

AN ANALYTICAL VERIFICATION TEST SUITE FOR MULTI-ZONE BUILDING FABRIC AND CONTROL MODELS IN WHOLE BUILDING ENERGY SIMULATION PROGRAMS

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ABSTRACT

An analytical verification test suite has recently been developed to test building fabric models for whole building energy simulation programs. (Rees, et al. 2002) The test suite covers a range of heat transfer mechanisms, including convection, conduction, thermal radiation, solar radiation, and infiltration in a single building zone. However, none of the existing tests cover inter-zone heat transfer or airflow. Nor does it cover any situations where a thermostat in one zone controls the heating or cooling provided to other zones.

This paper presents an extension of the analytical verification test suite to multi-zone situations. Analytical tests of inter-zone heat transfer by steady-state convection, steady-state and transient conduction, and inter-zone airflow have been developed. In addition, tests of master-slave control of the air system – of particular relevance in residential buildings – have been introduced. Results from verification of two building simulation programs – BLAST and ESP-r are also presented.

INTRODUCTION

The evaluation of building energy simulation programs becomes important with the increasing use of building energy simulation programs. Significant effort has been made in the last two decades to verify and validate whole building energy simulation programs. Reviews of a number of these efforts appear in the literature (Judkoff 1988; Judkoff and Neymark 1995; Bloomfield 1999).

It is generally considered that results from one program can be compared with results from other sources in three ways: analytical verification, inter-model comparison and empirical validation (Judkoff *et al.*, 1983). Of these approaches, analytical tests are most abstracted from the full complexities of real building simulation problems but offer the most certain form of reference or ‘truth’ model with which comparisons can be made.

Differences between the results of energy simulation program calculations and other reference data may arise for three reasons:

- Uncertainty in the input data
- The model algorithms
- Errors in the computer implementation of the model algorithms (bugs).

Analytical tests, being necessarily simplified in nature, can minimize the uncertainty in the input data. Being slightly abstract in nature, they may be of least interest to the end user of simulation codes but are arguably the most useful to the code developer in that they offer the clearest path to diagnosis of specific problems with the algorithms or their implementation.

An analytical verification test suite (ASHRAE research project 1052-RP) has recently been developed to test building fabric models for whole building energy simulation programs (Rees, et al. 2002). Trials of the test suite were initially made by carrying out the analytical tests with BLAST. The test suite has subsequently been used in the development of EnergyPlus (2002) and may become part of the ASHRAE standard method of test for whole building energy simulation programs (ASHRAE Standard 140).

The existing test suite is comprised of tests that use a single building zone (generally a 3x3x3m cube). None of the existing tests cover inter-zone heat transfer or airflow. Nor do the existing tests cover any situations where a thermostat in one zone controls the heating or cooling provided to other zones (master-slave control). This situation is of particular interest for residential building simulation. In general, most single-family detached houses use a single central thermostat to control the temperatures of all rooms. This type of temperature control necessarily results in inter-zone heat transfer and airflow and usually produces load-moderating effects in residences. The ability of an energy analysis program to model this type of control and the interaction between the thermal zones is

thought to be necessary to accurately model residential buildings.

In recognition of this need, most of the convection, conduction and infiltration tests in the existing single zone analytical verification test suite have been extended to cover multi-zone situations. This paper presents the extension of the single zone test suite by the introduction of three new control tests applicable to multi-zone cases. The development of the control tests is summarized and the results of applying these tests to two building simulation programs – BLAST and ESP-r are also presented.

THE SINGLE ZONE TEST SUITE

The ASHRAE 1052-RP single zone test suite consists of sixteen individual tests. The sixteen individual tests are organized into five groups according to particular heat transfer phenomena: convection and conduction; solar gains and shading; infiltration; long wave radiation; and other miscellaneous tests (ground coupling and internal gains).

All the tests have been implemented as a series of Fortran90 subroutines. Source code has also been written to generate weather file in either TMY2, WYEC2, or IWEC formats. The Fortran source code itself allows the test parameters to be modified and can be compiled and used by interested users and developers. To enable further convenience of running the tests using different parameter values, the Fortran source code has been compiled as dynamic link libraries (“DLLs”) that can be called from a graphical user interface created for the test suite.

The usefulness of the test suite may be limited by the clarity of the test documentation. Accordingly, care has been taken in providing complete and precise documentation for each test. The documentation for each test is in a standardized format and describes the objective and basis of the analytical model, the test zone and parameters, and includes test results from one or more given set of parameters, the derivation of the analytical solution and references for the particular test.

Tests are carried out by comparing the output of the energy simulation program with the analytical solution. The tests have been designed so that they require the use of the input and output mechanisms available to program users i.e. no custom coding. The output data required for comparison with the analytical test results is typically a steady-state zone load, heat flux, or hourly loads over one or more days of simulation.

In order to isolate heat transfer mechanisms and thus minimize the number of heat transfer models involved in each test, special zones with rather different construction and surface properties are used. For most cases, the test zone is based on a cube cell with 3x3x3 m internal size. The aspect ratio of the test cell is only modified in the tests involving internal long wave radiation, internal solar distribution and stack effect infiltration. Windows and openings are present only if needed by an individual test. As is usually assumed in building energy analysis tools, the zone air has no thermal mass. It is generally necessary to construct zone models that eliminate certain heat transfer paths. This may be accomplished in different ways depending on the program to be tested. For example, in several cases long wave and/or solar radiation need to be eliminated. This might be done explicitly or by modifying the surface properties. The tester therefore needs to have some familiarity with the models and input data structure of the program to be tested.

It is worth noting that in the case of convection coefficients, there has not been one universally acceptable “truth” coefficient or correlation. Although for some tests it would be desirable to eliminate convective heat transfer, this is generally not possible in energy simulation programs using conventional input mechanisms. Convection is therefore present in all the tests and consequently the convection coefficients have to be well defined. It is consequently recommended that the tests involving steady state convection are conducted and analyzed before any others. The test suite allows specification of the correlation in the form of constants and exponents applied to the surface to zone air temperature difference. This allows the user to specify a correlation consistent with that used by the energy simulation program.

Further details of each analytical test and initial test results from an in-house comparison of BLAST to analytical solutions can be found in the project final report (Spitler *et al.*, 2001) and the accompanying paper (Rees *et al.*, 2002).

THE MULTI-ZONE TEST SUITE

Extension of the Single Zone Test Suite

The single zone test suite has been extended to cover multi-zone situations for the purpose of verifying the ability of the test program to model the effect of inter-zone heat transfer and inter-zone airflow. Four tests of the convection and conduction group in the single zone test suite have been adapted to deal with inter-zone heat transfer. These are the tests for Inter-zone steady-state

convection, inter-zone steady-state conduction, transient conduction with an adiabatic wall, and inter-zone transient conduction with a conducting wall. The transient conduction tests employ a driving temperature profile consisting of three steps. The two infiltration tests in the single zone test suite have been similarly adapted to test models of inter-zone airflow. These tests use a fixed mass flow in one case, and flow driven by stack effect in the other.

In all the tests extended from the single zone test suite, there are two test zones connected by the inter-zone wall (as shown in Fig. 1). Each test zone generally has the same geometry, construction and surface properties as that of the test zone in the single zone test suite. In the single zone test suite heat transfer is driven through an exterior surface using different boundary conditions applied by manipulating external environmental data and introducing infiltration of outside air.

In the inter-zone tests, convection and conduction heat transfer is via an interior wall common to both zones. In the convection and conduction tests, heat transfer is driven by changes in the drybulb temperature of one of the interior zones rather than the external drybulb temperature. The boundary conditions and parameters applied to the external wall in the single zone test cases correspond to those applied at the inter-zone wall in the new test cases. Similarly, the solution for airflow between zones is analogous to the solution for airflow between the single zone and the outside environment.

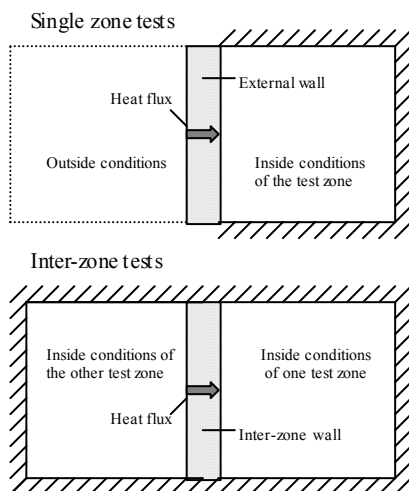


Figure 1 Comparison of test zones and boundary conditions in the single zone and inter-zone tests

The analytical solutions to the test cases extended from the single zone test suite are the same as those of the corresponding single zone tests. Accordingly, the corresponding documentation of the single zone tests

can be used except for the changes in the test zones mentioned above.

The two test zones in the inter-zone tests are isolated thermally from the environment, and so, rather than manipulate the environmental boundary conditions, it is necessary to be able to specify the drybulb temperature and humidity ratio in one of the zones. This requires the program to be tested offer a way to keep a constant dry bulb air temperature for most cases, and may require the ability to schedule changes in temperature to apply the stepped driving temperature used in the transient conduction tests.

Development of Control Tests

For residential buildings, it is desirable to be able to model inter-zone thermal interactions and multi-zone temperature control. In this case, a central thermostat in one zone usually controls the heating or cooling to other zones. This may be referred to as master-slave zone control – the zone containing the central thermostat is the “master” zone, and the other zones are called “slave” zones.

Three tests are designed to verify the ability of building energy analysis programs to simulate the master-slave zone control in three modes of operation: constant volume; variable air volume; and on-off constant volume. It should be understood that all of the tests are somewhat idealized compared to real buildings with real thermostats. In particular, the only thermal mass is the mass of the zone air, which leads to very short on-off cycles in the on-off control test.

All three control tests employ geometries consisting of two cubic zones, with internal dimensions 3×3×3m. With one exception, both zones have mass-less, adiabatic walls. In the ideal control test, each zone has one mass-less, but not adiabatic, external wall. (Since, for this test, the system is an idealized heat source, it is necessary that the zone be thermally connected to a temperature source in order for the solution to be determinable.) No windows are present for the test cubes. The effects of solar irradiation, long wave radiation, radiative internal gains and infiltration must be eliminated. Different convective internal gains to the two cubes can be scheduled hourly for a simulation day in the form of an input file to the analytical implementation of the tests.

Documentation similar to that of the single zone test suite has been developed for each control test (Xiao and Spitler 2002). Fortran90 subroutines for the implementation of the control tests have also been written. However, the graphical user interface for the

three control tests has not been created yet. As the test parameters are clearly described in the documentation and can be easily modified in the test program for each subroutine, this should not be a problem for interested users.

Ideal Control Test

The objective of the ideal control test is to find the response in the slave zone air temperature when the master zone air temperature is maintained at a constant value with an ideal system. This ideal system supplies whatever heat input the master zone needs to maintain its set point. The same amount of system heat input as supplied to the master zone is sent to the slave zone simultaneously.

The analytical solution is based on the steady state heat balance of the zone air and the steady state convective heat transfer through the exterior wall, given in Equation 1.

$$q_{sys}'' + q_{conv,i}'' + q_{conv,e}'' = 0 \quad (1)$$

where,

q_{sys}'' = the system heat input, W

$q_{conv,i}''$ = the convective internal heat gain, W

$q_{conv,e}''$ = the convective heat transfer through the exterior wall, W

The zone loads are made different by the convective internal heat gains to the master and slave zone. The difference in the zone loads should be reflected in deviations of the slave zone temperature from the setpoint. The master zone will maintain a constant air temperature.

VAV Control Test

The VAV control test is designed to find the response in the slave zone air temperature when the master zone air temperature is maintained constant with a variable air volume system. In this test, the air flow rate is varied to both zones, in response to the demands of the master zone. The air flow rate to the slave zone is controlled to be equal or proportional to the air flow rate to the master zone. The air flow to both zones has the same supply temperature and humidity. With different convective heat gains to the two zones, deviations from the setpoint temperature in the slave zone may be observed. The analytical solution is based on a transient heat balance of the zone air, given in Equation 2.

$$q_{conv,i}'' + \dot{m}_{sys} C_{pa} T_{sys} = M_z C_{pa} \frac{dT_z}{dt} + \dot{m}_{sys} C_{pa} T_z \quad (2)$$

where,

$q_{conv,i}''$ = convective internal gain to the zone, W

\dot{m}_{sys} = the mass flow rate of the system air, kg/s

$\dot{m}_{sys} = \rho_{air} v_{sys}$

ρ_{air} = density of the air, kg/m³

v_{sys} = volume flow rate of the system air, m³/s

M_z = mass of the zone air, kg; $M_z = \rho_{air} V_z$

V_z = volume of the zone, 27.0 m³

C_{pa} = specific heat capacity of the air, J/kg.K

T_{sys} = system air temperature, C

T_z = zone air temperature, C

t = time, s

In the case of the master zone, equation (2) reduces to an algebraic equation as we assume the zone air is held constant. The differential equation is solved for the slave zone temperature by evaluating the analytical solution over a series of short time intervals (assuming the mass flow rate is constant over this interval.) The volume flow rate is recalculated at the end of each time interval.

On-Off Control Test

The on-off control test uses a constant air volume system operated with on-off control. The on-off control of the system is based on the master zone air temperature. The on-off control has a user-specified setpoint temperature and deadband range. Like the VAV control test, the analytical solution assumes only the air has thermal mass, and is based on a transient heat balance of the zone air. Other assumptions like the system air humidity and volume flow rate ratio between the master and slave zone are also similar to that of the VAV control test.

The analytical solution is similar to that of the VAV control system. By evaluating the new zone air temperature over a series of short time intervals it is possible to detect the times at which the system switches either on or off, with sufficient accuracy.

TEST RESULTS OF BLAST AND ESP-R

The multi-zone test suite has been initially tested against two whole building energy simulation programs: BLAST (1995) and ESP-r (ESRU 2000). A

brief summary of the test results and a discussion of the principal problems diagnosed follows.

For the inter-zone heat transfer and airflow test results shown below, zone loads and surface temperatures are reported for the test zone that corresponds to the inside condition of the single zone test.

Inter-zone Convection and Conduction Test results

The test results of the steady state inter-zone heat transfer cases with BLAST and ESP-r are summarized in Table 1. The convection coefficients stated in the BLAST documentation – which have already been verified in the single zone tests (Rees et al. 2002) – were used both in the analytical solution and ESP-r so that comparisons are based on the same conditions. In steady state tests, both BLAST and ESP-r show very close agreement with the analytical results.

Table 1: Percentage differences between the BLAST and ESP-r programs and the analytically calculated loads for the steady-state inter-zone heat transfer test cases.

Steady-State Test	Percentage difference in zone load	
	BLAST	ESP-r
Convection	0.17	0.67
Conduction	0.26	0.17

The test results of the transient inter-zone heat transfer cases with BLAST and ESP-r are summarized in Table 2. Test results for the resultant surface temperatures are plotted in Fig. 2 for the test with a conducting wall. In these cases, the greatest differences occur when the driving temperature shows a step change. The deviation in wall temperature from the analytical solution corresponds to a 1~3% difference in zone loads. ESP-r has a relatively large maximum difference because only half the change in driving temperature is applied at the time of the step change, due to the way in which ESP-r interpolates time varying boundary conditions (a six minute time step was used and the interpolation appears to be unavoidable). As found in the single zone tests for BLAST (Rees et al. 2002), the default convergence

criteria used by BLAST needs to be improved to achieve these results.

Inter-zone Airflow Test Results

An application of the infiltration test under fixed flow rate to inter-zone airflow test showed a difference of 4.45% between the analytically calculated zone load and that predicted by BLAST. For ESP-r, the difference is 5.78%. These differences are thought to arise from differing treatment of the thermal properties of air in the two programs and the analytical solution.

In both BLAST and ESP-r infiltration loads are calculated using an air infiltration conductance that is calculated using constant value for the volumetric heat capacity (ρC_p). The values of the constants in each code differ. The analytical solution, however, is calculated using a correlation for the density of air at varying temperatures. Hand calculation using the constants found in the BLAST and ESP-r codes agrees with the corresponding program outputs.

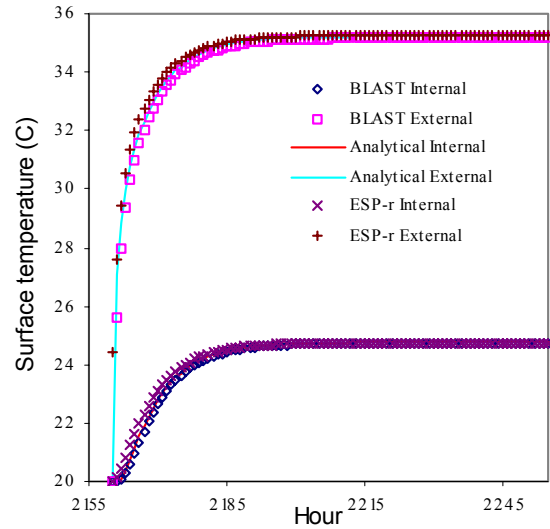


Figure 2 Analytically calculated surface temperatures and those predicted by BLAST and ESP-r for the inter-zone transient conduction test case with a conducting wall: first step change

Table 2: differences between BLAST and ESP-r programs and analytically calculated surface temperatures for the inter-zone transient conduction cases

Transient Test		Inside surface temperature differences (K)				Outside surface temperature difference (K)			
		Mean		Maximum		Mean		Maximum	
		BLAST	ESP-r	BLAST	ESP-r	BLAST	ESP-r	BLAST	ESP-r
Adiabatic wall		0.16	0.12	0.34	0.58	0.16	0.14	1.45	4.43
Conducting wall	First step	0.05	0.07	0.18	0.49	0.12	0.14	1.45	4.43
	Second step	0.08	0.14	0.33	1.00	0.13	0.26	2.82	8.83

Satisfactory results were obtained by applying the stack-effect driven infiltration test in ESP-r, with a difference in the zone load of 1.13% between the analytical and ESP-r calculations. This might be expected, as the default stack effect calculation in ESP-r (Hensen, 1991) is similar to the approach by Walton (1989) that is the basis of the analytical solution. The infiltration test of stack effect is beyond the scope of BLAST program, so no result can be reported.

A perhaps minor problem was discovered in the course of carrying out the stack-effect driven infiltration tests in ESP-r. To apply this test in ESP-r rather than scheduling the inter-zone airflow rate, an airflow network has to be defined. The equations of the airflow network are solved within ESP-r using an iterative solution method (Hensen 1991), for which it is necessary to check for successful convergence. An error in the formatting of the program output (an integer formatted as a real number) results in a failure to converge being indicated in all cases – whether the solution has converged or not.

Control Test Results

There is no way to simulate the master-slave zone control situation in the BLAST program since it does not allow the control of heating or cooling to one zone based on the temperature in another zone. Accordingly, the following discussion is based on the control test results from the ESP-r program.

One of ESP-r's great strengths is its high degree of flexibility. Two completely different approaches to modeling of the master-slave zone control situation have been tried to date, and approaches where the plant is explicitly modeled have not yet been attempted. The two approaches tried to date are based on 1) heating/cooling control functions in the zone model or 2) an airflow network. Results with both approaches are described below.

ESP-r with Heating/Cooling Control Function

The first approach, which appeared to be the most obvious approach to the authors, involved the use of the heating/cooling control functions in the zone models. Two ideal basic heating/cooling control functions were initially defined. The sensors of both functions measure the air temperature in the master zone. The actuator of one function actuates the heat input in the master zone; the other actuates the heat input in the slave zone. Both control functions have the same set point. Simulation results under these two control functions showed that the master zone air temperature is controlled as expected, but the slave zone air temperature is not

controlled. Instead, it appears to float as if there is no system heat input.

Tracing the zone air energy balance in the simulation showed that the sensor of the slave zone control function always measures the perfectly controlled master zone air temperature. Thus, the control function maintains the heat input to the slave zone at zero. Failing to achieve satisfactory results with this approach, an alternative approach, using an airflow network, was developed.

ESP-r with Airflow Network

Two types of airflow networks, corresponding to the master-slave zone VAV control and On-Off control tests, were carried out with ESP-r by defining an airflow network. An additional zone (the “mixing” zone) was defined to serve as a surrogate for a cold deck.

A schematic representation of the airflow network, as applied in the VAV case is shown in Figure 3. Standard ESP-r flow components were used to create the network. The four Type 15 power law flow components are specified with:

$$\dot{m} = a\Delta P^b \quad (3)$$

The flow exponent, b , is set to 1, and the values of a are set to cause the flow in the slave zone to be (nearly) exactly proportional to the flow to the master zone. The constant airflow rate fan, Type 35, is combined with a bypass loop (containing a Type 40 common orifice flow element) and a Type 410 damper.

A thermostat in the master zone controls the damper, and the specified airflow network is able to maintain the ratio between the airflow to the two zones within 2% of the desired ratio. This results in a close approximation to the test case.

The On-Off control test uses an airflow network that is similar, but simpler, than the VAV network. The bypass loop and damper are eliminated, and the constant airflow rate fan is merely switched on and off.

Both configurations may experience some flow between the master zone and slave zone. For the On-Off case, this is eliminated by applying on-off control to some of the flow junctions. In the VAV control test, some transient inter-zone airflow re-circulation occurs. The hourly-average error in the zone loads is on the order of 6-8%. It is possible that this error could be reduced by a different choice of airflow network and/or components.

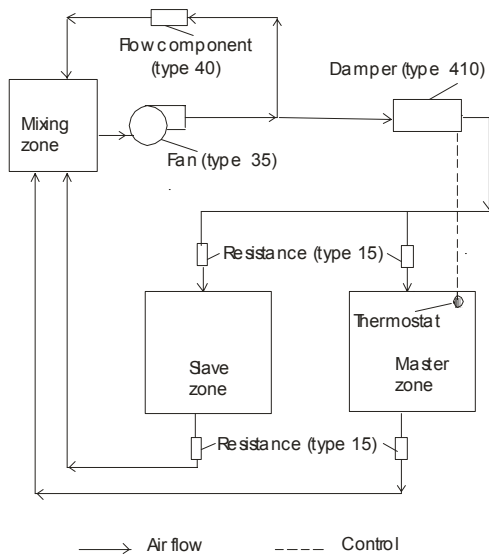


Figure 3 Airflow Network Used in ESP-r for VAV control tests with ESP-r program.

Test results for the VAV and On-Off control tests under convective heat input profiles scheduled as in Fig. 4 are given in Fig. 5 and Fig. 6. Both results were obtained using a simulation time step of six seconds. For both cases, ESP-r gives a reasonable approximation to the analytical solution. The differences in the zone air temperature may correspond to a 5~6% difference in instantaneous zone loads or a 1~2% difference in hourly average zone loads.

For the VAV test, the maximum differences between ESP-r and the analytical solution in the master and slave zone temperatures are 0.34 °C and 0.48 °C respectively. It might also be noted that some offset occurs in the master zone—this is caused by the proportional control of the airflow rate. Also, some system cycling is observable in the ESP-r solution for the first and fourth period of the day. During these times, the master zone is operating with a relatively low cooling load, and the damper controller appears to have too much gain. However, given the fact that the test zones have no thermal mass, except for the zone air, it is probably unrealistic to expect that perfect control be achieved.

For the On-Off control test, both the analytical solution and the ESP-r show the cycling behaviour expected for an on-off control. In the third period, the master zone is at its design capacity, and therefore, the system stays on all the time. ESP-r continues to cycle the system on and off. This may be due to the different value of c_p used by ESP-r. It might also be noted that ESP-r and the analytical solution appear to cycle at different frequencies. Although the ESP-r simulation is performed with six second time steps, results are only

output every minute. This may mask some of the cycling behaviour. Again, given the atypically low thermal mass in the zone, it is probably unrealistic to expect a perfect match of the cycling frequency.

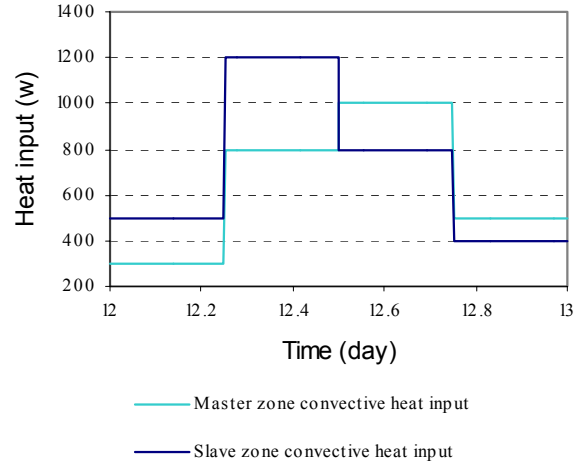


Figure 4 The convective heat input profiles of the test zones used in the VAV and On-Off control tests with ESP-r program.

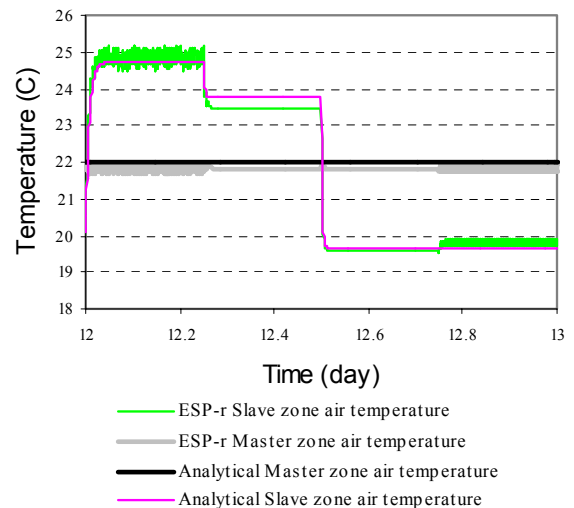


Figure 5 Zone air temperatures predicted by ESP-r program and analytical solution for the VAV control test.

CONCLUSIONS

An analytical verification test suite for multi-zone building fabric and control models of whole building energy simulation programs has been developed. The test suite consists of a series of nine tests covering inter-zone steady state and transient heat transfer, inter-zone airflow, and master-slave zone controls. Tests regarding inter-zone heat transfer and airflow are an extension of the test suite developed in the ASHRAE

1052-RP project. Tests regarding the control models are of particular interest in residential building simulations. Standardized documentation for each test contains the derivation of the analytical solution, the definition of the test zones and test parameters, and test results for one set of test parameters. Fortran 90 source code for implementation of the analytical solutions has been developed.

Initial testing of the multi-zone test suite has been undertaken by carrying out the tests with two whole building energy simulation programs (For four test cases that cannot be done with BLAST, only comparison with ESP-r is reported). The test suite has been found useful in verifying the algorithms used by the test programs and diagnosing some problems in their implementation. It is hoped this multi-zone analytical verification test suite will be useful in helping verifying building fabric and control models and diagnosing problems with multi-zone simulation cases in whole building energy simulation programs.

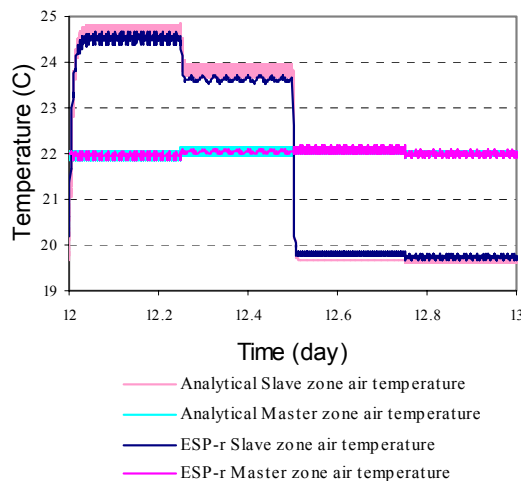


Figure 6 Zone air temperatures predicted by ESP-r program and analytical solution for the On-Off control test.

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