COMPARING TWO METHODS OF STOCHASTIC MODELING FOR BUILDINGS

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ABSTRACT

In buildings research Monte Carlo (MC) methods are used for uncertainty analyses. This paper presents an alternative approach for uncertainty analysis in buildings, coined the internal method. Here, the joint probability density functions (pdfs) of model outputs are directly computed from pdfs of the inputs. This method needs only a single simulation run to compute the output pdfs compared to significantly higher number of runs for MC methods. The application of the two competing techniques (MC and internal method) to the most elemental building problem (1R1C) reveals that MC methods prove to be much more appropriate for building problems.

INTRODUCTION

The relative influence of stochastic variations on the building energy consumption becomes higher for low energy buildings. This relationship is visualized in Figure 1. Here we compare a hypothetical traditional existing building with a mean consumption of 150 kWh/(m²a) to a very low-energy building, e.g. a Passivhaus, with an annual consumption of 15 kWh/(m²a).

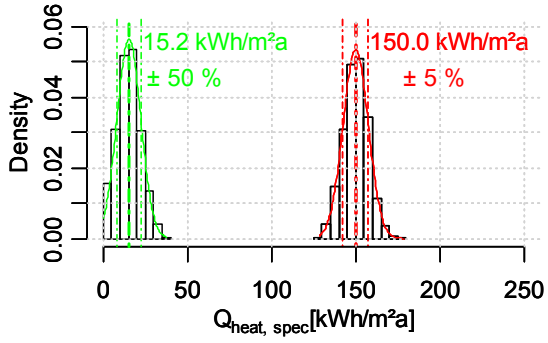

Figure 1: Comparison between two buildings with different annual energy densities with the same absolute uncertainty

It is assumed that the absolute stochastic variability in the consumption is independent of the consumption (in both cases ± 7.5 kWh/(m²a)). This results in a relative uncertainty for the traditional building of ± 5 % and for the Passivhaus of ± 50 %. Neglecting an uncertainty of ± 5 % may be justified in many cases but neglecting ± 50 % uncertainty is certainly not. Neglecting high levels of uncertainty in building design, retrofit or building operation can lead to disappointments and has a high potential for discrediting new and valuable technologies.

To date, uncertainty analyses are not common in the building sector (Macdonald, 2002), even for low energy buildings. Recently this has gained some attention (IEA ECBCS Annex 55; Jacob et al., 2010; Burhenne et al., 2010).

LITERATURE REVIEW

Xu et al. (2005) postulate, that for building optimization in particular uncertainties need to be considered. Possible approaches are shown e.g. in Jiang et al. (2007) and Jacob and Burhenne (2010). Uncertainty analysis has been examined in terms of both internal and external probabilistic approaches to quantifying the overall effect of parameter uncertainty in building simulations (Macdonald, 2002; Jensen et al., 1994; Lomas et al.; 1997). Attention has also been focused on how stochastic processes external to the building (e.g., meteorological events) influence the thermal processes in the building (Jiang and Hong 1993), or how knowledge of internal building processes (e.g., occupancy patterns) can lead to improved building operation. A study of the general problem of inference in buildings (Dodier 1999) showed how prediction, diagnosis, and calculating the value of information could be applied and verified. An example for the rarely treated internal stochastic building model is given in (Andersen et al., 2000). This work shows mathematically advanced techniques but the described stochastic differential equation is not used to calculate the probability density functions of the building model output variables but how to calibrate the stochastic model to measured data.

STOCHASTIC BUILDING MODELS

There are two different basic stochastic models, which are coined the external and internal method (the terms were introduced by Macdonald 2002):

1. External methods are Monte Carlo methods. In Monte Carlo (MC) methods classical deterministic building models are used. For all uncertain model inputs random samples are generated. Then the deterministic building model is executed or simulated for every element of

this input sample. From the individual model outputs, the joint probability distribution of the stochastic output variables can then be derived.

2. The internal method directly calculates the probability density functions (pdfs) of the stochastic output variables from the pdfs of the stochastic input variables by a stochastic equation or a stochastic differential equation in dynamic cases.

For this second technique – the internal method – almost no commonly known building simulation techniques can be used. Due to nonlinearities and other mathematical difficulties (like mode switching) in building simulations, some computationally expensive techniques including numerical integration and convolution will likely be necessary in more realistic examples.

This work has shown that the initial expectation of **shorter calculation times** with the internal method **could not be fulfilled**.

Reporting these results about the internal method still seems to be expedient:

1. For some special cases internal methods are superior to MC methods (e.g., if an analytical solution is possible such as for simple stationary building models),
2. the description seems to be mathematically more consistent,
3. and it is possible to gain some additional insights into the stochastic nature of the problem (e.g. dependencies between variables in a probabilistic sense) and
4. it shows, that the internal method seems not to be relevant for the building practice in the near future.

This last point is the reason for not having carried out realistic simulation studies using the internal method.

Building Model

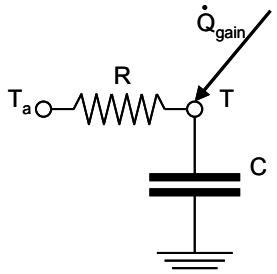

Figure 2: Most elemental dynamic building model

The two methods are applied to the most elemental dynamic building model (Figure 2) consisting of one resistance and one capacity (1R1C). A dynamic model was chosen to show how it is possible to treat a dynamic model employing the internal method. The resistance contains transmission and ventilation losses to the ambient and therefore is not constant in time. The term $\dot{Q}_{gain}$ contains internal, solar and heating and cooling equipment gains.

With an initial value this differential Equation (1) can be solved.

$$\begin{aligned} C\frac{dT}{dt} &= (H_T + \dot{V}_{ven}\cdot c_p)\cdot(T_a - T) + \dot{Q}_{gain} \\ \dot{Q}_{gain} &= \dot{Q}_{Int} + const_{Sol}\cdot z + \dot{Q}_{HC} \\ \frac{dT}{dt} &= f(T,...) \\ T(t=0) &= T_{start} \end{aligned} \tag{1}$$

Now T_i, $\dot{V}_{vent}$, $\dot{Q}_{Int}$, z and $\dot{Q}_{HC}$ are regarded as independent stochastic variables. To solve this stochastic differential equation in the framework of the internal method, Equation 1 needs to be integrated:

$$T(t) = T_{start} + \int_0^t f\left(T(t'),...\right)dt' \tag{2}$$

It can be solved with the following simple Eulerian discretization scheme.

$$\begin{aligned} T(t_{i+1}) &= T(t_i) + f\left(T(t_i),...\right)\cdot\Delta t \\ \Delta t &= t_{i+1} - t_i \\ T_{i+1} &= T_i + \left(\left(H_T + \dot{V}_{ven}c_p\right)\left(T_a - T_i\right) + Q_{gain}\right)\frac{\Delta t}{C} \\ Q_{gain} &= \dot{Q}_{Int} + const_{Sol}\cdot z + \dot{Q}_{HC} \\ T_0 &= T_{start} \end{aligned} \tag{3}$$

More complicated solution schemes are not compatible with the methods developed below for the internal method. This method also relies on stochastically independent variables, which implies that every stochastic variable must only appear once in the equation. For this elemental model this is possible by algebraic manipulations that lead to Equation (4)[1]. This also prohibits the use of temperature dependent heat transfer coefficients.

$$\begin{aligned} T_{i+1} &= T_a + \left(\left(H_T - \frac{C}{\Delta t} + \dot{V}_{ven}c_p\right)\left(T_a - T_i\right)\right. \\ &\quad \left. + Q_{gain}\right)\frac{\Delta t}{C} \\ Q_{gain} &= \dot{Q}_{Int} + const_{Sol}\cdot z + \dot{Q}_{HC} \\ T_0 &= T_{start} \end{aligned} \tag{4}$$

External Method (Monte Carlo Method)

The Monte Carlo-method (MC method) sometimes also called Monte Carlo simulation is a stochastic numerical method which involves repeated calculations on a random sample.

[1] Only if the outside temperature T_a is regarded as non-stochastic.

Generally, a MC analysis consists of the following steps to randomly sample the modeling domain (Saltelli et al., 2000):

1. Selection of the probability density function (pdf) for each uncertain input (X_i).
2. Generation of a sample from each pdf.
3. Simulating the model for each element of the sample.
4. Analysis of the results.

Internal Method

Equation (5) shows the relationship between the Probability P of a stochastic variable X being smaller or equal to x and the cumulative distribution function $F_X(x)$ and the probability density function of X (pdf_X):

$$P(X \le x) = F_X(x) = \int_{-\infty}^{x} pdf_X(x')\,dx' \qquad (5)$$

Stochastic textbooks (e.g. Soong, 2004) give a general relationship for calculating the pdf_Y from the pdf_X if Y is the function $g(X)$:

$$Y = g(X) \Rightarrow X = g^{-1}(Y) \qquad (6)$$

$$pdf_Y(y) = \left|\frac{dg^{-1}(y)}{dy}\right| pdf_X\left(g^{-1}(y)\right) \qquad (7)$$

This relationship even holds for piecewise invertible functions (Soong, 2004). In the following this relationship is explicitly written out for the cases described in Equation (4).

For linear transformations one obtains:

$$Y = a \cdot X = g(X)$$

$$\Leftrightarrow \quad X = \frac{1}{a} \cdot Y = g^{-1}(Y) \quad a \neq 0 \qquad (8)$$

$$\Leftrightarrow pdf_Y(y)\,dy = \left|\frac{1}{a}\right| pdf_X\left(\frac{y}{a}\right) \qquad (9)$$

The sum of a stochastic variable X and a non-stochastic variable b is:

$$Y = b + X = g(X) \Leftrightarrow X = Y - b = g^{-1}(Y) \qquad (10)$$

$$\Leftrightarrow pdf_Y(y) = pdf_X(y - b) \qquad (11)$$

For the sum Z of two stochastic variables X and Y a general expression is given (Soong 2004) that also holds if X and Y are stochastically dependent variables:

$$Z = X + Y$$

$$\Leftrightarrow pdf_Z(z) = \int_{-\infty}^{\infty} pdf_{X,Y}(x, z - x)\,dx \qquad (12)$$

Here, $pdf_{X,Y}(x,y)$ is the joint probability density function of X and Y. In case of independent variables this becomes:

$$pdf_Z(z) = \int_{-\infty}^{\infty} pdf_X(x) \cdot pdf_Y(z - x)\,dx \qquad (13)$$

Equation (13) is called the convolution of pdf_X and pdf_Y. It is possible to calculate the convolution with a Fourier transformation (Equations 14-16). This is numerically very effective when Fast Fourier Transformation (FFT) is used.

$$FT(f_t)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_t(t) \cdot e^{-i\omega t}\,dt = f_\omega(\omega) \qquad (14)$$

$$FT^{-1}(f_\omega)(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_\omega(\omega) \cdot e^{i\omega t}\,d\omega \qquad (15)$$

$$\text{with } f_t(t) = FT^{-1}\left(FT(f_t)\right)(t)$$

As a result, the convolution formula becomes:

$$pdf_Z(z) = FT^{-1}\left(FT(pdf_X) * FT(pdf_Y)\right)(z) \qquad (16).$$

Now an algorithm for calculating the sum of two stochastic variables can be given:

Algorithm 1: Calculation of the pdf of the sum of two independent stochastic variables from their pdfs

1: Search for an adequate common domain of definition for both pdfs so that both have equal length and account for the periodicity in the FFT in the convolution. The domain must start at zero.

2: Discretize $pdf_X \rightarrow x_1, ..., x_m$ and $pdf_Y \rightarrow y_1, ..., y_m$ on equidistant points of support. The number of points is equal to a power of 2 ($m = 2^j$ with $j \in \mathbb{N}^+$).

3: Calculate the discrete Fourier transformation with FFT: $x'_1, ..., x'_m = FFT(x_1, ..., x_m)$ and $y'_1, ..., y'_m = FFT(y_1, ..., y_m)$.

4: Convolute in Fourier space: $z'_1, ..., z'_m = x'_1 y'_1, ..., x'_m y'_m$.

5: Inverse transformation: $z_1, ..., z_m = FFT^{-1}(z'_1, ..., z'_m)$.

6: If necessary convert the discrete values back to a function (e.g. with splines).

For the product Z of two stochastic variables X and Y a general expression is given (Soong, 2004) that also holds if X and Y are dependent variables:

$$Z = X \cdot Y$$

$$\Leftrightarrow \mathrm{pdf}_Z(z) = \int_{-\infty}^{\infty} \mathrm{pdf}_{X,Y}\left(x, \frac{z}{x}\right) \frac{1}{|x|} dx \qquad (17)$$

For stochastically independent variables this becomes:

$$\mathrm{pdf}_Z(z) = \int_{-\infty}^{\infty} \mathrm{pdf}_X(x)\,\mathrm{pdf}_Y\left(\frac{z}{x}\right) \frac{1}{|x|} dx \qquad (18)$$

For numerical efficiency it is desirable to seek a calculation method for Equation (18) based on the FFT technique. This can be done with the logarithm, a technique used for example in slide rules:

$$Z = X \cdot Y$$

$$\Leftrightarrow Z = \begin{cases} e^{\log(|X|)+\log(|Y|)} & X \cdot Y > 0 \\ -e^{\log(|X|)+\log(|Y|)} & X \cdot Y < 0 \\ 0 & X \cdot Y = 0 \end{cases} \qquad (19)$$

To perform this calculation we turn to Equation (6) and (7):

$$Y = \ln(X) = \mathrm{g}(X)$$

$$\Leftrightarrow \quad X = e^Y = \mathrm{g}^{-1}(Y) \text{ for } X > 0 \qquad (20)$$

$$\Leftrightarrow \mathrm{pdf}_Y(y) = \left|e^y\right| \mathrm{pdf}_X\left(e^y\right) \qquad (21)$$

and for:

$$Y = e^X = \mathrm{g}(X)$$

$$\Leftrightarrow \quad X = \log(Y) = \mathrm{g}^{-1}(Y) \qquad (22)$$

$$\Leftrightarrow \mathrm{pdf}_Y(y) = \left|\frac{1}{y}\right| \mathrm{pdf}_X\left(\log(y)\right) \qquad (23)$$

Now an equivalent algorithm for the product of two independent stochastic variables can be provided:

Algorithm 2: Calculation of the pdf of the product of two independent stochastic variables from their pdfs

1: Distinguish all cases of Equation (19) and separate the pdfs accordingly.

2: Apply Equation (21) on all pieces: $\mathrm{pdf}_Y(y) = \left|e^y\right| \mathrm{pdf}_X\left(e^y\right)$ and the same for pdf_Y.

3: Apply Algorithm 1 for all cases (summation of the log(x) and log(y) in (19)).

4: Apply the inverse transformation (Equation 23): for all cases.

5: Combine all cases by summing the individual pdfs up (possibly a normalization is needed).

6: If necessary convert the discrete values back to a function (e.g. with splines).

The description of algorithm 2 might not be detailed enough for a technical implementation but it serves the purpose of estimating a lower limit on the computational resources needed for the internal method.

It appears to be important to note again that this is only valid for independent variables in a stochastic sense. If the stochastic variables depend on each other, the convolution technique is no longer applicable here. Then computationally significantly more laborious numerical integration techniques are needed for Equation (12) and (17).

RESULT: ESTIMATION OF THE COMPUTATIONAL RESOURCES NEEDED FOR THE TWO APPROACHES

With the above description of the two methods, it is now possible to estimate the necessary computational resources for both the external and internal method for the most elemental dynamic building model given in Equation (4). This estimation will be done for one time step and will assume that all calculations for the internal method are carried out on the basis of the same discretization. In more realistic applications, this will likely not be the case and further raise the necessary computational effort for the internal method. Later in this section this lower limit estimation for the computational resources needed for the internal method proves to be reasonable. For this estimation the variables T_i, $\dot{V}_{vent}$, $\dot{Q}_{Int}$, z and $\dot{Q}_{HC}$ are taken to be stochastic and independent. Furthermore, an ideal numerical coprocessor is assumed to be present which reduces the calculation of the basic algebraic operations (+, *, exp, log, sin, cos or complex exp) to one computational step.

$$T_{i+1} = T_a + \left(\left(H_T - \frac{C}{\Delta t} + \dot{V}_{ven} \cdot c_p \right) \cdot \left(T_a - T_i \right) + \dot{Q}_{Int} + const_{Sol} \cdot z + \dot{Q}_H \right) \frac{\Delta t}{C}$$

Figure 3 Equation (4) marked with color boxes for counting operations

In Figure 3, Equation (4) is marked with boxes in different colors to count the necessary operations. The result is shown in Table 1:

Table 1 Count of operations in Equation (4) according to Figure 3

Operation	count
sum of a constant and a stochastic variable:	3
product of a constant and a stochastic variable	3
sum of two stochastic variables	3
product of two stochastic variables:	1

External Method

For the Monte Carlo method, only one calculation step is needed for each of the operations listed in Table 1 under the above stated assumption of an ideal numerical coprocessor. If the stochastic input variables are time invariant or slowly changing in time compared to the time step in Equation (4) the sample generation can be neglected here.

Table 2 Estimate of the numerical resources needed for one time step of the Monte Carlo method

Operation	comp. resources
sum of a constant and a stochastic variable:	3
product of a constant and a stochastic variable	3
sum of two stochastic variables	3
product of two stochastic variables:	1
total count of ops per time step for each element of the MC sample	**10**
total count of operations per time step for the whole MC sample	$\mathcal{O}(10n)$

In Table 2 the necessary computational resources are estimated. From this it can easily be understood that the total numerical resources needed for one time step of the external method are of the order of ten times the sample size as shown in the last line of Table 2.

Here the numerical effort for setting up the models and those required to analyze the outputs for the external method is neglected. This is mainly done because that is only necessary at the beginning and the end of the simulation. Therefore the effort for that per time step is divided by the number of time steps, which is for normal building simulations a big number, especially for the Euler solving scheme needed for the internal method.

Internal Method

Doing the same exercise for the internal method is more complicated. The individual operations in Figure 3 need to be distinguished more closely.

According to Equation (11) for one summation between a stochastic variable X and a non-stochastic variable a, there are m subtractions of a necessary. Where $m = 2^j$ with $j \in \mathbb{N}^+$ is the discretization.

According to Equation (9) for one multiplication between a stochastic variable and a non-stochastic number a there are $2m$ multiplications with $1/a$ necessary.

The computational resources for one summation between two stochastic variables can be estimated from Algorithm 1. Since we assumed that we do not have to change the discretization, steps 1., 2. and 6. are not applicable here. Remaining are two FFTs and one pairwise multiplication of all Fourier transformed values and one inverse FFT. The computational resources needed for one FFT (or inverse FFT) are of the order $\mathcal{O}(m\log(m))$ for the assumed equidistant discretization with $m = 2^j$ and $j \in \mathbb{N}^+$ (Stoer, 1972). This adds up to $\mathcal{O}(m+3m3\log(m))$ operations for on summation of two stochastic variables.

For the multiplication of two stochastic variables (only occurring one time as denoted by the orange box in Figure 3) the necessary computational resources can be estimated from Algorithm 2. For the sake of simplicity we assume here, that all distribution domains are strictly positive, so no case distinctions according to Equation (19) are necessary. This adds up to $\mathcal{O}(7m+3m3\log(m))$ operations for on multiplication as shown in Table 3.

Table 3 Estimation of the necessary computational resources for the multiplication of two stochastic variables

operation	count	necessary computational resources	
		per operation	total
taking the logarithm	**2**	$\mathcal{O}(2m)$	$\mathcal{O}(4m)$
FFT	**2**	$\mathcal{O}(m\log(m))$	$\mathcal{O}(2m\log(m))$
convolution	**1**	$\mathcal{O}(m)$	$\mathcal{O}(m)$
FFT^{-1}	**1**	$\mathcal{O}(m\log(m))$	$\mathcal{O}(m\log(m))$
taking the exponential	**1**	$\mathcal{O}(2m)$	$\mathcal{O}(2m)$
total			$\mathcal{O}(7m+3m\log(m))$

Table 4 Estimation of the total necessary computational resources for the internal method according to Figure 3

operation	count	necessary computational resources	
		per operation	total
sum of a constant and a stochastic variable:	**3**	$\mathcal{O}(m)$	$\mathcal{O}(3m)$
product of a constant and a stochastic variable	**3**	$\mathcal{O}(2m)$	$\mathcal{O}(6m)$
sum of two stochastic variables	**3**	$\mathcal{O}(m+3m\log(m))$	$\mathcal{O}(3m+9m\log(m))$
product of two stochastic variables:	**1**	$\mathcal{O}(7m+3m\log(m))$	$\mathcal{O}(7m+3m\log(m))$
total			$\mathcal{O}(19m+12m\log(m))$

Now it is possible to compare the two methods – internal and external – with the information from Table 2 and Table 4. Nonetheless, the results from the two methods cannot be compared directly because the sample size n of a Monte Carlo method and the number of discretization points m of the internal method have different implications.

Härdle et al. (2004) show that in estimations of (output-) pdfs from Monte Carlo results with sample size n with so-called kernel density estimators the approximate mean integrated squared error is declining with $n^{-4/5}$.

For the internal method the error estimation is more complicated and involves more assumptions. Here, both the discretization error for the discrete Fourier transformation and convolution and the interpolation error for the inverse transformation have to be taken into account. Considering the interpolation error of the inverse transformation interpolation error and assuming a similar behavior for the convolution part. Stoer (1972) gives approximations for the asymptotic behavior of the integrated squared error for different interpolation functions. For linear interpolation the error is declining with m^{-1} and for spline interpolation with m^{-2} for sufficiently mathematically well-behaved functions[2].

[2] Basically the second derivative must be bounded.

If there are no other sources of errors present (which needs to be further investigated) this leads to the following observation:

> The decline of the estimation error seems to be faster for the internal method than the external method.
>
> Due to the computationally extremely efficient FFT transformation, the rise in the necessary numerical resources for the internal method is slower than the decline in the error compared to the external method.
>
> That means that the internal method has the potential to be superior for very high accuracy applications.

Now the question is: Is it possible to use this advantage in today's building science problems? The answer will be given by looking at the elemental building model from Figure 2, which is described by Equation (4). The necessary computational resources are contrasted for sample sizes n for the external method and discretization points m. As a start a sample size $n = 80$ for the Monte Carlo method is looked at as it is suggested for building simulations by Macdonald (2002). According to Table 2, the necessary computational resources for one time step with the Monte Carlo method are of the order of $\mathcal{O}(800)$. If the same amount of computational resources are spent on the internal method, an amount of 16 discretization points is derived from the

result in the last row of Table 4. For $m = 16$ this gives necessary computational resources of the order of $\mathcal{O}(836)$. Tests with the internal method, however, showed that a discretization of 16 is not sufficient.

In these tests a number of discretization points of $m = 128$ seemed to work reasonably. From Table 4 necessary computational resources of the order of $\mathcal{O}(9{,}500)$ are deduced. If the same amount of computational resources are spent on the external method it is possible to realize a sample size of $n = 950$. In our experience this is already an extensive sample size for Monte Carlo methods in building problems.

Another important point is that it is straightforward to parallelize Monte Carlo methods. Computers with 10 CPUs are not rare anymore and with cloud computing techniques it is possible to distribute Monte Carlo methods on hundreds of processors. With these techniques, the sample size of the Monte Carlo method is multiplied by 10 or rather 100 within the same order for the computation time. This is giving effective sample sizes of 9,500 or rather 95,000 for the example given in the preceding paragraph.

Another important aspect is that modern building simulation solvers are generally capable of using higher order schemes, implicit solving and variable time step techniques. For this reason the number of time steps can be reduced compared to the simple Euler integration scheme used in the internal method. Additionally fixed time step solvers are inapplicable to stiff differential equations, like for couple heat and airflow simulations or for coupled building and control simulations. This is an additional advantage of Monte Carlo methods.

CONCLUSION

Based on the presented findings on the necessary computational resources and computational runtimes, Monte Carlo methods are preferable for the accuracy desired of results from stochastic building simulations. An additional important advantage of the external method is the possibility to easily parallelize this method. Furthermore, it is possible to harness current highly advanced building simulation techniques directly with Monte Carlo methods; whereas for internal methods entirely new stochastic simulation engines would have to be developed. It is more important that stochastic effects in building simulations are considered at all, and for this purpose Monte Carlo methods are more applicable.

Due to the better convergence behavior of the internal methods they seem to be more appropriate for pdf calculations demanding very high levels of accuracy. Furthermore internal methods may also be more appropriate for linear time invariant systems used during controls design. The intention of this work is not to discourage further development of internal methods but to inspire a lively discussion on them and initiate research projects taking these ideas further.

ACKNOWLEDGEMENT

This study was funded by the German Federal Ministry of Economics and Technology (BMWi) (project number: BMWi 0327893A).

NOMENCLATURE

C	total zone heat capacity
$const_{sol}$	constant containing summation over product of radiation times area times solar transmission of all windows
c_p	specific heat capacity of air
i	number of time step
H_t	building loss coefficient
m	number discretization points of internal method
n	MC sample size
$\dot{Q}_{gain}$	heat gains (power) to zone
$\dot{Q}_{HC}$	heat gains (power) from heating and cooling equipment
$Q_{heat,spec}$	specific heating energy demand
$\dot{Q}_{Int}$	internal heat gains
R	total resistance to ambient
T	zone temperature
T_a	ambient temperature
T_{start}	initial zone temperature
$\dot{V}_{vent}$	volume flow of ventilation
z	solar transmission factor of blinds
FT	Fourier transformation
FFT	Fast Fourier Transformation
MC	Monte Carlo
pdf	probability density function
$\mathcal{O}$	Landau order symbol / big Oh notation

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