Digital Twin simulation of a Danish school building

Jiyuan Cui1, Konstantin Filonenko2, Alberto Gallottini2, Christian Thilker2, Rongling Li1, Peder Bacher2,
1Department of Civil and Mechanical Engineering, Technical University of Denmark, Kgs. Lyngby, Denmark
2Department of Applied Mathematics and Computer Science, Technical University of Denmark, Kgs. Lyngby, Denmark

Abstract
An auto-tuning algorithm based on Model Predictive Control (MPC) is developed for radiator thermostats embedded in a Danish school that can generate and replace the experimental thermal data of buildings through a Functional Mockup Unit (FMU) exported form Modelica. To achieve this, the estimation of relevant parameters is conducted through an algorithm that compares the experimental data, the simulated FMU data, and the data obtained from the AutoRegressive eXogenous (ARX) statistical models implemented in Python. The models are used to implement a digital twin of a school building integrating model predictive controller with low-order plant. After validating the estimated parameters, different controllers are tested and tuned.

Highlights
• The estimation methods provide a good tradeoff between computational performance and prediction accuracy in digital twin simulation.
• The plant model can be tuned based on ARX reference values.
• The performance of the digital twin can be tuned using a single parameter per zone.

Introduction
Smart operation of buildings can be achieved by utilizing advanced control algorithms such as MPC (Zhan and Chong (2021); Yang et al. (2020)). In this type of algorithm, the readings of the smart sensor are used to predict the consumption of energy based on a physical model that describes the dynamics of the system under investigation. The obtained predictions are subsequently used to lower the building energy consumption subject to constraints such as human comfort, building security, and operation costs. In the literature, a vast amount of statistical and mathematical models exist for the control and estimation of the heat dynamics inside buildings. Many of those are classified as white-box, black-box, and grey-box models, depending on how the information from the physical system is correlated with the information from sensor data.

A combination of the simplified governing equations with the information from sensor data using statistical methods leads to the grey-box models (Nielsen and Madsen (2006); Hollick et al. (2020); Thilker et al. (2021)). The physical equations describing the dynamics of the systems are expressed as a set of continuous in-time stochastic differential equations (SDE). The observation equation for the output variable investigated (i.e the temperature inside the building) is expressed by a discrete in-time measurement equation, whose unknown parameters are calculated from data using estimation techniques. The estimation of the thermal dynamics of buildings with water-based heating has been source of relevant research works in the past few years, all aimed at achieving the flexibility needed for the future smart energy system. Some of the relevant works are related to the development of accurate predictions of the thermal dynamics inside building such as schools.

A non-linear grey-box model based on SDEs of a Danish building school has been presented by Thilker et al. (2021) and validated using experimental data for one week and included proportional-integral (PI) control of the thermostatic valve changing water flow inside the radiator in form of a sigmoid function. A following study on non-linear MPC to predict the optimal radiator set-points has been addressed in Thilker et al. (2021). The authors, taking into account the solar irradiance and the outdoor temperature, reported a heating cost saving of 10%. Thilker et al. (2022) proposed a non-linear third-order ARX model to capture two of the non-linear thermal effects embedded in each room of the building for a one-day simulation. The time-dependent solar gain effects have been estimated in each room of the building using de Boor B-splines De Boor (1972) basis expansion from solar power experimental data, achieving predictions in agreement with the physical diurnal solar variation. The authors also improved the previous sigmoid function using cubic Hermite polynomials, capturing sudden changes in thermal dynamics. Compared to previous works found in the literature Cui et al. (2019), they reduced the root-mean-square error (RMSE) of the temperature predictions.

This research applies the methodology established in previous works to build a light-weight Digital Twin (DT) of a Danish school zone and use it to predict the effect of MPC on the cost of the existing proportional integral radiator controllers and to to evaluate the performance of the FMU model in terms of estimation, temperature prediction, and real-time operation. The resulting autotuning algorithm finds the best conditions for meeting the radiator set-points while reducing the cost and predicts
the best value of the MPC controller tuning parameter $R$ equal to the relative weight of the controller objectives.

The novelty of this work includes the following aspects:
- Fast nonlinear estimation algorithm based on SDEs, B-splines and FMU.
- Digital Twin that supports customizable parameter estimation and MPC.
- Comparison of different SDE-based estimation models for historical and simulation data.

Methods
This section describes the digital twin: algorithms used to tune and control the MPC and the plant models to investigate possible alternatives to conventional PI controller. It gives a brief introduction to the case study and how the methodology is applied to provide the relevant background for discussing the performance of the system under conventional and advanced controls.

ARX room temperature estimation

ARX model for the output variable $y_t$, linear with respect to $x_{i,t}$, inputs $x_{i,t}$ at time $t$, is

$$ y_t = \sum_{m=1}^{M} \phi_m B^m y_t + \sum_{i=1}^{N_x} \beta_i B x_{i,t} + \varepsilon_t $$

(1)

In the above equation, $B^m$ is the back-shift operator of order $m \in \mathcal{R}$, such that $B^m y_t = y_{t-m}$ (Bacher and Madsen (2022)), $\beta_i$ is a parameter associated with the $i$th input, and $\varepsilon_t \sim N(0, \sigma^2)$ is an independent and identically distributed white noise process with standard deviation of $\sigma$.

The main objective of fitting ARX models to data is to estimate the parameters of the model $\theta = ([\phi_1, \beta_1], \ldots, [\phi_M, \beta_M])$ in Eq.(1) such that the function relating the inputs to the outputs best approximates the data.

The ARX model in Eq.(1) can be re-written in a vector form as

$$ y = X \theta + \varepsilon $$

(2)

where $X$ is the so-called design matrix, $\theta$ is the vector of the unknown parameters and $\varepsilon$ is the error vector.

The estimation of the parameters is addressed by finding the $\theta$ that minimized the Sum of Squared Error (SSE) defined as

$$ S(\theta) = (y - X \theta)^T (y - X \theta) $$

(3)

For linear ARX models, the minimizer of the SSE are found analytically in a closed-form as the solution of the normal equations

$$ (X^T X) \hat{\theta} = X^T Y $$

(4)

The estimator is thus given by

$$ \hat{\theta} = (X^T X)^{-1} X^T y $$

(5)

When the parameters of the model are non-linear, the normal equations are not anymore valid and another strategy for the minimization of Eq.(3) is needed. Following the algorithm proposed by Thilker et al. (2022), the non-linear inputs are initially transformed into linear regression problems. Once the SSE of this linearized problem is evaluated, the $\theta$ values that minimize the SSE are approximated by the Sequential Least Squares Programming (SLSQP) optimization function built in the SciPy library GitHub (2023) in Python with a computational time of 0.16 s, a tolerance of $\varepsilon = 10^{-5}$ and a maximum of $10^4$ iterations.

The first ARX model given in Eq.(6) for the room temperature prediction $y_t$ at time $t$ is linear with respect to the parameters $\theta = [\phi_1, \beta_1]^T$.

$$ y_t = \phi_1 y_{t-1} + \beta_1 x_{1,t-1} + \varepsilon_t $$

(6)

The input of the model is given by

$$ x_{1,t} = T_{a,t} - x_t $$

(7)

where $T_{a,t}$ is the ambient temperature.

Due to linearity, the parameter estimation problem can be solved with the normal Eq.(4).

In addition to the ambient temperature, the effect of the thermostatic valve that regulates the flow in the radiators has been modeled as a non-linear sigmoid function Eq.(8), following the previous work of Thilker et al. (2021).

$$ x_{2,t} = \frac{T_{f,t} - y_t}{1 + \exp(-\alpha_1 (T_{set,t} + \alpha_2 - y_t))} $$

(8)

The inputs are now given by the temperature set-point $T_{set,t}$, the forward temperature to the radiators $T_{f,t}$ as well as the ambient temperature $T_{a,t}$.

The non-linear ARX model is now

$$ y_t = \phi_1 y_{t-1} + \beta_1 x_{1,t-1} + \beta_2 x_{2,t-1} + \varepsilon_t $$

(9)

where the linear parameters are $\theta = [\phi_1, \beta_1, \beta_2]^T$, and the $\theta = [\alpha_1, \alpha_2]^T$ are the non-linear parameters optimized with the numerical optimization function built in Python.

With the inclusion of the solar radiation input $x_{solar,t-1}$, the ARX model now becomes

$$ y_t = \phi_1 y_{t-1} + \sum_{i=1}^{3} \beta_i x_{i,t-1} + \varepsilon_t $$

(10)

where inputs $x_{1,t}, x_{2,t}, x_{s,t}$ represent heat gains from the outside air, HVAC system and direct solar radiation, respectively, and the linear parameters are now $\theta = [\phi_1, \beta_1, \beta_2, \beta_3]^T$.

The input $x_{3,t}$ can be transformed into a linear combination of de Boor B-splines (De Boor (1972)) interpolating functions of order $k$ such as

$$ x_{3,t} = \sum_{i=1}^{M} \eta_i B_{i,k}(x) $$

(11)

where $\eta_i$ is the scaling factor of the $i$th B-spline, which is defined as

$$ B_{i,k} = \frac{z_i - z_{i-k}}{z_{i+k} - z_i} B_{i-1,k} + \frac{z_{i+k+1} - z_i}{z_{i+k+1} - z_{i+k}} B_{i+1,k-1} $$

(12)

on a finite interval $[a, b] \in \mathcal{R}$ with non-decreasing knot placements $a = z_0 \leq z_1 \leq \cdots \leq z_{N+k+1}$.

The following indoor temperature ARX estimation models have been set up to see the influence of different inputs on the dynamics of the classroom zone:...
• Simple estimation of indoor temperature from ambient temperature (M1).
• Simple estimation from ambient temperature and the thermostatic valve function (M2).
• Simple estimation from ambient temperature, thermostatic valve function, and solar gain (M3).
• B-splines estimation from ambient temperature, thermostatic valve function, and solar gain (M4).

Case study
The case study investigated in this project is a classroom in a Danish public school. The building consists of three floors and a basement and has a total of 10 classrooms. District heating is used for space heating, domestic hot water, and the ventilation system that causes the air to circulate in the classrooms. Each room is heated by a water-based radiator where a thermostatic valve regulates the water flow to keep a predefined set-point temperature. A heat exchanger is used as the interface between the space heating and the district heating system. The experimental data used in this work for one of the school classrooms are taken from the previous work of Thilker et al. (2022). In particular, the ambient temperature $T_a$, the indoor air temperature $Y_t$, the set-point temperature $T_{set}$, the solar power $P_e$, and the forward temperature $T_f$ of the water in the radiator have been used. All the temperatures are expressed in [$^\circ$C], while the solar power is expressed as [kW/m²]. The total number of observations is 1294, each sampled every 30 minutes, from 2021-03-01 to 2021-03-27.

![First-order zone model in Modelica.](image)

Figure 1: First-order zone model in Modelica.

A first-order ARX model of a school classroom, built in Python, is created using Danish case data, while the plant model of the classroom is firstly implemented by using Modelica language Mattsson et al. (1998) in OpenModelica and then exported as an FMU for co-simulation in MATLAB(Simulink). A FMU is a self-contained file that encapsulates a dynamic model, along with its associated data and algorithms. It provides a standardized interface for connecting models and allow interoperability between different simulation tools. The thermal equations are described as an RC (Resistance Capacitance) state space model, with one resistor ($R_e$ represents the external wall resistance), and one capacitor ($C$ combines the heat capacity of air and the walls) as shown in Figure 1.

![Digital twin simulator in Simulink.](image)

Figure 2: Digital twin simulator in Simulink.

The objective function $J$ of this optimization problem includes both these terms as shown below, the first term represents the error, and the second term represents the energy cost:

$$\min J = \sum_{k=0}^{N} Q e_k^2 + R c_k (u_k \cdot T_s / 3600 / 10^6)$$

s.t. \hspace{1cm} x_{k+1} = A_d x_k + B_d u_k + E_d u_{2k} + G_d u_{3k}$$

$$y_k = C_d x_k$$

$$0 \leq u_k$$

$$288.15 \leq x_k \leq 298.15$$

$$x_{k+1} - x_k \leq 0.5$$

$$u_{k+1} - u_k \leq 1000$$

where $N = 48$ represents the prediction horizon. $Q$ and $R$ are the weights of different terms in the objective function which could be used to adjust their importance. The RC model inputs are: the ambient temperature $T_a$, the solar power $P_e$. The corresponding state space model in the discrete-time domain for the room temperature $T_t$ used in the optimization algorithms is

$$y_t = \left(1 - \frac{\tau_s}{R_e C}\right)y_{t-1} + \frac{\tau_s}{R_e C} T_{set} + k_s P_{st}$$

(13)

Initially, the parameters of the above model, solar coefficient $k_s$ and lumped conductivity of the external construction $\frac{1}{R_e}$ have been estimated using the M4 ARX model. Although simulations were performed, the results were found to be inaccurate. As a solution, the parameters were adjusted based on the recommendations outlined in Bacher and Madsen (2011), which yielded a more reasonable output from the simulation.

MPC autotuning algorithm
The PI controller aims at minimizing the error between the temperature set-point and the actual average temperature of a zone reflected in one of the objectives of the DT controller. On the other hand, the economic control must minimize the total energy cost to make the building operate in a flexible way. The MPC autotuning algorithm as shown in Figure 2 uses the discrete-time domain state-space model Eq.(13) to predict the future outputs, an objective function that minimizes both the error between the predicted outputs and reference (temperature set-point in this case) as well as the electricity cost for operating the HVAC while keeping the indoor room temperature in prescribed intervals.

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electricity prices enter the optimization problem as the cost coefficients \( c_k \). \( u_k \) is manipulated variables representing the input power, \( u_{2k} \) and \( u_{3k} \) are measured disturbances, representing the ambient temperature \( T_a \) and the solar irradiation \( P_s \) respectively. Constraints are set based on physical limitations and control objectives.

**Digital twin**

In this study, four DT control scenarios were investigated, three of them using MPC and the last one using PI control. Comparative analysis of all scenarios based on the results of temperature and energy cost separately. The MPC aims to minimize the total energy cost within a given period while keeping the indoor temperature. The PI controller is set for only tracking the temperature set-point without taking energy cost into account. All the scenarios were simulated by using day-ahead electricity prices from Nordpool, the Nordic power exchange market. These electricity prices are available in one hour intervals. The continuous system is discretized by using a sample time of 1800s (30 minutes). The given period is the second week in March 2021 of 7 days (604800 s).

### Table 1: Estimated parameters of linear ARX.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.002</td>
<td>0.005</td>
<td>0.0008</td>
<td>-0.0004</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-</td>
<td>0.002</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Results

#### Parameter estimation

![Figure 3: Estimation of ARX models from measurements.](image)

In Figure 3 the predictions from the different ARX models are compared. The M1 linear model with ambient temperature as the only input follows the trend of the lowest values of the room temperature dataset due to neglect of solar radiation and HVAC inputs. Additionally, this model is not able to capture the variability of the temperature between the different days. As the non-linear sigmoid function for the thermostatic valve is added to the models, the predictions are considerably improved, with minimal differences between the M2, M3, and M4 models. Nevertheless, the predicted temperatures from all the models investigated underestimate the observed temperature at the end of the simulation, where the measured room temperature has a sharp increase. The linear and non-linear parameters estimated from the different models are shown in Table 1 and in Table 2, respectively.

### Table 2: Estimated parameters of non-linear ARX.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>-</td>
<td>1.08</td>
<td>1.06</td>
<td>0.937</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-</td>
<td>-0.723</td>
<td>-0.744</td>
<td>-0.781</td>
</tr>
<tr>
<td>RMSE [°C]</td>
<td>-1.71</td>
<td>0.92</td>
<td>0.94</td>
<td>0.85</td>
</tr>
<tr>
<td>MBE [°C]</td>
<td>-5.19</td>
<td>0.02</td>
<td>0.03</td>
<td>-1.09</td>
</tr>
</tbody>
</table>

The ARX model reproduces the overall trend in a more accurate way than the grey-box FMU model, the latter captures the dynamics of the process better without introducing noise. When the noise is introduced, the estimation is capable of reproducing similar result. From these tables, it is possible to infer that the predicted data are strongly influenced by the data obtained by the previous time step (\( \phi_1 \) close to 1 for all the simulations). This is reasonable since there is no factor that could influence the room temperature in a short time step of half an hour. The \( \beta_1 \) is close to zero for all the simulations: it starts from positive values in the M1 model and reaches a negative value in the last model. Even though a negative value is physically unrealistic, the predictions from this model are the most accurate among all the predictions observed. The \( \beta_2 \) parameters are close to zero for all the simulations investigated, while the \( \beta_3 \) are positive for both the two models with the solar radiation.

To compare the accuracy of the models, the following root-mean-square error and mean bias error

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N}(Y_{exp} - Y_{pred})^2}{N}}
\]

\[
MBE = \frac{\sum_{i=1}^{N}(Y_{exp} - Y_{pred})}{N}
\]

between the measured and simulated temperatures have been used. The values of RMSE and MBE for each model are shown in Table 2. The parameters estimated by SSE algorithm and the measured time-series are used as a reference to calibrate the plant model consisting of a thermal zone model packed in a Modelica FMU, see Figure 1. Comparison of the calibrated FMU model is shown in Figure 4 with the MBE of -0.515 and RMSE of 1.089 °C.
Dynamics of the Digital Twin

The dynamics of the DT is defined by the MPC model dynamics (ARX) and that of the plant (FMU). The stochastic nature of the SDE approach makes this dynamics dependent on the process and measurement noise in the system. In Figures 5a and 5b the predictions for M3 with the three two levels of noise are shown. The noise is not visible in the ARX predictions for \( \sigma^2 = 0.01 \), as shown in Figure 5a. As the noise is increased to \( \sigma^2 = 0.1 \) (Figure 5b), some minor changes in the predictions are visible. Similar to the previous model, as the noise is increased, \( \phi_1, \beta_1, \alpha_1, \) and \( \alpha_2 \) decrease, while \( \beta_2 \) and \( \beta_3 \) increase. In Figures 5c and 5d the predictions for the M4 with the three different levels of noise are shown. The noise is slightly visible in the ARX predictions for \( \sigma^2 = 0.01 \) as shown in Figure 5c. Variations in the predictions become evident as the noise is increased to \( \sigma^2 = 0.1 \) (Figure 5d), where the predictions are damped from the set-point temperature.

Control and autotuning of the Digital Twin

The results of the four DT scenarios are presented in Figure 6 and Tables 3 and 4. Comparative analysis of all scenarios based on the results of temperature and energy cost separately.

In order to test the effect of different weights of the cost term in the objective function of MPC on the results, three scenarios which are R=0, R=50 and R=100 have been investigated separately. We also checked the scenarios of R=0.01 and R=1, however the results are almost identical to the result of scenario of R=0, so these two scenarios are not shown here. The weight of the error term is fixed to Q=1. The relative error between the simulation result of room temperature and temperature set-point was chosen to represent thermal comfort and compare different scenarios.

Table 3: Ratio of temperature relative error for different scenarios.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Error &lt; 0.05</th>
<th>Error &lt; 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>R=0</td>
<td>0.56</td>
<td>0.78</td>
</tr>
<tr>
<td>R=50</td>
<td>0.45</td>
<td>0.80</td>
</tr>
<tr>
<td>R=100</td>
<td>0.15</td>
<td>0.49</td>
</tr>
<tr>
<td>PI</td>
<td>0.62</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Figure 6 (a) and Table 3 illustrate that the small relative error ratio is higher in the scenarios of PI control and R=0 of MPC, which means that they are thermally comfortable most of the time during the whole simulation period. The opposite is true for the case of R=100.

Table 4: Total energy consumption and cost for different scenarios.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Consumption, MWh</th>
<th>Cost, EUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>R=0</td>
<td>0.66</td>
<td>148.21</td>
</tr>
<tr>
<td>R=50</td>
<td>0.56</td>
<td>120.92</td>
</tr>
<tr>
<td>R=100</td>
<td>0.48</td>
<td>98.32</td>
</tr>
<tr>
<td>PI</td>
<td>0.68</td>
<td>150.92</td>
</tr>
</tbody>
</table>

Figure 6 (b) shows the electricity price variation and power consumption for different MPC scenarios. It could be found that by using MPC controller, most of the electricity use occurs at times when electricity prices are not so high, thus reducing costs. Table 4 shows total energy consumption and cost for the four scenarios respectively.

Overall, among the four scenarios, PI control tracks the temperature most accurately but consumes the most energy and costs. For the three MPC scenarios, as the weight of the cost term gradually increases, the performance of MPC tracking temperature gradually decreases but within the constraints. However, the energy consumption and cost are significantly saved. Cost savings of up to 49.89 euros (34%) can be achieved in this study.

Discussion

ARX based on measured data

The results of the ARX models comparison with experimental data showed that the M1 model with the ambient temperature has the highest RSME of 1.71 consistent with the poor visual agreement of the room dynamics. The poor accuracy of this model can be explained by the fact that it is based only on one linear measurement signal (the difference between the ambient temperature and the room temperature).

The introduction of the thermostatic valve function by M2, leads to much higher accuracy with the non-linear optimization solver reaching an RSME of 0.92. The improvements in the prediction mismatch are obtained by supplementing the solar inputs with the B-splines interpolating functions (RSMEs of 0.94 and 0.85 for the M3 and M4 models, respectively). It should be noted that more accurate ARX models with lower RSME are present in the literature, cf. RSME of 0.6 °C in the background work by Thilker et al. (2022) and 0.77 °C with Artificial Neural Networks by Mba et al. (2016)).

In spite of a lower RSME, M4 is not able to reproduce the fast dynamics of the room. By this reason, the existing DT implementation uses the calibrated Modelica model and its discretized version for both emulation and control that provide much higher accuracy results.
(a) $\sigma^2 = 0.01$, M3

(b) $\sigma^2 = 0.1$, M3

(c) $\sigma^2 = 0.01$, M4

(d) $\sigma^2 = 0.1$, M4

Figure 5: Temperature predictions from M3 and M4 with the FMU data and added Gaussian noise.

(a) Indoor temperature of the zone.

(b) Power consumption and electricity prices.

Figure 6: Control and autotuning of the Digital Twin.

**ARX estimation based on DT simulation data**

All the ARX models based on simulated data have shown an RMSE of 1.089 °C compared to the measured data and the majority of the parameters estimated with the simulation data were in agreement with the parameters found in the ARX models based on the experimental data, with few variations for the solar radiation parameter ($\beta_3$) in both the simple model and the one with the splines. In addition, the offset parameter in the thermostatic valve function ($\alpha_2$) was different from the ARX estimated parameters. When some noise is included in the simulation data, the estimated parameters are altered and some variations are visible in the predictions. Overall, the estimated ARX and FMU models provide a good tradeoff between computational performance and prediction accuracy in digital twin simulation.
Digital Twin and MPC autotuning

The DT simulation shows that the different models used in the optimization and measurement loop are consistent and can follow the set-point with accuracy close to experimental under both proportional-integral and predictive control. Compared with PI control, which has the characteristics of high accuracy tracking set-point and high energy consumption and cost, MPC control can optimize the input power by adjusting the weight of the error term and energy cost term to achieve the lowest possible cost in the corresponding temperature interval. The results show that the application of MPC control is able to find the optimal tradeoffs between conflicting objectives. A suitable controller should be designed according to the purpose of control.

Conclusion

In this paper, we set up a digital twin simulation of a Danish school consisting of customizable parameter estimation algorithm, grey-box plant model and an economic MPC algorithm that autotunes the set-point controller based on the Nord Pool price signal. The work extends the approach of the described parameter estimation algorithm to models that include FMUs. After testing several black-box models, the best parameter fit has been identified and used to calibrate the plant model implemented in Modelica language exported as FMU. Summarizing, it can be concluded that the simplest model that can serve as a DT plant model is the FMU model in 3 and it cannot be replaced by the best performed ARX model(s) that it is compared to in the results section. The digital twin has been setup including either parameter estimation or MPC algorithm communicating with the plant model. It has been tested in the software-in-the-loop simulation with added Gaussian noise and control simulation with simultaneous tuning of the tracking and economic parameters of the objective function. The comparison of the advanced control with normal PI-based controller shows the benefit of implementing a DT-based smart actuation. Since this approach is open-source and is not computationally demanding, it creates new opportunities for easier and faster development of advanced controls in school buildings. ARX model is better to use in MPC because it is a black-box model which is fast and does not require “empirical” knowledge relying solely on measurements. Therefore, the existing algorithm will be improved outside this paper to outperform the Modelica-based solution in accuracy.

Nomenclature

- \( k_s \): Solar gain coefficient [W/m²]
- \( MBE \): Mean bias error between two time series
- \( N(\mu, \sigma^2) \): normal distribution with mean \( \mu \) and standard deviation \( \sigma \)
- \( N, N_x \): M number of points inside the time series for \( Y_{exp}, x_{i,t}, Y_t \)
- \( P_s \): Solar radiation [W]
- \( Q, R \): Weights of the dynamic setpoint tracking and energy cost objectives
- \( R_e \): Lumped resistance of the external construction [K/W]
- \( RMSE \): Root mean square error between two time series
- \( S(\theta) \): Sum of squared errors
- \( T_{a,t} \): the ambient temperature of the outside air
- \( T_{f,t} \): the forward temperature of the hot space heating water entering the zone radiator
- \( x_{i,t} \): i’th input in discrete time
- \( Y_{exp} \): measured historic time-series for the average indoor air temperature of the building
- \( Y_t, Y_{pred} \): the average indoor air temperature of the building in discrete time predicted by the ARX model
- \( z_i \): Coordinate of a knot with index i [m]

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