The admittance method for calculating the internal temperature swings in free running buildings

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Abstract

In order to describe the dynamic thermal response of buildings, the dynamic transfer properties, such as the thermal admittance and the decrement factor, can be used. These parameters, firstly introduced in the Seventies by some British researchers, allow us to quantify the response of the building fabric to sinusoidal temperature variations occurring on both sides of the fabric. Their use has been recently suggested in some international standards, such as the EN ISO 13786:2007.

However, little reference is made in the scientific literature to a further dynamic transfer property, the surface factor, that describes the response of the opaque components of a building to sinusoidal radiant heat fluxes occurring on their internal surface. The surface factor is worth being exploited, since the effect of wall inertia on the radiant heat gains is relevant in the study of the thermal behaviour of a building, especially in summer.

In this paper, an operational formulation of the surface factor is firstly provided starting from its conceptual definition. Afterwards, based on all the previously mentioned dynamic transfer properties, a comprehensive procedure for the assessment of the cyclic temperature swings in an enclosed space is introduced. Thanks to the use of the Fourier analysis, this procedure allows tackling any periodic driving force, and not only sinusoidal ones. The reliability of the procedure is validated against the test cases proposed in the Standards EN ISO 13791:2012 and EN ISO 13792:2012.

Finally, the results shown in the paper enable us to realize the size of the approximation introduced by calculating the dynamic transfer properties only with the first harmonic, i.e. based on sinusoidal driving forces with a period P = 24 hours, as suggested by the simplified approach proposed in the EN ISO 13792:2012 standard.

1. Introduction

The *thermal admittance* and the *decrement factor* are suitable concepts to describe the thermal response of the building opaque components in dynamic conditions. Such parameters were firstly introduced by Loudon at the Building Research Station, UK (Loudon, 1970), and further developed by (Millbank et al., 1974), (Davies, 1973) and (Davies, 1994). Thanks to this pioneering work, the foundations of the *Admittance Procedure* (AP) were laid: it is a technique for estimating energy transfers through the building envelope, which balances simplicity and accuracy and is mainly used for calculating temperature swings inside buildings. More details about the Admittance Procedure can be found in the CIBSE guide on the thermal response of buildings (CIBSE, 2006).

As far as the *thermal admittance* is concerned, it measures the heat flow rate entering the internal surface of a wall as a response to a unit cyclic temperature fluctuation of the air occurring at the same side (Millbank et al., 1974).The *decrement factor,* on the other hand, relates the amplitude of the cyclic external temperature swing acting on the wall to the periodic heat flux released to the indoor air. The decrement factor is normally cited together with the *time lag*, i.e. the time shift between the cyclic energy input and the corresponding response of the wall. Thus, the decrement factor provides information about the dampening of the periodic thermal signal passing from outside to inside, whereas the time lag gives the delay between a peak in the outdoor temperature profile and the corresponding peak in the heat flux released to the indoor air.

It should be remembered that, even if they only

apply to sinusoidal heat fluxes, the dynamic thermal properties can be used to characterize the response of the envelope to any real forcing condition. As a rule, any periodic function can be decomposed, by means of the Fourier analysis, in a series of sinusoidal functions, called *harmonics*, whose frequency is a multiple of the first one, the so-called *fundamental harmonic*. By summing up the response to each harmonic it is possible to obtain the response to the original periodic excitation. When studying the energy performance of buildings, a daily variation occurs for the main forcing conditions, thus the period of the fundamental harmonic is set to P_1 = 24 hours.

The interest in the dynamic thermal properties is also testified by some recent international Standards: the ISO Standard 13786:2007 recommends their adoption for the characterization of the thermal behaviour of the envelope, whereas the ISO Standard 13792:2012 proposes a simplified procedure for the determination of the internal temperature in summer, based on the use of the dynamic properties.

2. Methodology

2.1 Decrement factor and thermal admittance

Let us consider an homogeneous slab of finite thickness, subject to sinusoidal temperature variations θ_{si} and θ_{so} on its internal and external surface, respectively. Let $\bar{\theta}_{si}$ and $\bar{\theta}_{so}$ be the mean values, whereas $\tilde{\theta}_{si}$ and $\tilde{\theta}_{so}$ are the respective cyclic fluctuations around the mean value.

Under the hypothesis of unidirectional conductive heat transfer through the slab thickness in the direction normal to its surfaces, the cyclic heat fluxes $\tilde{q}_i$ and $\tilde{q}_o$ occurring at the two surfaces of the slab can be written as a function of the surface temperature in the following form:

$$\begin{bmatrix} \tilde{\theta}_{si} \\ \tilde{q}_i \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix} \begin{bmatrix} \tilde{\theta}_{so} \\ \tilde{q}_o \end{bmatrix} \quad (1)$$

Here, the elements of the transmission matrix can be calculated as follows (Davies, 1994):

$$z_1 = z_4 = \cosh(t + i\,t) \quad (2)$$

$$z_2 = \frac{\sinh(t + i\,t)}{\xi \cdot (1 + i)} \quad (3)$$

$$z_3 = \xi \cdot (1 + i) \cdot \sinh(t + i\,t) \quad (4)$$

In Equations (2) to (4), *i* is the imaginary unit ($i^2 = -1$). Only two parameters appear in the definition of the matrix, namely the cyclic thickness *t* and the thermal effusivity ξ, defined in Eqs. (5) and (6), that collect all the data concerning the thermal properties of the material, the slab thickness *L* and the period *P* of the cyclic energy transfer:

$$t = \left[\frac{\omega}{2 \cdot \lambda/(\rho c)} \right]^{1/2} \cdot L = \left[\frac{\pi}{P \cdot 3600} \cdot \frac{\rho c}{\lambda} \cdot L^2 \right]^{1/2} \quad (5)$$

$$\xi = \left[\frac{2\pi \cdot \lambda \cdot \rho \cdot c}{P \cdot 3600} \right]^{1/2} \quad (6)$$

In practice, it is more useful to obtain an equation which involves the air temperatures $\tilde{\theta}_i$ and $\tilde{\theta}_o$ instead of the temperatures on the wall surface. In this case, the film thermal resistances R_{si} and R_{so} must be introduced, and the final expression for a multi-layered construction made up of *n* different homogenous layers becomes:

$$\begin{bmatrix} \tilde{\theta}_i \\ \tilde{q}_i \end{bmatrix} = \begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix} \begin{bmatrix} \tilde{\theta}_o \\ \tilde{q}_o \end{bmatrix} \quad (7)$$

The transmission matrix Z of the multi-layered wall is obtained through the product of the matrices related to the each layer, including the transmission matrix containing the film resistance:

$$\begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix} = \begin{bmatrix} 1 & R_{si} \\ 0 & 1 \end{bmatrix} \prod_{k=1}^{n} \begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix}_k \begin{bmatrix} 1 & R_{so} \\ 0 & 1 \end{bmatrix} \quad (8)$$

In Eq. (7), the sol-air temperature can be used in place of the outdoor temperature if the effect of the solar radiation absorbed on the outer surface of the wall has to be taken into account.

According to the presented methodology, one can introduce the so-called periodic thermal transmittance X, defined as the ratio of the cyclic heat flux released on the internal surface of the wall to the cyclic temperature excitation on the other side of the wall, while holding a constant indoor temperature ($\tilde{\theta}_i = 0$, see Eq. 9). The decrement factor *f* is defined as the amplitude of the periodic thermal transmittance, normalized with respect to the steady thermal transmittance U; moreover, the time lag ϕ is the phase of the complex number X, measured in hours and referred to a solicitation having a period *P* (see Eq. 10).

Finally, the admittance Y is conceptually similar to the periodic thermal transmittance, but in this case the temperature excitation $\tilde{\theta}_i$ and the wall response $\tilde{q}_i$ are measured by the same side (see Eq. 11).

$$X = \left.\frac{\tilde{q}_i}{\tilde{\theta}_o}\right|_{\tilde{\theta}_i=0} = -\frac{1}{Z_2} \quad (9)$$

$$f = \left|\frac{X}{U}\right| \qquad \varphi = \frac{P}{2\pi}\cdot\arctan\left(\frac{\mathrm{Im}(X)}{\mathrm{Re}(X)}\right) \quad (10)$$

$$Y = \left.\frac{\tilde{q}_i}{\tilde{\theta}_i}\right|_{\tilde{\theta}_o=0} = -\frac{Z_4}{Z_2} \quad (11)$$

According to the ISO Standard 13786:2007, the behaviour of the building envelope is fully described by the values of the dynamic transfer properties for P = P_1 = 24 h. However, all the relations previously introduced can be applied to any harmonic of order *n*, i.e. having a period $P_n = P_1/n$.

2.2 The surface factor

Despite the interest shown in the scientific literature towards the dynamic transfer properties, little reference is made to the thermal response of the opaque components to the radiant heat fluxes occurring on their internal surface, such as those associated to solar heat gains through the windows or to internal radiant loads.

Some attempts were made in the past in this sense. For instance, worth mentioning is the *thermal storage factor* defined in the Carrier method (Carrier, 1962) as the ratio of the rate of instantaneous cooling load to the rate of solar heat gain. This factor has to be determined through appropriate tables depending on the weight per unit floor area of the opaque components and the running time. Therefore, its use requires interpolation among table data, it is rather rough because it does not account for the actual sequence of the wall layers, and it lacks any theoretical basis, as it comes from numerical simulations.

A substantially different approach can be found in the framework of the Admittance Procedure, laid down in the early Seventies (Millbank et al, 1974), where these contributions are taken into account by means of the so called *surface factor*. Nonetheless, the surface factor has been deserved little attention: to the authors' knowledge, little reference is made to this parameter in the whole scientific literature (Beattie and Ward, 1999) (Rees et al., 2000), while its definition has been only recently recovered in the CIBSE guide (CIBSE, 2006) and in the international Standard ISO 13792:2012.

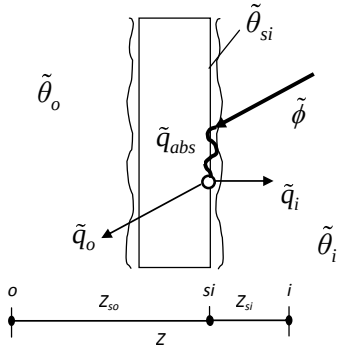

Fig. 1 – Energy balance on the internal surface for the definition of the surface factor

According to the definition provided by (Millbank et al., 1974), the *surface factor* F quantifies the thermal flux released by a wall to the environmental point per unit heat gain impinging on its internal surface, when the air temperatures on both sides of the wall are held constant and equal. With reference to Fig. 1, let us call $\tilde{\phi}$ the cyclic radiant thermal flux acting on the internal surface of the wall, as a result of the radiant energy released by internal sources or transmitted through the glazing; the following definition holds:

$$F = \left.\frac{\tilde{q}_i}{\tilde{q}_{abs}}\right|_{\tilde{\theta}_i=\tilde{\theta}_o=0} = \frac{\tilde{q}_i}{\alpha\cdot\tilde{\phi}} \quad (12)$$

Here, α is the fraction of the radiant thermal flux that is absorbed by the wall. This amount of thermal energy will be then re-emitted to the internal (q_i) and to the external environment (q_o): the ratio between such contributions equals the inverse ratio of the corresponding thermal impedances. This leads to the following expression:

$$\tilde{q}_i = \alpha\cdot\tilde{\phi}\cdot\left(\frac{Z-Z_{si}}{Z}\right) = \tilde{q}_{abs}\cdot\left(1-\frac{Z_{si}}{Z}\right) \quad (13)$$

At this stage, one must consider that the thermal impedance Z_{si} between the surface of the wall and the indoor air is purely resistive: thus, $Z_{si} = R_{sF}$, being R_{sF} the inner side thermal resistance. Moreover, the inverse of the wall thermal impedance Z corresponds to the thermal admittance Y. Such positions imply the following expression for the surface factor:

$$F = \frac{\tilde{q}_i}{\tilde{q}_{abs}} = 1 - Y \cdot R_{sF} \quad (14)$$

The previous definition corresponds to the one provided in the CIBSE guide and in the ISO 13792:2012 international standard. The value calculated according to Eq. (14) is a complex number, to quantify in terms of amplitude and argument. The latter can be assessed through Eq. (15), and will always result negative, which means that it describes a delay of the wall response to the radiant heat flux acting on it. For this reason, since now on this term will be named *time shift* and will be assigned a positive sign.

$$\varphi_F = \frac{P}{2\pi} \cdot \arctan\left(\frac{Im(F)}{Re(F)}\right) \quad (15)$$

However, it is important to underline that the internal surface resistance R_{sF} is not necessarily the same as that used for in Eq. (8), i.e. for the definition of X and Y, since the definition framework of such parameters is not the same. The Standard ISO 13792:2012 suggests to use $R_{sF} = 0.22$ m²·K·W⁻¹.

2.3 The energy balance for a room

According to the definition of the previously mentioned dynamic transfer properties, the response of an opaque component subject to external periodic forcing conditions can be written as follows.

Equation (16) describes the density of heat flux released by a wall to the indoor environment as a response to the *n*-th harmonic component of the forcing conditions. Here, θ_o is the sol-air temperature, which accounts for both the outdoor temperature and the solar irradiation absorbed on the outer surface. On the other hand, Eq. (17) is the stationary term, obtained by considering the average value of each forcing condition. The periodic response of the wall is obtained by recombining such contributions as in Eq. (18); the summation of the harmonics is truncated to the harmonic of order N_H.

$$\tilde{q}_{i,n} = X_n \cdot \tilde{\theta}_{o,n} - Y_n \cdot \tilde{\theta}_{i,n} + F_n \cdot \tilde{\phi}_n \quad (16)$$

$$\bar{q}_i = U \cdot \left(\bar{\theta}_o - \bar{\theta}_i\right) + F \cdot \bar{\phi} \quad (17)$$

$$q_i(\tau) = \bar{q}_i + \sum_{n=1}^{N_H} \tilde{q}_{i,n} \quad (18)$$

Finally, the energy balance on the indoor environment can be written as in Eq. (19). Here, one can recognize the contributions due to :

- heat transfer through the windows;
- infiltration of outdoor air (na is the number of air changes per hour);
- convective part of the internal loads, Qint (people, lighting, appliances)

The thermal balance reported in Eq. (19) does not contain any contribution due to heating or cooling plants, since it refers to free-running buildings. It can be repeated at each time step τ (here, τ = 1 h); due to the negligible thermal capacity of the indoor air, the thermal balance assumes the same form as for steady state conditions.

$$\sum_{k=1}^{6} q_{i,k,\tau} + \left(U_w A_w + 0.34 \cdot n_a V\right)\left[\theta_{o,\tau} - \theta_{i,\tau}\right] + Q_{int,\tau} = 0 \quad (19)$$

Equation (19) can be used to calculate the time profile of the indoor air temperature θ_i. Starting from such information, one can also calculate the temperature of the inner surface of the *k*-th wall as:

$$\theta_{si,k}(\tau) = \theta_i(\tau) + q_{i,k}(\tau) \cdot R_{si} \quad (20)$$

The last point to address is the determination of the radiant thermal flux ϕ coming from external agents and acting on the inner surface of each wall. This term appears in Eq. 17 (mean value) and Eq. 16 (cyclic variation around the mean value), and is basically due to the solar radiation transmitted through the glazing and to the radiant component of the internal loads. However, its evaluation is not easy, as it implies the knowledge of the distribution of such radiant flows in the indoor environment.

In this study, the authors chose to adopt a simplified approach, based on the Ulbricht hypothesis, i.e. the uniform distribution of the radiant heat gains ψ over all the surfaces that form the boundary of the enclosure. According to this model, the radiant flux acting on the generic surface can be calculated as:

$$\phi = \frac{\psi}{\left(1 - r_m\right) \cdot A_{tot}} \quad (21)$$

Here, r_m is the mean reflectivity of the enclosing surface. In its evaluation, it is suitable to split short-wave (sw) and long-wave (lw) radiant fluxes, as the reflectivity of walls and glazing to such contributions is not the same. Thus, Eq. (21) can be written as:

$$\phi = \frac{1}{A_{tot}} \cdot \left[\frac{\psi_{lw}}{\left(1 - r_{m,lw}\right)} + \frac{\psi_{sw}}{\left(1 - r_{m,sw}\right)}\right] \quad (22)$$

Here, ψ_{sw} relates to the solar radiation transmitted through the glazing, whereas ψ_{lw} is mainly related to

internal radiant sources and to the fraction of solar energy absorbed by the glazing and re-emitted towards the indoor environment. All of the data needed in Eq. (22) are usually known.

In the following, the validation of the formulation presented so far is discussed, by using the test cases reported in the EN ISO 13791 and EN ISO 13792 standards, which concern the calculation of the internal temperature of a room in summer without mechanical cooling. It should be remembered that, despite the admittance procedure being well established in the literature, some novel features are introduced in the present study:

- the use of the surface factor for describing the response of the walls to internal radiant fluxes;
- the adoption of the Ulbricht hypothesis for evaluating the distribution of such radiant fluxes in the indoor environment.

The validation procedure will evaluate the reliability of this approach. It will also be helpful in identifying the most appropriate value to be used for the term R_{sF} (see Eq. 14). Furthermore, the approximation introduced by truncating the sum in Eq. (19) to the first harmonic (P = 24 h), as suggested by the simplified approach proposed in the Standard EN ISO 13792:2012, will be also discussed.

3. The Validation Procedure

The validation of new mathematical codes for the dynamic simulation of buildings can be performed by comparing the simulated results to appropriate reference values, obtained through experimental measurements or by means of well established codes.

The EN ISO 13791 and EN ISO 13792 international standards propose a procedure that allows the validation of mathematical models for the calculation of the summer internal temperature in enclosures without mechanical cooling. In both standards, the procedure consists in the calculation of the hourly profile of the operative temperature for a test room; the minimum, average and maximum values shall then be compared to the reference values indicated in the standard. Actually, the two documents use the same validation procedure and the same test cases; the difference lies in the reference values and in the range of allowed acceptable discrepancy for the calculated results. Indeed, according to ISO 13791, a difference of less than 0.5 K between reference values and simulated values can be accepted. On the other hand, ISO 13792 introduces three classes on the basis of the difference Δ between the calculated values and the reference values:

Class 1: $-1\ K < \Delta < 1\ K$

Class 2: $-1\ K < \Delta < 2\ K$

Class 3: $-1\ K < \Delta < 3\ K$

The model is classified according to the worst result.

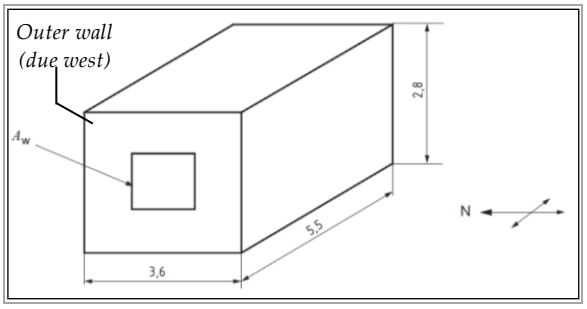

Fig. 2 – Geometry proposed by the ISO Standards

	Y	X	ϕ	F	ϕF	U
Wall (all)	4.66	0.08	-11.7	0.35	-2.0	0.49
Partition (all)	0.77	0.34	-1.1	0.91	-0.6	0.35
Ceiling (2)	5.14	0.12	-9.6	0.20	-3.1	0.71
Floor (2)	5.41	0.12	-9.6	0.45	-3.1	0.71
Ceiling (1)	0.61	0.01	-11.5	0.92	-0.4	0.24
Floor (1-3)	5.40	0.01	-11.5	0.45	-3.1	0.24
Roof (3)	5.10	0.08	-7.4	0.20	-2.9	0.43

Table 1 – Dynamic transfer properties (in brackets, the ISO test cases)

Figure 2 shows the geometry of the test room. Two different cases are available, which differ in the size A_w of the window (3.5 m² for case A, 7.0 m² for case B). Furthermore, in Case A there is a single pane window provided with an external shading device, whereas Case B implies a double pane glazing. The climatic conditions are also different: case A applies to warm climates (latitude 40°N), whereas case B applies to temperate climates (latitude 52°N)

For each room geometry, different sub-cases are possible, distinguished by a test number (from 1 to 3, according to the type of floor/ceiling) and by a second letter, associated to ventilation rate (*a* = 1 ACH, *c* = 10 ACH). Case *b* applies a variable ventilation rate, but it is not considered in the

framework of this study, since the proposed model only allows constant parameters. All the surfaces, except the outer wall and the roof in the test number 3, are bounded by similar rooms.

Table 1 collects the dynamic transfer properties for the vertical and horizontal envelope components, calculated for the fundamental harmonic (P = 24 h) with the relations introduced in the previous sections, as required by the standards. Concerning the glazed element and the shading device, the external, internal and cavity thermal resistances are assigned, as well as the short-wave reflectance and transmittance. The standards also prescribe the time profile of the internal sources to be used in the simulations, and an equal proportion of the heat flow transferred to the room by convection and radiation (50% each).

Thus, all the input values needed for the simulations are assigned, and they can be easily implemented in the calculation procedure shown in the previous section. The calculation was carried out by using N_H = 6 harmonics, since a preliminary analysis showed that no significant variation of the results would be introduced by the addition of further harmonics.

The calculation provides the time profile for the indoor temperature and for each surface temperature; these results can be finally used for the determination of both the mean radiant temperature and the room operative temperature according to Eq. (23) and (24).

$$\theta_{mr}(\tau) = \sum_{k=1}^{6} A_k \cdot \theta_{si,k}(\tau) \Big/ \sum_{k=1}^{6} A_k \qquad (23)$$

$$\theta_{op}(\tau) = 0.5 \cdot \left[\theta_{mr}(\tau) + \theta_i(\tau)\right] \qquad (24)$$

4. Results and discussion

Before discussing the validation outcomes, it is convenient to clarify something about the surface resistance R_{sF}. The values of this parameter to be used for the calculation of F (see Eq. 14) should not be the same as for the calculation of X and Y (see Eq. 8). In fact, while F rests on the hypothesis of constant air temperature on both sides of the wall, for X and Y this constraint does not hold, and any suitable value of the surface thermal resistance can be adopted.

The ISO Standard 13792 prescribes R_s = 0.22 $m^2 \cdot K \cdot W^{-1}$ (i.e. $h = R_s^{-1}$ = 4.5 $W \cdot m^{-2} \cdot K^{-1}$) for all the dynamic properties (X, Y, F) and for any wall. However, due to the relevance of this point, the authors conducted a parametric analysis aimed at assessing the influence of the coefficient h on the outcomes of the validation procedure. In the following two values will be considered: h = 4.5 $W \cdot m^{-2} \cdot K^{-1}$ (default value) and h = 2.5 $W \cdot m^{-2} \cdot K^{-1}$.

It is also important to outline that, although the approach to the validation procedure is common in ISO 13791 and ISO 13792, some differences still stand. For instance, there are some discrepancies between the values of the solar radiation impinging on the west wall, and - above all - different reference values are assigned for the room operative temperature.

Figure 3 shows the discrepancy Δ between the results of the simulations and the reference values provided in the ISO Standards, for each test case considered in this study. The compliance of the calculation procedure to the standards has to be assessed by looking at the minimum, the average and the maximum room operative temperature; thus, the diagrams report the discrepancy Δ for each one of these parameters. Two ranges are highlighted: the narrow one (-0.5 K < Δ < 0.5 K) is the range that assures the compliance to ISO 13791, whereas the largest one (-1 K < Δ < 1 K) must be respected in order for the calculation procedure to be classified in Class 1 according to ISO 13792.

However, in most of the unfavourable cases, Δ keeps within the range 0.5 - 1 K, and only in very few cases Δ is higher than 1 K. On the whole, h = 4.5 $W \cdot m^{-2} \cdot K^{-1}$ (hollow triangles) seems to introduce a lower discrepancy than h = 2.5 $W \cdot m^{-2} \cdot K^{-1}$ (filled triangles). On the contrary, when looking at the comparison between simulations and reference values provided in ISO 13792, the discrepancy Δ is very often negative. However, this standard is less strict than ISO 13791, as it allows a higher discrepancy (-1 K < Δ < 1 K). Thus, almost all cases comply to the Standard: if h = 4.5 $W \cdot m^{-2} \cdot K^{-1}$ (hollow squares), only case B.1.a shows a discrepancy higher than 1 K. However, this is sufficient to classify the calculation procedure in Class 2. When h = 2.5 $W \cdot m^{-2} \cdot K^{-1}$ (filled squares) the condition required for Class 1 is always met.

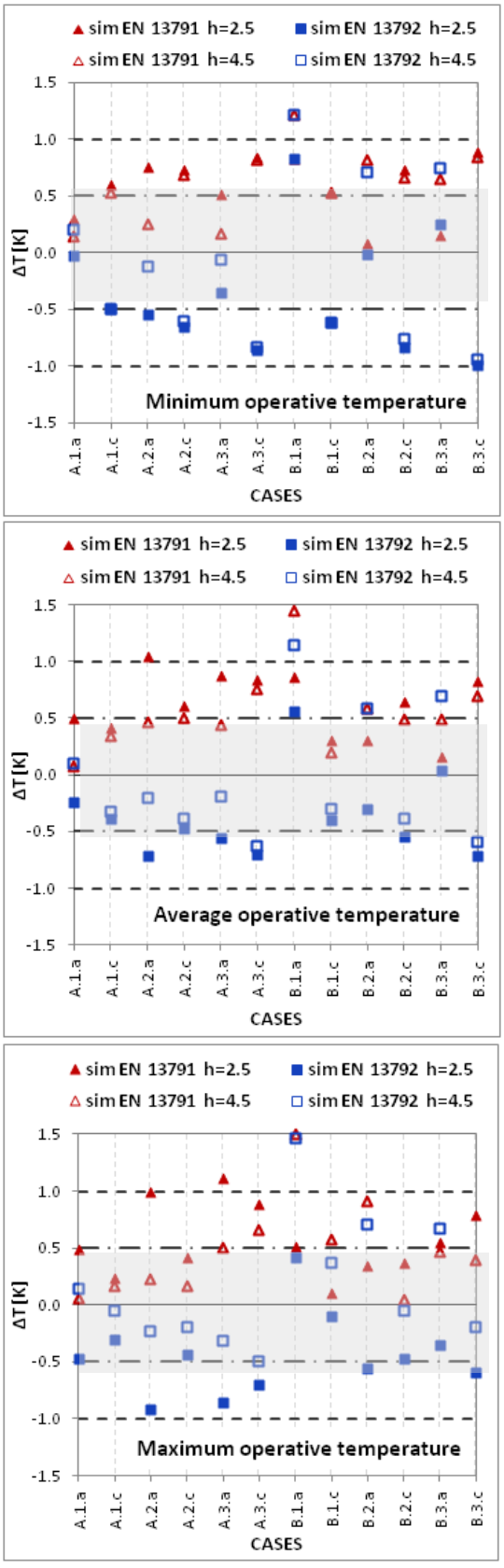

Fig. 3 – Verification of the compliance to the ISO Standards: discrepancy with the reference values (a: min Top, b: average Top, c: max Top)

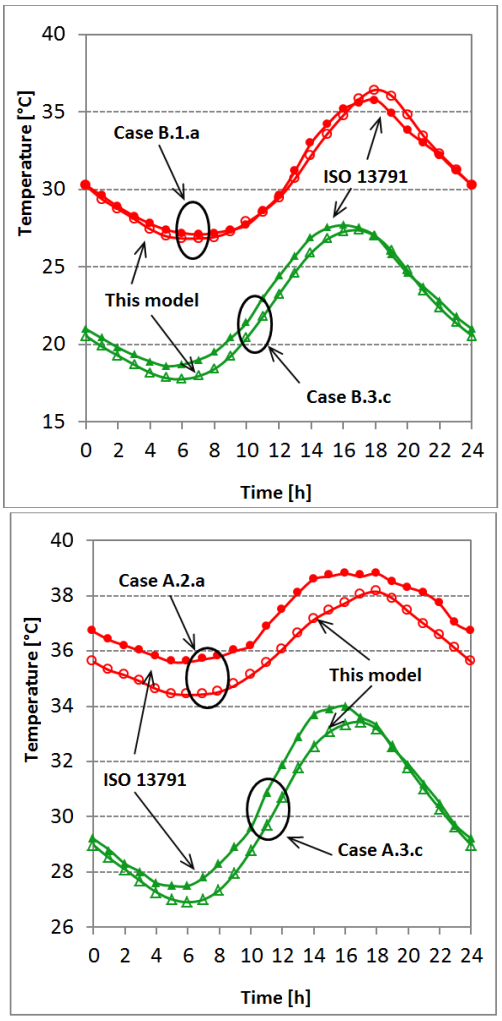

Fig. 4 – Comparison between simulated results and reference values for the operative temperature

Figure 4 shows the comparison between simulated and reference daily profile of the operative temperature for the most unsatisfactory cases emerging from Fig. 3. The comparison is based on ISO 13791, since ISO 13792 does not provide any reference daily profile.

4.1 The influence of the number of harmonics on the building response

In the previous section, it was shown that the mathematical model discussed so far proves sufficiently reliable if adopting $N_H = 6$ harmonics in the Fourier analysis. However, one must remark that, both in the scientific literature and in the international standards, the dynamic transfer properties are normally used by looking only at the fundamental harmonic ($N_H = 1$).

With reference to the case B.1.c of ISO 13792, Fig. 5 shows that, if comparing the room operative temperature calculated by truncating the sum of Eq. (18) to N_H = 1 and N_H = 6, the results are quite different. Indeed, the difference between the two profiles can also be higher than 1 K during some hours of the simulated day.

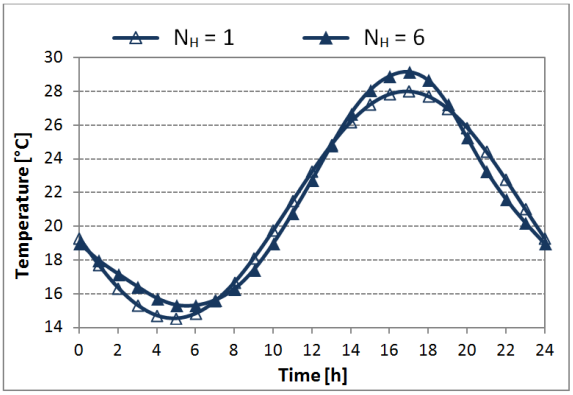

Fig. 5 – Simulated results for NH = 1 and NH = 6 (test case B.1.c)

Test cases	Discrepancy with EN 13792 (NH = 1)			Class for NH = 1	Class for NH = 6
	ΔT_{min}	ΔT_{avg}	ΔT_{max}		
A.1.a	0.1	0.1	0.7	**1**	**1**
A.1.c	0.8	0.4	0.8	**1**	**1**
A.2.a	0.6	0.7	1.1	**2**	**1**
A.2.c	0.9	0.5	0.8	**1**	**1**
A.3.a	0.3	0.5	1.0	**2**	**1**
A.3.c	1.1	0.8	1.0	**2**	**1**
B.1.a	0.6	0.8	0.4	**1**	**1**
B.1.c	1.2	0.5	1.1	**2**	**1**
B.2.a	0.2	0.2	1.1	**2**	**1**
B.2.c	1.2	0.6	1.1	**2**	**1**
B.3.a	0.1	0.2	0.8	**1**	**1**
B.3.c	1.3	0.8	1.2	**2**	**1**

Table 2 – Effect of the number of harmonics on the classification of the mathematical procedure

It is then interesting to evaluate such a difference for some other representative cases. The results of this calculation are reported in Fig. 6, where the curves refer to the most unfavourable cases addressed during the validation procedure. Here:

$$\Delta\theta_N = \theta_{op}\big|_{N_H=1} - \theta_{op}\big|_{N_H=6} \qquad (25)$$

If the validation procedure were carried out by using N_H = 1, the discrepancy highlighted in Fig. 6 would lead the model to be downgraded to Class 2, as shown in Table 2, whereas for N_H = 6 it deserves Class 1. The misleading effect of considering only the first harmonic in the evaluation of X and Y was already observed by (Gasparella et al., 2011).

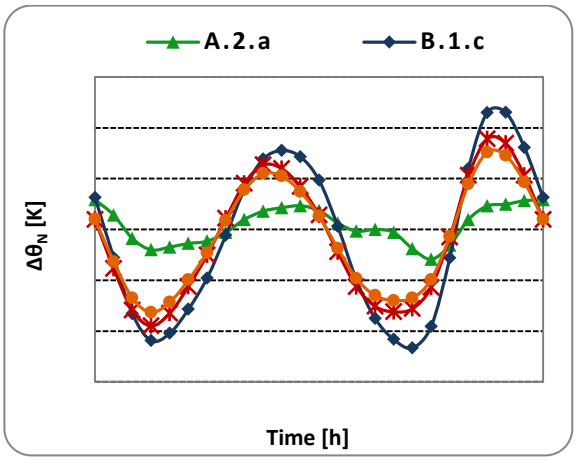

Fig. 6 – Difference between the simulations with NH = 1 and those with NH = 6

5. Conclusion

In this study, the validation of an original calculation code for the evaluation of the dynamic thermal response of buildings was addressed. This code is based on the Admittance Method and is supported by the use of the *surface factor*, a concept rarely exploited so far in the literature, that allows the explicit consideration of the radiant heat gains acting on the surfaces of the enclosure. These effects are normally accounted for through numerical methods or by means of *thermal storage factors* determined through appropriate tables, like in the Carrier method. On the contrary, the use of the *surface factor* allows a rigorous analytical approach having general validity.

The overall mathematical model discussed so far resulted sufficiently reliable on the basis of the validation procedure proposed in the ISO Standards 13791 and 13792. The results also showed that, in order to obtain more satisfactory results, an appropriate value must be used for the inner surface resistance R_{sF} in the calculation of the surface factor. Even if the Standard EN 13792 suggests to use R_{sF} = 1/4.5, the paper underlines that in some cases R_{sF} = 1/2.5 provides better results, leading to the achievement of Class 1.

Finally, it was also shown that, when using methods based on the harmonic analysis, the adoption of the only fundamental harmonic may be inadequate for a reliable prediction of the building thermal response. As an example, according to the ISO 13792, the proposed model, if based on the first harmonic (N_H = 1), would be classified in Class 2, whereas it deserves Class 1 if N_H = 6.

6. Nomenclature

Symbols

A	area (m^2)
c	specific heat capacity ($J\ kg^{-1}\ K^{-1}$)
f	decrement factor (-)
F	surface factor ($W\ m^{-2}\ K^{-1}$)
h	heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
L	thickness (m)
n	order of the harmonic (-)
N	total number of harmonics (-)
P	time period (h)
q	density of heat flux ($W\ m^{-2}$)
$\tilde{q}$	cyclic density of heat flux ($W\ m^{-2}$)
Q	thermal power (W)
r	reflectivity (-)
R	thermal resistance ($m^2\ K^1\ W^{-1}$)
t	cyclic thickness (-)
U	thermal transmittance ($W\ m^{-2}\ K^{-1}$)
V	volume (m^3)
X	periodic transmittance ($W\ m^{-2}\ K^{-1}$)
Y	thermal admittance ($W\ m^{-2}\ K^{-1}$)
Z	thermal impedance ($m^2\ K\ W^{-1}$)

Greek letters

α	absorption coefficient (-)
ϕ	time shift (h)
ϕ	radiant heat flux ($W\ m^{-2}$)
λ	thermal conductivity ($W\ m^{-2}\ K^{-1}$)
ψ	radiant thermal power of a source (W)
ρ	density ($kg\ m^{-3}$)
θ	temperature (K)
$\tilde{\theta}$	cyclic temperature variation (K)
ω	angular frequency ($rad\ h^{-1}$)
ξ	thermal effusivity ($W\ m^{-2}\ K^{-1}$)

Subscripts

a	air
c	convective
H	harmonic
i	indoor
lw	long wave
m	mean
mr	mean radiant
o	outdoor
op	operative
si	inner surface
so	outer surface
sw	short wave

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