Abstract

Several factors, such as building envelope, density, and scene shape, have been identified as influencing the thermal comfort of urban environments. However, the role of thermal inertia at larger scales beyond individual buildings remains poorly studied. The objective of this paper is to investigate the impact of intelligent distribution of thermal inertia and albedo in a district, based on solar paths, on improving thermal comfort. To conduct a detailed analysis of thermal comfort indicators related to conductive and radiative phenomena, we used a coupling of the finite element method and the radiosity method. Our findings demonstrate that the distribution of thermal inertia and albedo, based on solar paths, improve significantly both inhabitants and pedestrians’ thermal comfort.

Highlights

- Coupling finite element method and radiosity method for urban thermal study.
- Analyzing the relation between thermal inertia, albedo and thermal comfort.
- Interpreting the distribution of thermal and radiative properties according to solar paths.
- Improving both inhabitants and pedestrians’ thermal comfort.

Introduction

In recent years, there has been a growing interest in studying how urban morphology affects thermal comfort (Kawai 2014; Taleghani 2015; Sobstyl 2018) and solar potential (Sarralde 2015; Carloseda 2020).

While several factors that influence thermal comfort, such as building envelope, density, and scene shape, have been identified (Voogt 2002), the role of thermal inertia in buildings has been emphasized but little studied beyond the scale of individual buildings. Thermal inertia helps to limit temperature increase in buildings (diffusivity) and ensures a sufficient phase shift to remove accumulated heat (effusivity) during the night.

In most studies on solar gain, the reflected part of radiation has been neglected due to the widespread use of the ray-tracing method (Ward 1994). Only recently have inter-reflections been considered in calculations using the radiosity method (Goral 1984, Musy 2015). However, the method’s slow democratization is due to the reliance on a scene discretization into a mesh and the limitation of reflection modes considered. Recently, this limitation has been lifted (Bugeat 2020) using extended view factors (Sillion and Puech 1989).

The coupled study of thermal inertia and albedo influences the thermal comfort of both inhabitants and pedestrians, which are intimately linked to the scene’s geometry. The finite element method (Lewis 2004) allows for the consideration of the scene as an integral part at all stages of calculation, with the only limitations being CAD modeling difficulties. Coupling with the radiosity method (unrestricted reflection types), the distribution of radiation exchanges is precisely accounted for.

Thus, this coupling is used to study the thermal behavior of an idealized neighborhood under the Mediterranean climate of the city of Cordoba in Spain. Four scenarios are compared: the first one with high inertia and high albedo, the second one with high inertia and low albedo, the third one with low inertia and high albedo and finally the fourth one with low inertia and low albedo. Results are analyzed for three quantities of interest in thermal simulations: surface temperature, indoor air temperature (inhabitant comfort) and mean radiant temperature (pedestrian comfort). For greater clarity, the first two are indicated in Celsius degree and the third in Kelvin since it is closely linked to the radiative phenomenon.

For this, the method used in the calculations and the urban model are first presented, followed by an analysis of the results from the different scenarios. Finally, a discussion is conducted on the interest of distributing thermal inertia and albedo according to solar paths to improve the comfort of both inhabitants and pedestrians.

Methods

Calculation methods

First, the radiative exchanges between the sky and the skin of the 3D model are computed. Then, the thermal simulations are carried out in a generic finite element method software adapted to deal with urban problems.

To compute the sky radiation that is absorbed in shortwave and longwave, the radiosity method is used. The sky vault is divided into 2000 equal-area cells with the same aspect ratio. The radiance of each sky cell is computed using two anisotropic sky models: the Perez (1993) model for shortwave and the Martin and Berdhal
(1984) one for longwave. The second allows to consider the directional sky emissivity as a function of the zenith angle.

Then, classical view factors are replaced by the extended view factors. They incorporate translucent and specular faces. Their computation is accelerated using Embree®, a high-performance ray-tracing kernel developed by Wald (2014).

To calculate the irradiance received at each patch of the skin from the sky, the systems of linear equations are solved for shortwave \( E_{oc,i} \) and longwave \( E_{ol,i} \):

\[
E_{oc,i} = \sum_j (F_{ij}^{ext} M_{oc,j} + F_{ij}^{ext} \rho_{d,j} E_{oc,j})
\]

\[
E_{ol,i} = \sum_j (F_{ij}^{ext} M_{ol,j} + F_{ij}^{ext} \rho_{d,j} E_{ol,j})
\]

Where \( E_{oc,i} \), \( E_{oc,j} \) and \( E_{oc,j'} \) are the shortwave energy irradiances of patch \( i \), \( j \) or \( j' \) (Wm\(^{-2}\)); \( M_{oc,j} \) is the shortwave emission of patch \( j \) (Wm\(^{-2}\)); \( \rho_{d,j} \) and \( \tau_{d,j'} \) are the diffuse reflectance and direct shortwave transmittance of patch \( j' \), respectively; \( F_{ij}^{ext} \) is the extended view factor between patch \( i \) and \( j \). Patch \( j \) and \( j' \) represent a "twin" pair of thin transparent elements, such as glazing.

To solve both systems, we use a Jacobi iterative algorithm, which allows for controlling the number of reflections considered (Cohen 1985). Based on presimulation tests, we set 10 reflections for both shortwave and longwave computations. Finally, we obtain the flux absorbed by each surface of the skin by multiplying the received irradiance by its absorptance.

To solve the thermal problem (3), the finite element program Cast3M (Le Fichoux 2011) is used.

\[
C T + (K_{cond} + K_{conv} + H_{rad}(T)) T = Q + H_{rad}(T) T_{radeq}
\]

\( C \) is the capacitance matrix (JK\(^{-1}\)); \( K_{cond} \) is the conductivity matrix (WK\(^{-1}\)); \( K_{conv} \) is the conductivity matrix for convection (WK\(^{-1}\)); \( Q \) is a term associated with the boundary conditions (WK\(^{-1}\)), which includes atmospheric radiation fluxes and convective fluxes (Wm\(^{-2}\)); \( H_{rad} \) is the conductivity matrix for radiation (WK\(^{-1}\)); \( T_{radeq} \) is the vector of the equivalent temperatures of the environment (K).

The outer air temperature and the ground temperature at 1.5-m depth are imposed, retrieved from Cordoba’s weather file. The urban model is surrounded by an infinite ground at the air temperature. Since the same geometry is used during all stages of the simulations, the precomputed view factor matrix is introducing in Cast3M, thus reducing computational times.

To compute the mean radiant temperature (Fanger 1972), the principle is to determine the amount of radiant energy arriving on a sphere. This sphere is discretized according to a partition where each tile represents the same solid angle. For each tile, we determine the amount of radiant energy in shortwave and longwave that arrives (elementary irradiance) by a ray tracing method. Then, we integrate all these irradiances on the sphere to obtain the total radiant flux. It allows us to calculate the mean radiant temperature at the center of the sphere. Since the equivalent partition (equal solid angles) of the sphere has been used, the total radiant flux is calculated as the average of the total elementary irradiances (4) and the mean radiant temperature is calculated according to the Stefan-Boltzmann law (5).

\[
\phi_r = \frac{\alpha_{oc} \sum_{i=1}^{n} E_{oc,i} + \varepsilon_{ol} \sum_{i=1}^{n} E_{ol,i}}{N_{sphere}}
\]

\[
T_{mrt} = \frac{4 \phi_r}{\alpha \varepsilon_{at}}
\]

The accuracy of the mean radiant temperature calculation depends on the number of rays launched. Based on presimulation tests, 50 000 rays are sufficient to obtain a convergence of the results.

**Figure 1:** Urban 3D model at the left and skin of the model with the different points of interest at the right.
Urban model

The simulation model, illustrated in Figure 1, depicts a rectangular neighborhood with dimensions of 64 m by 72.8 m. It consists of a square with dimensions of 32 m by 19.2 m, a street canyon with a length of 32 m (H/L = 0.4) oriented along the east-west axis, and eight pavilions facing east and west. The buildings have an average height of 10 m with a maximum height of 23.80 m for the tower situated between the square and the street canyon. The built-up area covers 46% of the district's right-of-way, with 121 rooms, each containing 20% glazed area, constructed by replicating the same room. The district mesh is split into three parts: the buildings (mesh of 50 cm side), the roadway (mesh of 1 m side), and the basement (mesh of 2 m side) connected through the mesh gluing method. The geometric model comprises 424,532 volume elements (97.5% for buildings, 1.6% for pavement, and 0.7% for underground) and 539,071 nodes (degrees of freedom).

To analyze the indoor air temperature, the results were obtained from three apartments as shown in Figure 1. The first (I1) is located on the second to last floor of the south-facing tower, the second (I2) is situated on the second floor of a building in the center of the south-facing street canyon, and the third (I3) is positioned on the second floor of a building overlooking the north-facing pavilions. To evaluate pedestrians' comfort, the mean radiant temperature is calculated for three points characteristics of the three configurations: point P1 located at the square's center, point P2 situated in the middle of the street canyon, and point P3 in the pavilion area.

The outer skin of the 3D model exchanges with the external environment, except the faces on the perimeter of the model, which are considered adiabatic. It contains 83,364 radiative faces, with 10% representing glazing. The view factor matrix has a fill rate of 7% and a memory size of 10.1 GB.

The Figure 2 shows the sky temperature and the shortwave irradiance for the study period. The outside air temperature is equal to the temperature of the sky at the horizon. This is why it is not shown explicitly. The shortwave and longwave radiative properties of the skin model are shown in Figure 3. The glazings are perfectly specular in both shortwave and longwave.

Two materials are used for the buildings based on the simulation scenarios: heavy concrete with high thermal inertia ($\alpha_{\text{heavy}} = 9.17\times10^{-7}$) and light concrete with low thermal inertia ($\alpha_{\text{light}} = 5.23\times10^{-7}$) (Figure 3). These values were extracted from the databases of the EnergyPlus application site (Crawley 1999).

Regarding convective flows, the outer air temperature is obtained from the climate data file, and the convection coefficient is set to 10 Wm$^{-2}$K$^{-1}$, considering a low wind speed (Mirsadeghi 2013). The indoor air temperature of the three apartments is unrestricted, with a convection coefficient of 2.5 Wm$^{-2}$K$^{-1}$ for walls, 5 Wm$^{-2}$K$^{-1}$ for ceilings, and 0.7 Wm$^{-2}$K$^{-1}$ for floors (CSTB 2012). The air renewal rate is constant and equal to 1 volh$^{-1}$.

The study was conducted over a 24-hour period from September 26th at 6:00 am to September 27th at 6:00 am, on a clear and warm day. The zenithal height of the sun close to the September equinox provides more uniform sunlight on facades throughout the day than on a day closer to the summer solstice. At this time of year, facade color and thermal inertia will have an even greater influence on the thermal comfort of residents and pedestrians. The thermal simulation uses a time step of one hour and initial temperatures are calculated in a steady state for the first-time step. The subsurface temperature at a depth of 1.5 meters is set to 23°C based on the climate data file. The calculation of the extended view factors takes approximately 30 minutes using Embree, while the calculation of the absorbed irradiances takes less than two minutes. The thermal calculation is carried out using Cast3M and takes about 1 hour. Each simulation takes around 1.5 hours to complete.

![Figure 2: Sky temperature at the left and shortwave irradiance at the right.](image-url)
Results

Figure 4 presents the surface temperatures obtained for the four scenarios at 14h, at solar noon. The scenario with low inertia and dark facades shows the highest surface temperatures (42.8°C on average), while the scenario with high inertia and clear facades displays the lowest surface temperatures (29.5°C on average). The other two scenarios exhibit surface temperatures that are relatively close, with an average of 36°C for the scenario with low inertia and clear facades, and 33.4°C for the last scenario. However, the difference between these two scenarios is more pronounced in pavement temperature, where the clear facades of buildings reflect more solar radiation, resulting in higher pavement temperatures.

For buildings with high inertia, dark facades lead to surface temperatures that are on average 4.5°C warmer at 14h than clear facades. For buildings with low inertia, dark facades result in surface temperatures that are on average 6.8°C warmer at 14h than clear facades.

Dark facades have more influence on buildings with low inertia than on buildings with high inertia. Specifically, for buildings with high inertia and dark facades, the sunny surfaces are warmer by 9°C and the shaded surfaces by 2°C on average compared to the same surfaces in the scenario with high inertia and clear facades at 14h. For buildings with low inertia and dark facades, the former is higher by 14.7°C and the latter by 2.9°C than in the scenario with low inertia and clear facades.

The surface temperature averages for the four scenarios at different times during the simulation are shown in Table 1. The green color values represent the minimum average surface temperature, while the red color values represent the maximum.

During the day, the scenario with high inertia produces lower surface temperatures than the scenario with low inertia. In the late afternoon, the average surface temperatures tend to balance out until an inversion occurs during the night, with buildings with low inertia being less warm than buildings with high inertia. At 14h, the scenario with low inertia and dark facades results in a 9.4°C increase over the scenario with high inertia and clear facade. At 4h, the scenario with low inertia and clear facades causes a decrease of 1.8°C compared to the scenario with high inertia and dark facades.

Thus, the color change has a greater impact on buildings with low inertia during the day (+6.8°C at 14h between low inertia and clear facades and low inertia and dark facade), while it has a greater impact on buildings with high inertia during the night (+0.7°C at 4h between high inertia and clear facades and high inertia and dark facades).

In summary, the scenario with high inertia and clear facades provides the best compromise on surface temperatures over the entire simulation period, while the scenario with low inertia and dark facades provides the worst results during the day. After analyzing the surface temperatures, the comfort indicators for both inhabitants (air temperature) and pedestrians (mean radiant temperature) are studied.

The results for indoor air temperature for the three apartments and mean radiant temperature at three points of interest are presented in Figure 5 for the four scenarios.

For all three apartments, the air temperature is consistently higher with low inertia compared to high inertia, regardless of facade color. Specifically, the

Table 1: Mean temperatures of buildings surfaces at different time of the simulation.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>10h</th>
<th>14h</th>
<th>18h</th>
<th>22h</th>
<th>4h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low inertia, clear facades</td>
<td>28.5</td>
<td>36.0</td>
<td>25.8</td>
<td>19.2</td>
<td>19.1</td>
</tr>
<tr>
<td>Low inertia, dark facades</td>
<td>33.0</td>
<td>42.8</td>
<td>27.6</td>
<td>19.6</td>
<td>19.3</td>
</tr>
<tr>
<td>High inertia, clear facades</td>
<td>22.9</td>
<td>29.5</td>
<td>25.6</td>
<td>21.5</td>
<td>20.2</td>
</tr>
<tr>
<td>High inertia, dark facades</td>
<td>25.2</td>
<td>33.4</td>
<td>28.6</td>
<td>22.8</td>
<td>20.9</td>
</tr>
</tbody>
</table>
average air temperature is higher by 2.7 °C for I1, 1.2 °C for I2, and 1.5 °C for I3.

For apartment I1 located in the tower, dark facades result in higher air temperatures compared to clear facades, regardless of thermal inertia. This is because the tower is exposed to direct sunlight and thus absorbs more energy when facades are dark. For I2 and I3, the color of the facades has no significant influence on indoor air temperature with high inertia, while it has a slight impact for clear buildings. Specifically, the indoor air temperature in clear buildings is reduced by 0.3 °C at 16 h for I2 and 0.5 °C at 17 h for I3, resulting in higher indoor air temperatures when multiple reflections are prevalent (canyon street).

Overall, high inertia is beneficial for the comfort of inhabitants in all scenarios. The impact of facade color on indoor air temperature depends on the geometrical configuration and solar paths.

The average amplitude of mean radiant temperatures for the three points of interest are 75 K, 15.5 K, and 68 K, respectively. This difference in amplitude is due to the exposure of points P1 and P3 to direct sunlight during part of the day, while point P2 remains in the shade. During the day, clear facades generate higher mean radiant temperatures than dark facades for all three points studied. Therefore, mean radiant temperature is more sensitive to the change of color than to the change of thermal properties of the buildings. This is because clear facades reflect more shortwave radiation than dark facades, resulting in 80 W, 27 W, and 76 W more radiation received by points P1, P2, and P3, respectively, with clear facades compared to dark facades at 14h. For all three points, regardless of facade color, low inertia generates a higher mean radiant temperature than high inertia because low inertia results in higher surface temperatures, leading to greater longwave radiation emission. During the night, the mean radiant temperature is similar for all scenarios, with differences much smaller than those observed during the day, as seen with surface temperatures.

In conclusion, the most optimal compromise for the entire simulation period is achieved by using high inertia combined with dark facades. Conversely, the worst compromise is attained when using low inertia with clear facades due to their significant shortwave reflection and longwave emission. While high inertia generally yields better results than low inertia, the color of the facade also plays a significant role. The best compromise for surface temperatures is achieved by coupling high inertia with clear facades, while the worst compromise for mean radiant temperature is attained by coupling high inertia with dark facades. For air temperature, the appropriate facade color must be chosen in relation to the geometrical configuration and solar paths, while also using high inertia.

Ultimately, none of the four scenarios produced entirely satisfactory results across all three indicators of the study.

Discussion

To improve the comfort of both inhabitants and pedestrians, we aim to find a compromise between high and low inertia and clear and dark facades. To accomplish this, we compare a reference scenario with low inertia and clear facades to a new scenario (Figure 6).
In this new scenario, we seek to modify the reference scenario as minimally as possible while improving the comfort indicators’ performance. We achieve this by increasing inertia and decreasing albedo as solar radiation increases. Specifically, buildings that are most exposed to the sun’s radiation (i.e., facing south) are modeled with high inertia and dark facades (depicted in red in Figure 7). Buildings facing east and west are modified only in terms of facade color, transitioning from clear to dark (shown in gray in Figure 7). Buildings that are never exposed to direct solar radiation are unchanged in this new scenario. Figure 7 illustrates the indoor air temperatures for the three apartments studied under both scenarios. For all three apartments, the new scenario decreases the indoor air temperature. For apartment I1, which is in the south-facing tower, switching from low inertia with clear facades to high inertia with dark facades reduces indoor air temperature by a maximum of 4.3 °C at 15h. The clear facades allow for reflection of solar radiation, and the high thermal inertia limits the temperature increase of the walls. There is no change in apartment I2, as the same properties were used for the canyon street buildings in both scenarios. For apartment I3, the new scenario allows a maximum decrease of 0.7 °C in indoor air temperature, despite not modifying the apartment’s thermal and radiative properties between the two scenarios. This is due to the change in color of the buildings in front of this apartment (from clear to dark) and the thermal and radiative properties of the buildings at the back of the neighborhood facing south. This is a noteworthy finding, as changes in surrounding buildings’ properties can inadvertently impact the thermal comfort of apartments without directly modifying their properties.

The difference in mean radiant temperature between the reference scenario and the new scenario is presented in Figure 8 for three points of interest (P1, P2, and P3). The new scenario reduces the mean radiant temperature during the day and slightly increases it at night. The most significant changes occur at points with a higher sky view factor and exposure to direct sunlight (P1 and P3). For these points, the mean radiant temperature drops by as much as 7 K during the day and increases by up to 1 K at night. Although the canyon street buildings at point P2 have not been modified, the mean radiant temperature decreases slightly with the new scenario. This is because the buildings at the eastern and western ends of the neighborhood, which are visible from point P2, have changed color and reflect less shortwave energy back to this point. Therefore, the solar reflections at the ends of street canyon have been limited.

Figure 7: Indoor air temperatures of the three apartments studied, according to the two scenarios.
As geometry plays a crucial role of the developed simulation environment, it is possible to spatialize the representation of the mean radiant temperature. We created a mean radiant temperature map of the neighborhood for both scenarios at 14h, based on a grid of 2,352 points spaced 50cm apart with 50,000 rays/point, similar way to daylighting studies. As shown in Figure 9, the new scenario reduces the mean radiant temperature of the neighborhood compared to the reference scenario. Indeed, the mean radiant temperature of the hottest areas near the sunniest buildings has decreased by almost 11 K. The mean radiant temperature of the coldest areas, which are always in the shade, has also decreased due to the suppression of multiple shortwave reflections. Overall, the new scenario results in a decrease in the mean radiant temperature, leading to a better homogenization of comfort than the reference scenario, where the gradients of mean radiant temperature are more pronounced. Indeed, the amplitude of the mean radiant temperature of the neighborhood is 74 K in the reference scenario ($T_{mrt,max} = 383.6 \, K$, $T_{mrt,min} = 309.8 \, K$) while it is 65 K in the new scenario ($T_{mrt,max} = 372.8 \, K$, $T_{mrt,min} = 307.7 \, K$).

Finally, the new scenario leads to improved comfort for both inhabitants and pedestrians throughout the entire simulation, regardless of the geometrical configuration. Regarding pedestrian comfort, the new scenario significantly improves the mean radiant temperature during the day, with a slight increase during the night due to the longwave radiation of buildings with high inertia that have accumulated heat throughout the day. Thus, distributing thermal inertia with an appropriate albedo according to the solar paths enhances the comfort indicators, improving both the comfort of the inhabitants and pedestrians.

Conclusion

We conducted a thermal study on an idealized neighborhood in the city of Cordoba to investigate the influence of thermal inertia and albedo on the comfort of inhabitants and pedestrians. The study involved a model with half a million degrees of freedom and eighty thousand radiative faces. The calculation times are reasonable with half an hour for the calculation of the extended view factors and one hour for the thermal calculation. Four scenarios were compared with different levels of inertia and albedo. While none of these scenarios provided full satisfaction, we found that the high inertia gave better results than the low inertia, but the color plays a significant role. High inertia coupled with clear facades gave the best compromise on surface temperatures, while high inertia coupled with dark facades gave the best compromise on mean radiant temperature. Additionally, we found that high inertia must be paired with an
appropriate albedo depending on the geometrical configuration in relation to solar paths to achieve optimal air temperature.

To illustrate this, we built a new scenario in which the stronger the sunshine, the more the inertia increases, and the albedo decreases. This allowed us to improve simultaneously the comfort of the inhabitants and the pedestrians. Specifically, we were able to decrease the inner air temperature by up to 4°C and decrease the mean radiant temperature by up to 7°C on points of interest representing different configurations, as well as on the whole area, thanks to the use of a mean radiant temperature map.

This study has shown that considering the geometry in a precise way in the calculations is essential to evaluate the comfort of the inhabitants and pedestrians. In this sense, the coupling of the finite element method with the radiosity method provides satisfactory results regarding the fine analysis of the comfort indicators related to the conductive and radiative phenomena.

Moving forward, we aim to optimize the simulation environment, particularly the calculation of mean radiant temperature and extended view factors, which are now optimized for CPUs. We believe that transferring these calculations to GPUs, which are increasingly being developed, will significantly reduce calculation times, and provide greater freedom in graphical representation of results.

References
Le Fichoux E., 2011. Présentation et utilisation de cast3m. ENSTA-LME (http://wwwcast3m.cea.fr).