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Comparing Bottom-Up Energy Consumption Models Using The Wasserstein Distance Between Load Profile Histograms

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Abstract

This paper presents a comparison of bottom up models that generate appliance load profiles. The comparison is based on their ability to accurately distribute load over time-of-day. This is a key feature of model performance if the model is used to assess the impact of low carbon technologies and practices on the network. No work has yet assessed models on this basis. In this work, the temporal characteristics of load are captured using histograms, and similarity between the histogram representations of measured and generated data is assessed using the Wasserstein distance. This is then applied to compare the results of three models, which were developed here by adopting approaches used in previous research. One is based on occupant presence, one on occupant activity, and one on empirical data. Typical statistical tests showed that the comparison method is robust and can be used for this purpose.

Introduction

Accurate estimation of the distribution of load over time is required to quantify and assess the effects of low carbon technologies and practices on low voltage electricity distribution networks. In this work, we present a comparison of relevant appliance load modelling approaches on this basis.

There is a clear variety of views on what is the best approach to model appliance load profiles. Yamaguchi (2019) reviewed a range of approaches to modelling appliance usage and classified them into four categories. Inference of load from usage is generally independent of the approach to modelling usage, so this classification still applies here. Household occupancy-based switch-on probability models, including those proposed by Richardson et al. (2010) and Flett and Kelly (2017), relate the use of appliances to models of occupant presence, which are derived from time-use survey (TUS) data. Individual agent activity-based appliance use models, such as those introduced by Widén and Wäckelgård (2010) and Wilke et al. (2013), use TUS-based sub-models to simulate the activities from which appliance use can be inferred. Empirical data-based time-dependent switch-on probability models, which include those introduced by Paatero and Lund (2006), Page (2007), Gruber et al. (2014), and Yilmaz et al. (2017), treat occupant behaviour as a latent entity and sample the key features of appliance usage from low-dimensional distributions, such as the number of switch on events in a day, the timing of each switch-on event and the duration of each use. For conciseness, we refer to these categories as occupant presence driven (OPD) models, occupant activity driven (OAD) models, and empirical data driven (EDD) models, respectively. Yamaguchi (2019) also identified TUS-based time-dependent switch-on probability models, such as that proposed by Fischer, Härtl, and Wille-Haussmann (2015), which operate very similarly to EDD models but instead derive switch-on probabilities from TUS data. However, since models of this form are very similar to EDD models, we do not consider them any further in this work.

Yamaguchi (2019) and others have compared a range of these models based on various features of appliance usage and load. However, there is little research done on the accuracy with which a model captures the probability of an appliance exhibiting a certain load at a certain point in time. This is what this work does. Due to the availability of data and in line with other research this study is restricted to the duration of one day.

To allow the assessment of the ability of a model to accurately capture the distribution of appliance load over a day, we first mapped individual appliance load profiles to 2-dimensional histograms that give the distribution of load with respect to the time-of-day. We then assessed the similarity against measured load profiles using the Wasserstein distance between histogram representations.

On this basis, we formally defined and compared three models, which are modified versions of existing models proposed for the generation of appliance load profiles. The models differ in the way they determine switch-on events, and include: an OPD model based on the strategy proposed by Richardson et al. (2010), an OAD model based on the approach used by Widén and Wäckelgård (2010), and an EDD model based on the methodology of Yilmaz et al. (2017). For simplification, we focus on the modelling of washing machine load profiles on weekdays.

Experiment: comparison of models

Data: load measurements

The data used as ground truth and in calibrating the EDD model was the Household Electricity Survey (HES) (2014). In collection of this data, Wattmeter's were fitted to appliances in 250 households to record electrical load, which took place between May 2010 and June 2011. In 224 of these households, appliances were monitored at a resolution of 2-minutes, for a period that varied between 20 and 45 days. To expand the dataset as much as possible with the available equipment, different households were monitored over different periods. In addition to the load monitoring, the dataset includes several socio-demographic factors for each household, including the resident number. It should be noted that data on the location of households is not included.

For consistency between load profiles, the HES (2014) dataset was reduced to the 170 washing machines for which 1) 18 full weekdays of load measurements could be extracted and that 2) came from a household with <6 residents. This led to the removal of 14 washing machines Each load profile was then reduced to the first 18 weekdays.

Data: time-use survey (TUS)

The calibration of the OPD and OAD models involved the UK TUS (2001). This was collected in 2000 and involved 6,414 participants over the age of 8 from a wide range of demographics. Each participant completed two 24-hour diaries, one on a weekday and one on a weekend, documenting their location, activity and other criteria at 10-minute intervals.

Methodology

For each ground truth washing machine from the HES (2014), indexed by k, and for each model, we repeatedly used the model to reproduce the data and quantified the similarity of the synthetic data with the ground truth, repeating until we have a reliable approximation of the mean similarity value. This resulted in a score for the model's ability to simulate the particular ground truth washing machine, which we refer to as the ϕ_k-score.

For a model $M\colon \boldsymbol{f}_k \to \boldsymbol{g}_{k,\gamma}$, where $\boldsymbol{f}_k = [r_k,\ D_k, \boldsymbol{\theta}_k]$ is the set of features that define the appliance being simulated, where r_k is the number of residents, D_k is the number of days in the simulation and $\boldsymbol{\theta}_k$ is the day-type of each day (weekday or weekend day, however this was always set to weekday throughout the experiment), and $\boldsymbol{g}_{k,\gamma}$ is the γth generated load profile for that appliance, we define the ϕ_k-score:

$$\phi_k(M) = \frac{1}{n}\sum_{\gamma=1}^{n} W\big(\boldsymbol{\rho}_k^0, \boldsymbol{\rho}_{k,\gamma}^1\big) \qquad (1)$$

where n is the number of repetitions necessary for ϕ_k to be reliable, $W\big(\boldsymbol{\rho}_k^0, \boldsymbol{\rho}_{k,\gamma}^1\big)$ is the Wasserstein distance between $\boldsymbol{\rho}_k^0$ and $\boldsymbol{\rho}_{k,\gamma}^1$, $\boldsymbol{\rho}_k^0 = \boldsymbol{H}(\boldsymbol{l}_k)$ where $\boldsymbol{H}$ is a histogram representation of appliance load profiles and $\boldsymbol{l}_k$ is the kth real profile, and $\boldsymbol{\rho}_{k,\gamma}^1 = \boldsymbol{H}(\boldsymbol{g}_{k,\gamma})$.

The Wasserstein distance is a powerful measure of similarity between two probability distributions that can be effectively used with load profile histograms. The common intuition behind this measure is to consider each probability distribution representing the shape and position of a pile of dirt within a particular space. The Wasserstein distance is then the minimum amount of energy required to transform one pile of dirt into another, and it is therefore the optimal transport (OT) between probability distributions, as described by Monge (1781). In contrast to most measures of statistical similarity, this accounts for the underlying geometry of the space.

In simple discrete terms, for a set of sources $\boldsymbol{X}$ and a set of destinations $\boldsymbol{Y}$, with cardinality N and M respectively, the optimal transport plan $T\colon \boldsymbol{X} \to \boldsymbol{Y}$ is then the plan of transport that minimises the total cost c, which is the sum of energy across all movements of material:

$$c(T) = \min_{\pi} \sum_{x\in \boldsymbol{X}} \sum_{y \in \boldsymbol{Y}} \|x - y\|_p \, d\pi(x, y) \qquad (2)$$

$\|x - y\|_p$, $1 \le p \le \infty$ is the p-norm, which is the ground distance measure between x and y as defined by p, and $d\pi(x, y)$ is the amount of mass moved from x to y. In this work we use $p = 2$, making this the Euclidean distance.

Model inputs

Prior to generation, models are conditioned to generate load profiles based on the feature set $\boldsymbol{f}_k = [r_k,\ D_k, \boldsymbol{\theta}_k]$. Consistent with the measured data, we used a value for r_k that corresponds to the ground truth washing machine under consideration. To generate 18 days of data, we actually use $D_k = 19$, since all models initialize with the washing machine switched off, and by generating a 19 day long profile, we can then remove the first and overcome any associated issues. Finally, we set all values $\boldsymbol{\theta}_k$ to the value representative of a weekday, for all k.

Occupant presence driven (OPD) model

The OPD model is based on the model proposed by Richardson et al. (2010). This initially predicts the number of occupants in a household at 10-minute intervals using the time-use survey (TUS) calibrated first-order time-inhomogeneous Markov chain given in Algorithm 1. This calibration followed Richardson et al. (2008) and is defined in Equations (1)-(3). Here, $T_{10} = D_k \times 144$ (144 10-minute intervals in 24-hours), $\{t_{10} \in \mathbb{N} | 1 \le t_{10} \le T_{10}\}$ is the tth 10-minute time-step of the simulation, $\tau_{10} = 1 + ((t_{10} - 1)\ mod\ 144)$ is the τth 10-minute time-step since the previous midnight ($a\ mod\ b$ is the modulo operation and returns the remainder of the division of dividend a and divisor b), $x(t_{10})$ is the state at t_{10}, and $\boldsymbol{x}_{k,\gamma} = \big[x_{k,\gamma}(1), x_{k,\gamma}(2), \ldots, x_{k,\gamma}(T_{10})\big]$. $x_{k,\gamma}(t_{10})$ is determined probabilistically using the transition matrix $\boldsymbol{P}(r_k, \tau_{10})$, where:

$$\boldsymbol{P}(r_k, \tau_{10}) = \begin{bmatrix} p_{1,1}(r_k, \tau_{10}) & \cdots & p_{1,S}(r_k, \tau_{10}) \\ \vdots & \ddots & \vdots \\ p_{S,1}(r_k, \tau_{10}) & \cdots & p_{S,S}(r_k, \tau_{10}) \end{bmatrix} \qquad (3)$$

$$p_{i,j}(R,\tau_{10}) = \frac{N_{i,j}(r_k,\tau_{10})}{\sum_{i=1}^{N_s} N_{i,j}(r_k,\tau_{10})} \quad (4)$$

where $\boldsymbol{s_k} = [s_1, s_2, \dots, s_{N_s}]$ is the possible states, and $N_{i,j}(R, t_{10})$ is the total number of times households of size r_k transitioned from s_j to s_i during the τth 10-minute time-step since midnight. $\boldsymbol{P}(r_k, \tau_{10} = 1)$ is a special case where there is no previous state to consider, and therefore:

$$p_{i,j}(R,\tau_{10} = 1) = \frac{N_{i,initial}(r_k)}{\sum_{i=1}^{N_s} N_{i,initial}(r_k)} \quad (5)$$

Where $N_{i,initial}(r_k)$ is the number of times households of size r_k began in state s_i.

Algorithm 1: First-order time-inhomogeneous Markov chain-based simulation performed in step 1 of OPD and OAD models

Input: Resident number r_k, Simulation period T_{10}
Initialize $t_{10} = 1$, $\tau_{10} = 1$
Obtain $x_{k,\gamma}(t_{10} = 1)$ using $\boldsymbol{P}(r_k, \tau_{10} = 1)$
While $t_{10} < T_{10}$ **do**
    $t_{10} = t_{10} + 1$
    $\tau_{10} = 1 + ((t_{10} - 1)\ mod\ 144)$
    Obtain $x_{k,\gamma}(t_{10})$ using $\boldsymbol{P}(r_k, \tau_{10})$
End
Output: Occupancy series $\boldsymbol{x}_{k,\gamma}$

For the OPD model, $N_s = r_k + 1$ (occupant presence is 0 when the house is empty), and the values of $\boldsymbol{s}$ are the possibilities for occupant presence.

The model then steps through the day at 2-minute resolution using switch-on probabilities to determine whether a switch-on event will occur, accounting for active occupancy. The probability of a laundry activity being performed under the condition of a certain level of occupant presence is taken from TUS data. To estimate the switch-on probability of a washing machine from this, the probability of the laundry activity being performed is then multiplied by a calibration scalar. The original calibration scalar was determined by Richardson et al. (2010) accounting for the fact that a laundry activity is not necessarily linked to a switch-on event of a washing machine. The scalar is derived taking also into consideration the average total load of the washing machine and the load throughout each use. Since this original model is 1-minute resolution, and we are building a model of 2-minute resolution, we simply multiplied this value by 2 to maintain the same frequency of uses, resulting in a value of ≈ 0.103. When a switch-on event is predicted, the model skips to the end of the use, the duration of which is defined by the single-use load profile (SULP) discussed later on in this section, and continues in this way until the end of the simulation is reached. This procedure is given in Algorithm 2.

Algorithm 2: Simulation of usage in step 2 of the OPD model

Input: Resident number r_k, Simulation period T_2, Occupancy series $\boldsymbol{x}_{k,\gamma}$
Initialize $t_2 = 1$
While $t_2 \leq T_2$ **do**
    $t_{10} = 1 + \left\lfloor \frac{t_2 - 1}{5} \right\rfloor$
    $\tau_{10} = 1 + ((t_{10} - 1)\ mod\ 144)$
    Generate z
    If $x_{k,\gamma}(t_{10}) > 0$ **and** $z \leq \mu(x_{k,\gamma}(t_{10}), \tau_{10})$ **then**
        $t_{use} = t_2$
        While $t_2 - t_{use} < 69$ **and** $t_2 \leq T_2$ **do**
            $u_{k,\gamma}(t_2) = 1$
            $t_2 = t_2 + 1$
        End
    Else
        $u_{k,\gamma}(t_2) = 0$
        $t_2 = t_2 + 1$
    End
End
Output: Usage profile $\boldsymbol{u}_{k,\gamma}$

Here, $T_2 = D_k \times 720$ (720 10-minute intervals in 24-hours), $\{t_2 \in \mathbb{N} | 1 \leq t_2 \leq T_2\}$ is the tth 2-minute time-step of the simulation, $t_{10} = 1 + \left\lfloor \frac{t_2 - 1}{5} \right\rfloor$ ($\lfloor \dots \rfloor$ is the floor operation and rounds the contents downwards to the nearest integer), z is a randomly generated value in the interval $[0, 1]$, τ_{10} is defined and calculated as before, $\boldsymbol{u}_{k,\gamma} = [u_{k,\gamma}(1), u_{k,\gamma}(2), \dots, u_{k,\gamma}(3)]$, $u_{k,\gamma}(t_2)$ is the usage at time-step t_2 ($1 =$ on, $0 =$ off), and $\mu(x_{k,\gamma}(t_{10}), \tau_{10})$ is the probability, derived from the TUS, of the laundry activity being performed in a household with $x_{k,\gamma}(t_{10})$ active occupants at τ_{10}:

$$\mu(x_{k,\gamma}(t_{10}), \tau_{10}) = 0.103\, \frac{N_\mu(x_{k,\gamma}(t_{10}), \tau_{10})}{N_h(x_{k,\gamma}(t_{10}))} \quad (6)$$

Where $N_\mu(x_{k,\gamma}(t_{10}), \tau_{10})$ is the number of households with $x_{k,\gamma}(t_{10})$ active occupants at τ_{10} where at least one recorded a laundry activity at this time, and $N_h(x_{k,\gamma}(t_{10}))$ is total the number of households with $x_{k,\gamma}(t_{10})$ active occupants at τ_{10}.

Occupant activity driven (OAD) model

The approach followed by Widén and Wäckelgård (2010) to predicting appliance use has been applied in the OAD model. This approach begins by predicting the activities of each of a household's residents at 10-minute resolution using a TUS calibrated first-order time-inhomogeneous Markov chain based activity model, from which the timing and duration of uses, the combination of which is often referred to as the usage, are deterministically inferred. In the case of washing machines, for example, the model first predicts the occurrence of the laundry activity. This is assumed to involve loading the machine and preparing it for a cycle, which begins at the end of the laundry activity. Due to the resolution of the activity profile, switch-on events can therefore only be determined at 10-minute intervals. However, due to the resolution of the SULP used in this work, the output of the model is 2-minute resolution.

As with Widén and Wäckelgård (2010), the definition and calibration of the first-order time-inhomogeneous Markov chain used in step 1 of the OAD model is very much the same as that for the OPD model. We will therefore only highlight the differences. For the OAD model, $N_s = 9$, and the values of s correspond to the values used by participants to record their activity. We will denote the value indicative of the laundry activity as $s_{laundry}$. In line with Widén and Wäckelgård (2010), transition probabilities in the OAD model were averaged over the duration of an hour:

$$p_{i,j}(R, \tau_{10}) = \frac{\sum_{m_{10}=a}^{b} N_{i,j}(r_k, m_{10})}{\sum_{m_{10}=a}^{b} \sum_{i=1}^{N_s} N_{i,j}(r_k, m_{10})} \quad (7)$$

where $a = 1 + 6\left\lceil\frac{\tau_{10}-1}{6}\right\rceil$ and $b = 6\left\lceil\frac{\tau_{10}}{6}\right\rceil$. $\lceil\ldots\rceil$ is the ceiling function and rounds the contents upwards to the nearest integer.

Step 2 of the OAD model is given in Algorithm 3.

Algorithm 3: Simulation of usage in step 2 of the OAD model

Input: Resident number r_k, Simulation period T_2, Activity series $\boldsymbol{x}_{k,\gamma}$
Initialize $t_2 = 1$, $loading = false$
While $t_2 \leq T_2$ **do**
    $t_{10} = 1 + \left\lfloor\frac{t_2 - 1}{5}\right\rfloor$
    $\tau_{10} = 1 + ((t_{10} - 1)\ mod\ 144)$
    If $x_{k,\gamma}(t_{10}) = s_{laundry}$
        $loading = true$
        $u_{k,\gamma}(t_2) = 0$
        $t_2 = t_2 + 1$
    Else if $loading = true$
        $loading = false$
        $t_{use} = t_2$
        While $t_2 - t_{use} < 69$ **and** $t_2 \leq T_2$ **do**
            $u_{k,\gamma}(t_2) = 1$
            $t_2 = t_2 + 1$
        End
    Else
        $u_{k,\gamma}(t_2)$=0
        $t_2 = t_2 + 1$
    End
End
Output: Usage profile $\boldsymbol{u}_{k,\gamma}$

Empirical data driven (EDD) model

The approach the EDD model is based on is that developed by Yilmaz et al. (2017). In contrast to the OPD and OAD models which infer appliance use from TUS-based modelling of occupant behaviour, this model derives probabilities from the HES (2014). Step 1 involves predicting the number of switch-on events in each day at 2-minute resolution through inverse transform sampling through the procedure given in Algorithm 4.

Algorithm 4: Procedure followed in step 1 of EDD model to predict the number of uses in each day

Input: Days in simulation D_k
Initialize d= 1
While $d \leq D_k$ **do**
    $i = 1$
    Generate z
    While $z > P\left(e_{k,\gamma}(d) \leq \varepsilon_i\right)$ **do**
        $i = i + 1$
    End
    $e_{k,\gamma}(d) = \varepsilon_i$
    $d = d + 1$
End
Output: Number of switch-on events in each day $\boldsymbol{e}_{k,\gamma}$

Here, ε_i is the ith possibility for the number of switch on events in a day, $\boldsymbol{e}_{k,\gamma} = [e_{k,\gamma}(1), e_{k,\gamma}(2), \ldots, e_{k,\gamma}(D_k)]$, $e_{k,\gamma}(d)$ is the number of switch on events predicted for day d, $\{i \in \mathbb{N} | 1 \leq i \leq N_\varepsilon\}$ (N_ε is the number of possibilities for the number of switch-on events in a day) and $P(e_{k,\gamma}(d) \leq \varepsilon_i)$ is the probability that $e_{k,\gamma}(d)$ is less than or equal to ε_i, and is determined using:

$$P\left(e_{k,\gamma}(d) \leq \varepsilon_i\right) = \frac{\sum_{i=1}^{j} N_{\varepsilon_i}}{\sum_{i=1}^{N_\varepsilon} N_{\varepsilon_i}} \quad (8)$$

Where N_{ε_i} is the number of days in the calibration data where there are ε_i switch-on events.

The second step involves stepping through the simulation until a switch-on event is predicted, where the model skips to the end of the use. The model then continues stepping through the simulation until either another switch-on event occurs, where the model will again skip to the end of the use as done previously, or the end of the simulation is reached. This second step is repeated until the number of switch-on events predicted matches that of the first step. This procedure is given in Algorithm 5.

Here, $\eta(\alpha)$ is the number of switch-on events predicted in step 2 for day α, $CB(t_2) = 1 + 720\left(\left\lceil\frac{t_2-1}{720}\right\rceil - 1\right)$ is the cycle back function (there are 720 2-minute intervals in 24 hours), $\tau_2 = 1 + (t_2 - 1)\ mod\ 720$ is the τ th 2-minute time step since the previous midnight and $p(\tau_2)$ is the switch-on probability at τ_2. In contrast to the work by Yilmaz et al. (2017), here we calculate $p(\tau_2)$ considering the conditionality of the appliance not already being in use:

$$p(\tau_2) = \frac{b(\tau_2)}{a(\tau_2)} \quad (9)$$

Where $b(\tau_2)$ is the number of days in which the calibration washing machine was switched on during the τth 2-minute time step since the previous midnight, and $a(\tau_2)$ is the number of days in which the appliance was not in use at τ_2. This conditionality is necessary due to the model only considering the occurrence of a switch-on event when the appliance is not in use.

The procedure has been designed in such a way that, at the end of day α, if $\eta_{k,\gamma}(\alpha) \neq e_{k,\gamma}(\alpha)$, the program only cycles back to the start of the day, rather than the start of the simulation. Additionally, if at the start of the day that the model has cycled back to there is a use that started on the previous day and overlaps into the current day, the model then skips to the end of this use.

Algorithm 5: Simulation of usage in step 2 of the EDD model

Input: Number of switch-on events in each day $\boldsymbol{e}_{k,\gamma}$, Simulation period T_2
Initialize $t_2 = 1, d = 1, \alpha = 1$
While $t_2 \leq T_2$ **do**
  $\eta_{k,\gamma}(\alpha) = 0$
  While $\alpha = d$ **do**
    Generate z
    If $z \leq p(\tau_2)$ **then**
      $\eta_{k,\gamma}(\alpha) = \eta_{k,\gamma}(\alpha) + 1$
      $t_{use} = t_2$
      While $t_2 - t_{use} < 69$ **and** $t_2 \leq T_2$ **do**
        $u_{k,\gamma}(t_2) = 1$
        $t_2 = t_2 + 1$
      End
    Else
      $u_{k,\gamma}(t_2) = 0$
      $t_2 = t_2 + 1$
    End
    $d = 1 + \left\lfloor \frac{t_2 - 1}{720} \right\rfloor$
  End
  If $\eta_{k,\gamma}(\alpha) = e_{k,\gamma}(\alpha)$ **do**
    $\alpha = d$
  Else
    $t_2 = CB(t_2)$
    $d = 1 + \left\lfloor \frac{t_2 - 1}{720} \right\rfloor$
    If $d > 1$ **then**
      If $u_{k,\gamma}(t_2 - 1) = 1$ **and** $u_{k,\gamma}(t_2) = 1$ **then**
        While $u_{k,\gamma}(t_2) = 1$ **do**
          $t_2 = t_2 + 1$
        End
      End
    End
  End
End
Output: Usage profile $\boldsymbol{u}_{k,\gamma}$

In each assessment of the EDD model against a single ground truth washing machine indexed by k, the model was calibrated using data from the remaining 169 washing machines. As demonstrated by Yilmaz et al. (2017), models built on load measurements are best calibrated using the data for a single appliance. To incorporate this, while also refining our selection of an individual washing machine by some relevant detail, we selected a single appliance from the set of washing machines that come from a household for which the resident number is r_k. To mitigate any bias that could be introduced in this selection, we uniformly randomly selected the calibration washing machine, indexed by ν, from this set. This selection and calibration process was then repeated every repetition of the method.

The identification of switch on times can be difficult with washing machine data, due to periods of low load during mid-cycle. Yilmaz et al. (2017) addressed this by visually inspecting the profiles, however there is a significant number of profiles, and with some profiles it is difficult to distinguish multiple temporally close uses from one. Therefore, an alternative approach was used here, where a 10-minute (5 time-steps) moving average was first applied to the data:

$$\bar{\boldsymbol{l}}_\nu(t_2) = \frac{\sum_{i=t_2-2}^{t_2+2} l_\nu(i)}{5} \quad (10)$$

Experimenting with several profiles, this was seen to lift the periods of low load mid-cycle to a level that was distinguishable from the standby load. Due to the standby load varying between appliances, the minimum load value was then subtracted from the resulting profile:

$$\tilde{\boldsymbol{l}}_\nu(t_2) = \bar{\boldsymbol{l}}_\nu(t_2) - min\,(\bar{\boldsymbol{l}}_\nu(t_2)) \quad (11)$$

This provided a relatively robust basis to assess switch-on events against a simple rule, which was that a switch-on event for a washing machine that isn't already in use occurs at t_2 when $\tilde{l}_\nu(t_2) > 3W$, and the use is then assumed to continue until t_2 when $\tilde{l}_\nu(t_2) < 1W$.

Conversion from usage to load

Since the sequence of predictions associated with generating usage profiles is generally independent of the mapping from usage profiles to load profiles, we decided to focus on the sequence of steps involved with usage simulation, and maintain the same conversion from usage to load for all models. This involves the application of the typical washing machine single-use load profile (SULP) where a use has been predicted, which is shown in Figure 1. This SULP is that used by Richardson et al. (2010), which the authors obtained from a washing machine manufacturer, resampled from 1-minute to 2-minute resolution.

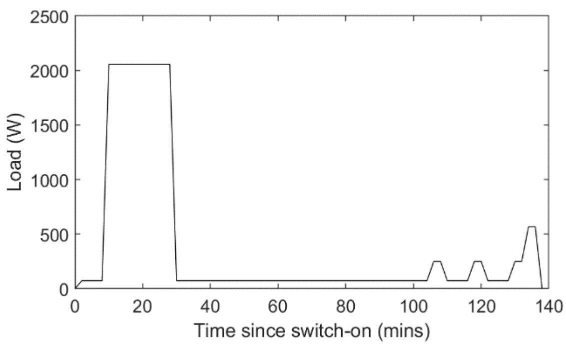

Figure 1: Typical washing machine SULP (Richardson et al., 2010) used in all three models, represented by $\boldsymbol{c}$

Converting from a usage profile to a load profile follows the procedure given in Algorithm 6. Here, $c(i)$ is the ith value in the SULP, which is represented by $\boldsymbol{c} = [c(1), c(2), \ldots, c(69)]$.

Algorithm 6: Procedure to convert predicted usage profile to load profile

Input: Usage profile U, Simulation period T_2, SULP $\boldsymbol{c}$
Initialize $t_2 = 1$
While $t_2 \leq T_2$ **do**
  If $u_{k,\gamma}(t_2) = 1$ **then**
    $t_{use} = t_2$
    While $t_2 - t_{use} < 69$ **and** $t_2 \leq T_2$ **do**
      $g_{k,\gamma}(t_2) = c(t_2 - t_{use} + 1)$
      $t_2 = t_2 + 1$
    End
  Else
    $g_{k,\gamma}(t_2) = 0$
    $t_2 = t_2 + 1$
  End
End
Output: Synthetic load profile $\boldsymbol{g}_{k,\gamma}$

Representation of load profiles as histograms

As discussed, the histogram representation of load profiles used here provides the distribution of load with respect to time-of-day, for the 18 weekdays considered. We established the load on a support of $[0\text{W}, 4320\text{W})$, and used a 144 × 144 grid, translating to a bin size of 10 min × 30W. For $\{i \in \mathbb{N} | 1 \leq i \leq 144\}$ and $\{j \in \mathbb{N} | 1 \leq j \leq 144\}$, we then computed the probability ρ_{ij} of the particular washing machine consuming power L_i at time-of-day t_j on a weekday using:

$$\rho_{ij} = \frac{N(L_i, t_j)}{12960} \quad (12)$$

Where $N(L_i, t_j)$ is the number of data points that fall in the bin defined by i and j, and $18 \times 720 = 12960$ is the number of data points in each load profile.

Since time-of-day is a cyclical dimension, the distribution would be best represented on a cylindrical grid rather than planar. However, we left this for future work to address, and found a planar grid to work reasonably well for our purposes. In doing so, we had to select a time-of-day to have as our start/end point of the dimension. The time selected was that at which the minimum load during the average weekday for all washing machines occurred, which was 02:22.

An example distribution for one ground truth washing machine is shown in Figure 2. This shows the probability of a load from a certain interval occurring at a certain interval of time of day, represented as a histogram.

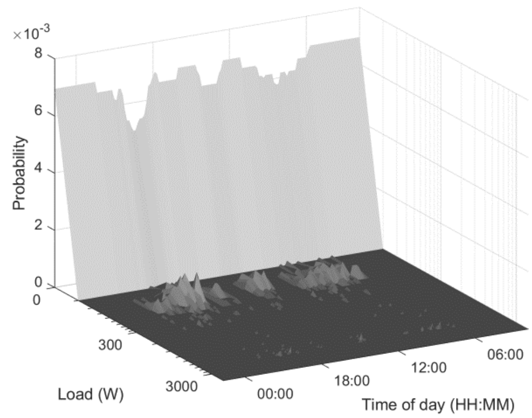

Figure 2: Example distribution of the load measurements for a single washing machine recorded over 18 weekdays, with respect to time-of-day, represented as a histogram.

The Wasserstein distance between load profile histograms

For computation of the Wasserstein distance between probability distributions, we use 1M Algorithm as proposed by Liu (2018), which is a fast implementation of the Chambolle and Pock algorithm (2013) that uses the Euclidean distance as a ground distance metric.

Because washing machines typically remain unused, there should always be a high probability for 0W to occur at all times of day, as can be seen in Figure 2. With 18 days of 2-minute resolution data, there are 12,960 datapoints, and therefore a difference of a single data point falling in a certain bin leads to a difference in the probability for that bin of $\frac{1}{12960} \approx 7.72 \times 10^{-5}$. Such values were found to be rounded to 0, and in cases where the maximum difference was a low value, which would sometimes occur, the computed OT would be 0. To overcome this, each element of both distributions was multiplied by a scaling factor prior to computing the OT, and then the resulting cost was divided by the same factor. The scaling factor is $\zeta = 10^a$, where a is the integer that leads to $1 < \zeta\delta \leq 10$, where δ is the sum of elementwise absolute differences for distributions ρ^1 and ρ^0 defined on an $m \times m$ grid:

$$\delta = \sum_{i=1}^{m} \sum_{j=1}^{m} \left| \rho_{ij}^{1} - \rho_{ij}^{0} \right| \quad (13)$$

Following the experiment, the results were inspected and with this modification the algorithm didn't output any zeros.

It should be noted that this algorithm maps the histogram to a support of [0,1] for all dimensions by dividing the support by the maximum value, leading to a unitless similarity value.

Assessment of reliability of ϕ_k-score

To assess the reliability of the mean of the set of similarity values generated for each model against each of the validation washing machines, the relative difference was used:

$$\Delta = \left| \frac{\phi_k(n) - \phi_k(n-\omega-1)}{\phi_k(n-\omega-1)} \right| \quad (14)$$

where $\phi_k(n)$ is the ϕ_k-score after n repetitions, and ω is the number of consecutive repetitions in which Δ has fallen below the acceptance threshold. The acceptance threshold was set to $\varepsilon = 5 \times 10^{-3}$, and the checking procedure was performed for $\{\omega \in \mathbb{N} | 0 \leq \omega \leq 6\}$. We therefore found the initial value of n for which Δ is smaller than ε for all ω.

Results and discussion

Boxplots of the scores for each model are shown in Figure 3, where the lower a score is, the better the model performed in representing the statistical distribution of load with respect to time-of-day. The median score, represented by the central line, indicates the central tendency of each models scoring, while the interquartile range and whiskers give the general distribution, and the outliers show extreme deviations from this distribution.

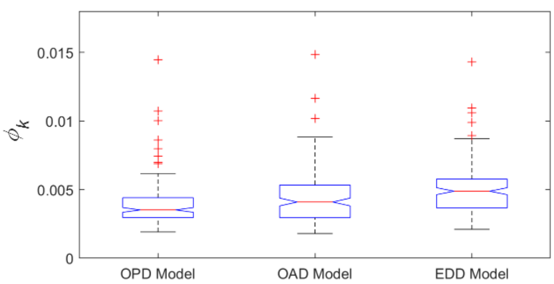

Figure 3 Boxplots of ϕ_k-scores for each model

Since there is large overlap in the whisker range of the ϕ_k-scores, it was necessary to check these distributions were significantly different before inferring anything from the results. To do this, we used a 2-sample T-test between each pairing of the three distributions, using Box-Cox transformations to correct for normality. The p-values obtained were far lower than the 0.05 significance value, providing confidence that the distributions are different.

It can be seen that the central tendency and interquartile range of the occupant presence driven (OPD) model is better than the other models, with the occupant activity driven (OAD) model second and the empirical data driven (EDD) model third. However, the range of scores between the bottom whisker and the highest outlier for all models is very similar. Interestingly however, the OAD model performed on average better than the EDD model, which is surprising due to the fact it can only predict switch-on events at 10-minute resolution, rather than 2-minute resolution, and that the cross-analysis by Yamaguchi (2019) indicated the model by Yilmaz et al. (2017) was one of the better approaches. Figure 4 shows boxplots for the sets of the values for the number of repetitions n which correspond to these ϕ_k-scores.

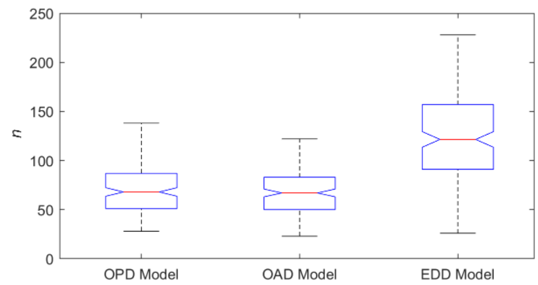

Figure 4 Boxplots of number of repetitions n for each model, as derived from (14)

The ϕ_k-scores were also visualised by plotting the values by the corresponding washing machine. For a sample of 20 random washing machine load profiles, each indexed k_i where $\{i \in \mathbb{N} | 1 \leq i \leq 20\}$, the score for each model is given in Figure 5. To see how representative this sample of ϕ_k-scores is, we also plotted an approximation of the probability distribution in Figure 6, based on the full 170 scores. We used kernel density estimation as suggested by Hill (1985) for this approximation, with Gaussian kernels and a bandwidth of 4×10^{-4}. From these figures, we can see that no model always performs better or worse than the others, and for each of the models there are cases in which the model performs best, and others in which it performs worst.

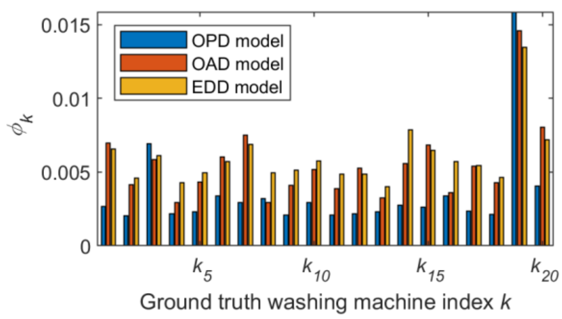

Figure 5 Sample of 20 scores for each model

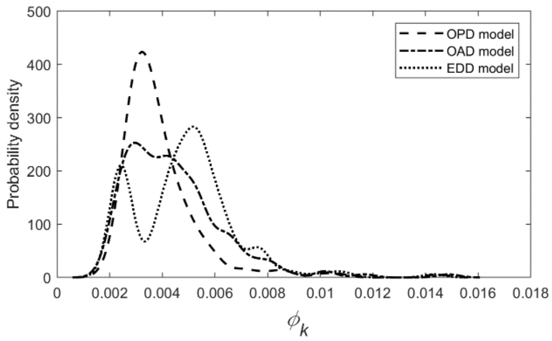

Figure 6 Estimated probability density functions for ϕ_k

For verification purposes, the entire experiment was repeated, and the results were consistent.

Conclusion

This paper compares appliance load models by assessing their ability to accurately capture the distribution of load with respect to time-of-day. We quantified this with respect to each measured sample as a ϕ_k-score. This is the mean similarity measure between the probability of load over time of day, for a range of generated samples and the measured sample. We represented this distribution using histograms and assessed the similarity between histograms using the Wasserstein distance. This is a novel approach to assessing models on a basis that has not been studied previously.

Three models were developed for this work, which are based on existing examples and representative of common approaches used in the bottom-up modelling of residential electrical appliance load. Boxplot comparisons of ϕ_k-scores generated for each model showed that the OPD modelling approach performed best overall, with OAD modelling second and EDD modelling third.

Further tests were performed to assess the robustness of this result. The results of a 2-sample T-test between each pairing of the three distributions of the ϕ_k-scores for each model provided confidence that these distributions can be considered different, indicating that the comparison method is robust.

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