kinds of basic units in this simulation system, one is surface units of opaque objects, and the other is body units of transparent objects, shown in Figure.1. These two objects constitute the basic units of radiation simulation and calculation systems.

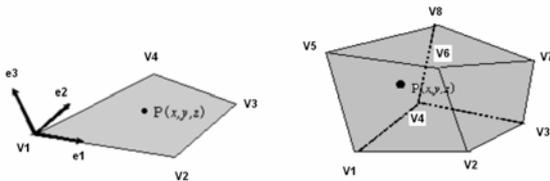


Figure 1 Basic units of radiation system (Left is opaque objects; Right is transparent objects)

The probability distribution of radials' directions radiating from the mesh surface should meet Lambert cosine law [3]. Therefore, in a certain auxiliary coordinate system of unit surface, the unit  $f$  of the radiating direction is calculated by:

$$f = \sqrt{1-p^2} \cos 2\pi R_\theta e_1 + \sqrt{1-p^2} \sin 2\pi R_\theta e_2 + p e_3 \quad (3)$$

$$p = \sqrt{1-R_\eta} \quad (4)$$

Where,  $R_\theta$  and  $R_\eta$  are random numbers between 0 and 1, respectively corresponding to the azimuth angle  $\theta$  and altitude angle  $\eta$ . The column vector  $f$  is the direction vector in the auxiliary coordinate system of unit surface, which could be changed into its equivalent in general coordinate system using formula (5):

$$f' = T f \quad (5)$$

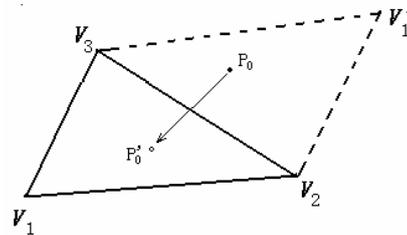


Figure 3 The generating method of random points in plane triangles

The direction  $f$  of the radials is determined by following formulas:

$$f' = \sqrt{1-p^2} \cos 2\pi R_\theta e'_1 + \sqrt{1-p^2} \sin 2\pi R_\theta e'_2 + p e'_3 \quad (6)$$

$$p = 1 - 2R_\eta \quad (7)$$

The traveling distance of ray beams in transparent objects could be calculated using Beer law:

$$l = -\ln(1 - \varepsilon_p) / kLAD \quad (8)$$

where  $\varepsilon_p$  is the emission rate of current layer in transparent objects (equal with absorption rate);  $LAD$  is the average porosity of transparent objects, which is the canopy's leaf area index density for plants ( $m^2 \cdot m^{-3}$ );  $k$  is the long wave (or short wave) dispersion coefficient of current layer in transparent objects ( $k_s$  and  $k_l$ ).

The traveling distance of ray beams from point P to point Q is:

$$\Delta l = |PQ| = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2 + (z_q - z_p)^2} \quad (9)$$

Different from only opaque objects exit, due to the existence of transparent objects, the ray beams would

be absorbed, reflected when meeting opaque objects (such as ground, external surface of walls), or be absorbed, reflected and transmitted through by transparent objects when the ray beams arrive at the grid elements of current layer of transparent objects. In fact, the outcomes of ray beams are listed as follows:

$$\left\{ \begin{array}{l} l < \Delta l : \text{being absorbed} \\ l \geq \Delta l : \text{reach the surface of the body} \\ \quad \text{opaque objects : being absorbed} \\ \text{transparent objects : transmit into adjacent objects} \end{array} \right. \quad (10)$$

When the ray beams transmitted through adjacent transparent objects, the radial-tracing starting point P would be replaced by point Q, and the distance of ray beams would be re-calculated:

$$l' = (l - \Delta l)kLAD / k'LAD' \quad (11)$$

where,  $l'$ ,  $LAD'$  and  $k'$  are the traveling distance, porosity and dispersion coefficient of ray beams when coming into new transparent body units. Evidently, the radiation system established in this paper can simulate complex transparent body radiation transmission process with different properties in the vertical direction.

### DIRECT EXCHANGE AREA CALCULATION

The total number of radials emitted from surface  $i$  is  $N_{tot}$ , and the number of ones absorbed by surface  $j$  is  $N_{ij}$ , then the direct exchange area  $D_{ij}$  is calculated by formula (12):

$$D_{ij} = \begin{cases} A_i N_{ij} / N_{tot} & : \text{opaque} \\ 4k_i LAD_i V_i N_{ij} / N_{tot} & : \text{translucent} \end{cases} \quad (12)$$

Although the direct exchange area calculated by Monte-Carlo method can meet the radiation integrity rules as shown in formula 13, it cannot meet the radiation symmetry rules as shown in formula 14:

$$\sum_{j=1}^n D_{ij} = A_i \quad (13)$$

$$D_{ij} = D_{ji} \quad (14)$$

Using this method to calculate the direct exchange area would lead to unreasonable results. From the aspect of thermodynamics rules, the integrity and symmetry rules should be met simultaneously. Therefore, this paper proposes new methods as formulas (15) and (16), making the direct exchange area meets the requirements of these two rules.

$$D_{ij} = D_{ji} = \begin{cases} \frac{D_{ij} A_j^m + D_{ji} A_i^m}{A_i^m + A_j^m} \\ \frac{D_{ij} (4k_j LAD_j V_j)^m + D_{ji} A_i^m}{A_i^m + (4k_j LAD_j V_j)^m} \\ \frac{D_{ij} (4k_j LAD_j V_j)^m + D_{ji} (4k_i LAD_i V_i)^m}{(4k_i LAD_i V_i)^m + (4k_j LAD_j V_j)^m} \end{cases} \quad (15)$$

$$D_{ij} = \begin{cases} A_i D_{ij} / \sum_{j=1}^n D_{ij} & : i = \text{opaque} \\ 4k_i LAD_i V_i D_{ij} / \sum_{j=1}^n D_{ij} & : i = \text{transparent} \end{cases} \quad (16)$$

To the right of formula (16), the corresponding calculation methods are presented top-down respectively for "different opaque surfaces", "opaque and transparent surfaces coexist" and "different transparent objects coexist".

Assuming the symmetric index  $m$  as 1, considering the plant canopy in radiation calculations, the direct exchange area calculation is shown in formula (16).

The entire exchange area could be calculated using Gebhart method, as described in the literature [2, 4]. In this method, the direct exchange area  $D_{ij}$  defined in blackbody space is replaced with the entire exchange area  $C_{ij}$  in grey-body space. Using  $C_{ij}$ , the radiation exchange heat  $Q_{i \rightarrow j}$  between solid surface grid  $i$  and  $j$  could be calculated.

### SOLAR RADIATION CALCULATIONS

The calculation method of the sun's direction vector

is shown in formula (17):

$$\vec{e} = \vec{i} \cosh \cos \alpha + \vec{j} \cosh \sin \alpha + \vec{k} \sinh \quad (17)$$

where  $h$  is solar altitude angel,  $\alpha$  is solar azimuth angle.

The external normal vector of any point on buildings or land surface is  $\vec{n}$ , assuming that the sunlight casts on the surface in parallel, and the unit vector directing to the sun from the surface along the sunshine is  $\vec{e}$ . This point could be irradiated by the sun when:

$$\vec{e} \cdot \vec{n} = \cos \theta > 0 \quad (18)$$

where  $\theta$  is the incident angel of sunshine. In the formula, under the conditions of formula (18), the radials emitted from the location of the Sun radiates along the sunshine vector, respectively searching various facets of the calculation area. If the surrounding buildings or trees are on the way of this trajectory, the sunlight would be blocked, absorbed and weakened, and ray beams would be reduced correspondingly.

The direct solar radiation heat emitted from sky grid  $i$  is:

$$G_i = A_i I_N \cos \theta \quad (19)$$

Assuming the number of particles radiating from the sky grid  $i$  is  $N_i$ , and the number of particles arriving at the surface grid  $j$  in calculation area is  $N_{ij}$ , then the solar radiation percentage  $\beta_i$  of surface grid  $j$  could be calculated using formula (22):

$$\beta_{ij} = N_{ij} / N_i \quad (20)$$

Therefore, the direct solar radiation heat absorbed by plane  $j$  is

$$E_{Dj} = \alpha_j \sum_i G_i = \alpha_j \sum_i A_i \beta_{ij} I_N \cos \theta \quad (21)$$

Where  $\alpha_j$  is the short-wave absorption rate of underlying surfaces, building surfaces or leaves;  $A_i$  is the area of sky grid  $i$  of radiating radials.  $I_N$  is the normal direct solar radiation when the light arrives at the ground surface:

$$I_N = I_0 P^{1/\sinh} \quad (24)$$

where  $I_0$  is the solar constant,  $h$  is solar altitude angel;  $P$  is transparent rate of air.

Sky scattering radiation heat  $E_{Sj}$  collected from surface grid  $j$  in calculation area is:

$$E_{Sj} = A_j F_{jS} I_{SH} \quad (23)$$

where  $F_{jS}$  is angle coefficient of surface grid  $j$  in calculation area to the sky, and could be calculated using the Monte-Carlo method described in previous chapters.  $I_{SH}$  is horizontal sky solar radiation intensity during sunny days, and could be approximately calculated using Berlage formula:

$$I_{SH} = 0.5 I_o \sinh \frac{1 - P^{1/\sinh}}{1 - 1.4 \ln P} \quad (24)$$

The short-wave absorption rate of surface grid  $j$  that receives sunlight is  $\alpha_j$ , the energy that is absorbed directly by grid  $j$  is  $Q_{Dj}$ , the reflection energy is  $Q_{Rj}$ , and the calculation method is as follows:

$$Q_{Dj} = \alpha_j (E_{Dj} + E_{Sj}) \quad (25)$$

$$Q_{Rj} = (1 - \alpha_j) (E_{Dj} + E_{Sj}) \quad (26)$$

The energy  $Q_{Rk}$  that is reflected by grid  $j$  is reflected for many times, and finally is absorbed by various surfaces including the sky. The sunlight reflected to the sky could be considered as radiations towards the universe. Then the reflected energy absorption amount of grid surface  $j$  to grid surface  $k$  is:

$$Q_{kj} = \frac{C_{kj}}{\alpha_k A_k} Q_{Rk} = \alpha'_{kj} Q_{Rk} \quad (27)$$

where  $C_{kj}$  is the entire exchange area of grid surface  $j$  to grid surface  $k$ ,  $\alpha'_{kj}$  is the absorption factor for short-wave radiations. If considering the mutual reflections of all surfaces, the total solar radiation energy absorbed by grid  $j$  is:

$$Q_j = Q_{Dj} + \sum_{k=1}^n Q_{kj} \quad (29)$$

Using above methods, the total received solar heat of surface grids (for transparent objects, it refers to the volume unit) of different objects could be calculated,

when the transparent and opaque objects coexist outdoor in any time.

### CALCULATION CASE

Figure 5 shows a simulation and calculation example of short-wave radiations received by the ground, when there is a tree on the open outdoor cement floor (Beijing, July 21).

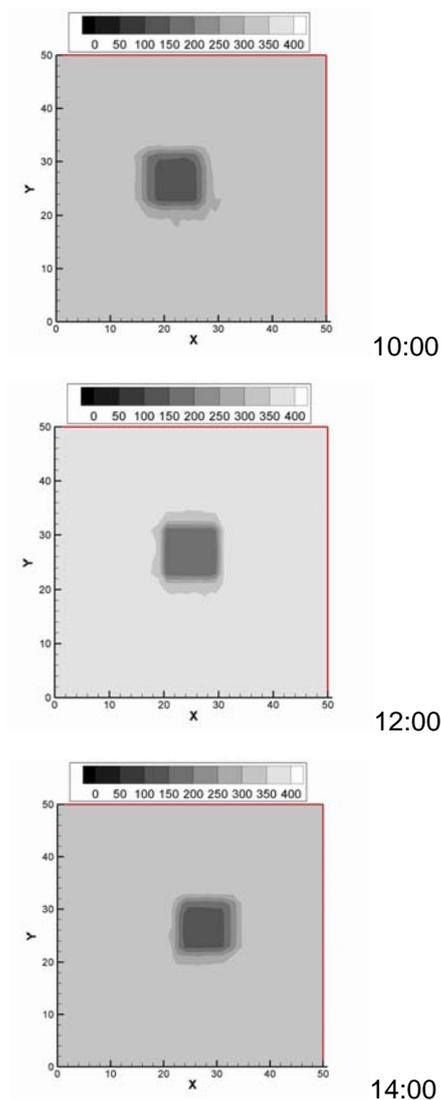


Figure 5 results of solar radiation heats received by the ground (unit:  $W/m^2$ )

In these figures, the plant canopy is three meters away from the floor, with the shape of  $5 \times 5 \times 3m$  and the  $LAD$  of  $2m^2 \cdot m^{-3}$ , its reflection rate 0.3 and its short-wave dispersion coefficient 0.6. The short-wave reflection rate and absorption rate is 0.4.

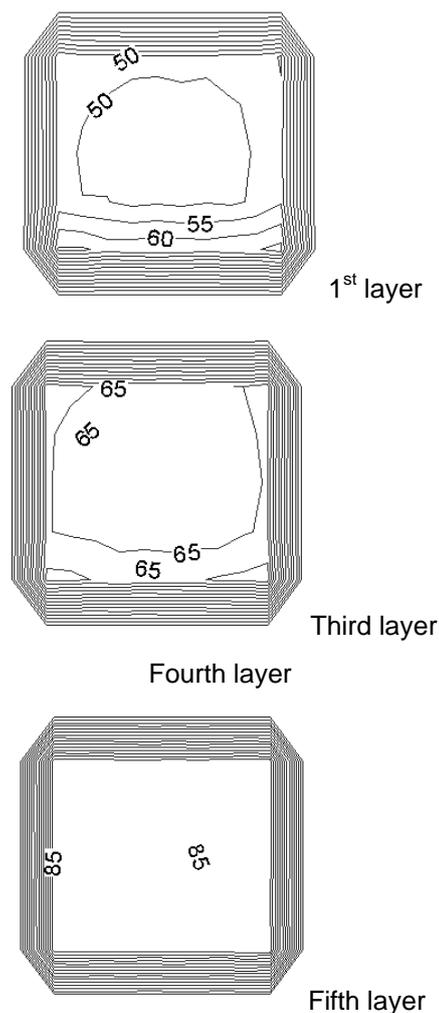


Figure 6 Comparison of solar radiation heats received by different layers of the tree

Figure 6 provides the received heat results of 5 layers bottom-up of plant canopies at noon (unit:  $W/m^2$ ). Obviously, the absorbed solar radiations of plant canopy at the bottom are fewer than that at the top, and the former accounts for approximately 60% of the latter.

### DISCUSSION

The general radiation calculation system established above could be applied to calculate the long-wave and short wave radiations when opaque objects (such as opaque building enclosure, ground, etc.) and transparent objects (including plant canopies such as trees, bushes, grass, etc., glass, sunshield, and so on). It can provide results of the received solar short-wave heat of various plant canopies (or glass and sunshield) and the received short-wave radiation heat of solid

surfaces (ground surface, building external surface, etc.). It can also provide the entire exchange areas ( $C_{ij}$ ) of various plant canopies with other solid surfaces in calculation areas, to provide boundary conditions for outdoor long-wave radiation and heat balance calculation. Simultaneously, it can provide data for indoor long-wave and short-wave radiation heat calculations of the atriums or transparent enclosures, and further provide data for thermal comfort conditions under such environments. Currently this system has been applied to the research on simulation and analysis of the effects of greening to outdoor thermal environment<sup>[5]</sup>.

Moreover, this simulation system could be integrated with building energy simulation software, to calculate and analyze the effects of vertical greening, roof greening and high trees shield surrounding buildings to building energy, enclosure structure heat balance and temperature distributions of internal and external surfaces of buildings.

## CONCLUSION

This paper aims to solve the long-wave and short wave radiation calculations of plant canopies in the green areas. Based on the exponential declining law of long-wave and short-wave radiation transmissions in plant canopies and Monte-Carlo, Gebhart methods, this paper establishes a general calculation system for both long-wave and short-wave radiations when the opaque and transparent objects coexist.

Therefore, with these parameters including the leaf area index of plant canopy ( $LAD$ ), shapes, reflection rate, dispersion coefficient, etc., the reflection, absorption, transmission of plant canopies to the solar short-wave radiations and its heat exchange with the environment through long-wave radiations could be simulated. In addition, it provides basis for studying the radiations and heat balance calculations of plant canopy, heat balance of district radiations, heat balance between air and underlying surfaces, and outdoor thermal environment radiation simulation and thermal comfort assessments.

Besides, this paper discussed about the generality of this radiation simulation systems, and points out its

feasibility in solving long and short wave radiation calculations when opaque objects (such as opaque building enclosures, ground, etc.) and transparent objects (including plant canopies, glass and sunshield, etc.) coexist, and in combination with building energy simulation.

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