MODELLING OF HYBRID BUILDING COMPONENTS WITH R-C NETWORKS IN MACRO ELEMENTS

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ABSTRACT

This paper presents an effective method for thermal analysis of special building constructions, hybrid systems, and the way in which the resulting mathematical models can be implemented in a commonly used dynamic computer simulation program. The advantage of this method is the fact that only a limited number of nodes are required to obtain reliable results for the simulation. The modelling method is described in detail and the work of two different case studies is presented. Research has been conducted on this method (Jóhannesson 1981, Mao 1997, Akander 2000), and it has been expanded and used on a different class of constructions, hybrid systems.

It is essential to estimate the thermal behaviour of the whole building as a system and to open up the research knowledge in this field to a broader audience of building designers, architects and engineers. The presented method could be one step further towards this goal.

INTRODUCTION

In the past decades, considerable efforts have been made to reduce energy consumption in buildings, for example, by constructing heavily thermally insulated buildings, by improving the quality of window glazing and by using the thermal storage of the construction itself. In order to make the energy use in buildings even more efficient, new low temperature heating and cooling systems are required. A promising solution is the development of hybrid systems.

Hybrid systems are building components or a combination of different building components that utilise the heat transfer and heat capacity properties of the whole construction to achieve room conditioning. Sometimes, they are also denoted as thermally activated building components. Embedded pipes in concrete slabs or hollow core slabs, where the entire slab construction is tempered to heat or cool the room, are typical examples. The heat transfer from all of these systems to the rooms takes place mainly via radiation. The room surfaces are kept warm and at a fairly uniform temperature, thereby, a good level of thermal comfort can be provided. Draught from air movements around, e.g. convectors, or burning of dust will not occur. For the dynamic simulation of such a system, several heat transfer mechanisms are involved and have to be treated simultaneously.

To carry out the implementation of hybrid building components in buildings, safe, efficient and reliable tools are needed to analyse their thermal performance. The often commonly used one-dimensional or steady state models are not sufficient in estimating the performance of those systems. Existing stand-alone-models depicting hybrid systems are not satisfactory for design purposes. More dimensional dynamic simulation models, including all possible heat transfer effects, have to be implemented in an effective way into known simulation programs. Efficient models for overall system studies with simulations are needed.

Some attention has been paid to the modelling of hybrid systems, especially during the Solar Heating and Cooling Programme and the Energy Conservation in Buildings and Community Service Programme performed by the International Energy Agency (Jørgensen 1984, Scartezzini et al 1987). Also, some generalised design-methods for hybrid systems have been developed (Rittelmann et al 1983, Evans et al 1985, Fort 1989) and a basis for further research has been given.

THE MACRO ELEMENT MODELLING METHOD (MEM)

A method has been developed for mathematical modelling, for the analysis of thermal conditions of a building component and for the description of multi-dimensional heat conduction. In this new method, the analysis procedure for wall and ceiling constructions has been extended to include a mass flow inside the construction. Therefore, it is possible to model even hybrid building components with optimised resistorcapacitor (RC) network with a limited number of macro elements. This MEM approach can be divided into the following three basic steps: analysis, transformation and implementation.

Analysis of the system

The analysis of the solid construction parts is done in the frequency domain and can be conducted in an analytical way. The system responses are directly calculated by analytical methods or, if this is not possible, the construction is implemented in a finite difference program to calculate the system responses. For the analysis, all possible simplifications (e.g. symmetry) are made.

The analysis result of the heat transfer balance between two arbitrary isothermals 0 and 1, for example, the surfaces on either side of a multi-layer wall, is in the form of a frequency dependent matrix (Carslaw, Jaeger 1959):

$$\begin{bmatrix} \tilde{T}_1 \\ \tilde{q}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\omega} \cdot \begin{bmatrix} \tilde{T}_0 \\ \tilde{q}_0 \end{bmatrix} \qquad (1)$$

The admittance can be seen as a measure for the heat exchange between the system and its surroundings on each side. It is defined as a relation between the heat flow and the temperature on the same side and can be expressed in the terms of the matrix in equation (1):

$$Y_1 = \frac{\tilde{q}_1}{\tilde{T}_1} = D/B \; ; \tilde{T}_0 = 0 \qquad (2)$$

Transformation

The heart of the MEM method is a transformation of the building construction's properties into a network of discrete resistances and capacities. An RC-network is optimised in such a way that, for a given nodal configuration, the system responses from the RC-network are, for a certain frequency band, as close as possible to the response from the former calculations of the building construction. For the chosen RC-network configuration of a given construction, a heat transfer matrix for each frequency can be expressed in terms of the resistive and capacitive parts (Beuken 1936, Rouvel and Zimmermann 1997, Akander 2000). The deviation of the system responses between the detailed model and the RC-network can then be calculated and the deviation for the simplified model can be studied in the frequency domain.

The heat balance of the mass flow inside the construction is described by a set of differential equations. Solving the equations for a linear change in (pipe/channel) surface temperatures in the direction of the flow in each calculation segment makes it possible to use considerably enlarged segments for a given discretization error.

(I'm not sure here what is meant!)

Implementation

The resulting simplified model is formulated in the Neutral Model Format (NMF) (Sahlin 1996). This format includes a model definition based on equations. A continuous component, which can be described by a system of differential-algebraic equations, can be modelled directly with its equations. The NMF-code is translated and the models thereby implemented in dynamic simulation programs such as TRNSYS or IDA (Sahlin 1996). This provides a method of examining hybrid systems and their interactions with the rest of the building. The resulting models can, in this way, be made available for a broader audience of researchers and designers.

CASE STUDIES FOR THE MEM APPROACH

To date, two different configurations have been studied within a larger project on a low exergy heating system for a building with a ground heat storage (LOWTE 2002). The first one is a double air gap wall and window construction. The second one is a borehole heat storage with a so called C-pipe.

Double Air Gap Wall Construction

The construction shown in Figure 1 on the right hand side is divided into an opaque wall segment at the bottom and a window segment at the top. Both segments have a double air gap with an upward air stream in the inner gap and a downward air stream in the outer gap. The airflow is generated by a combined fan and water to air heat-exchanger unit at the bottom of the wall. With this special set-up, an air stream is heated in the heat exchanger at the bottom of the inner air gap and heats the inner room surface, thereby providing heating for the room. The construction includes transparent window parts and the temperature distribution along the air stream are significantly influenced by solar radiation. The results from a steady state model for design purposes are presented in (Schmidt, Jóhannesson 2001).

A laboratory measurement project is currently under way to study the function of the system in more detail and to verify the simulation results.

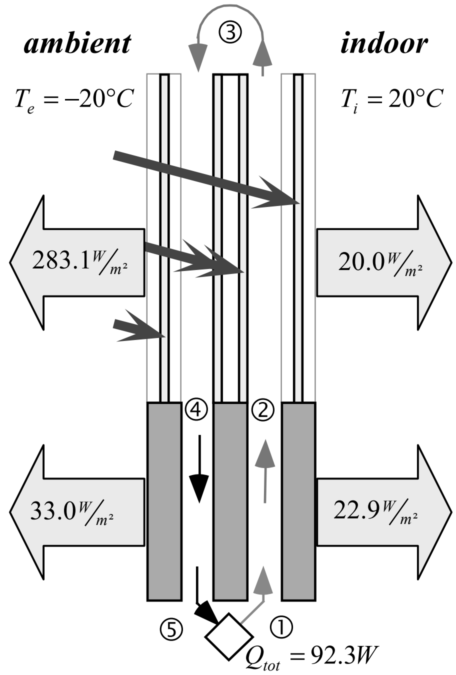

Figure 1: Results from the analytical steady state model of a double air gap wall and window construction with 4 panes.

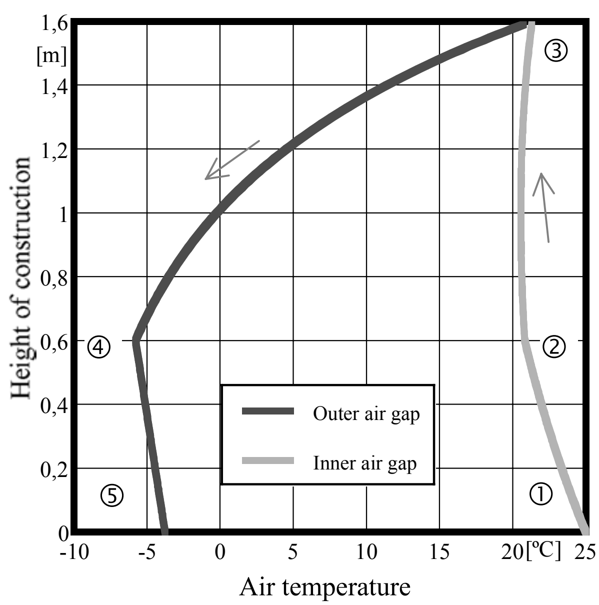

Figure 2: Results from the analytical steady state model

Bore hole heat storage

The objective of this case study is to investigate the dynamic properties of a heat store in the ground by means of a borehole equipped with a so called C-pipe, a coaxial flow pipe (Schmidt 2001). Water, as the chosen heat carrier, flows downward in the inner pipe and back upward in the outer one. Some insulation may be applied between the flows. The ground is regarded as a cylindrical piece of soil (see Figure 3)

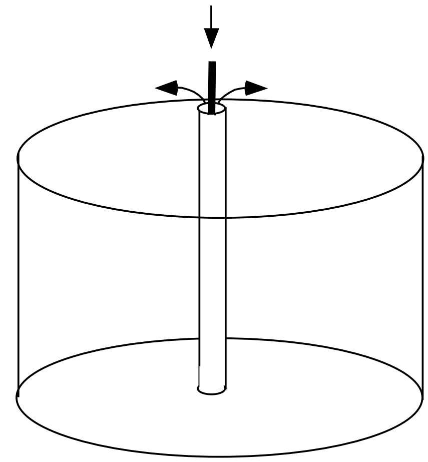

Figure 3: Borehole heat storage equipped with a C-pipe.

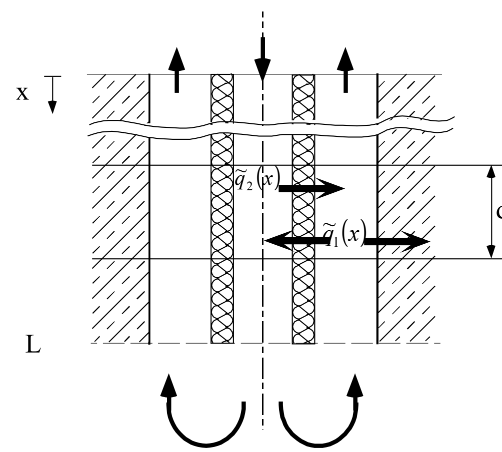

Figure 4: Borehole heat storage equipped with a C-pipe.

Analysis

This was carried out analytically. Thermal properties of the admittance hole perimeter were calculated as a solution for radial heat conduction in cylinders. The heat transfer function between the inlet and outlet at the ground surface were modelled as a system of differential equations (Schmidt 2001).

Figure 5 shows the analysis results for the entire heat storage.

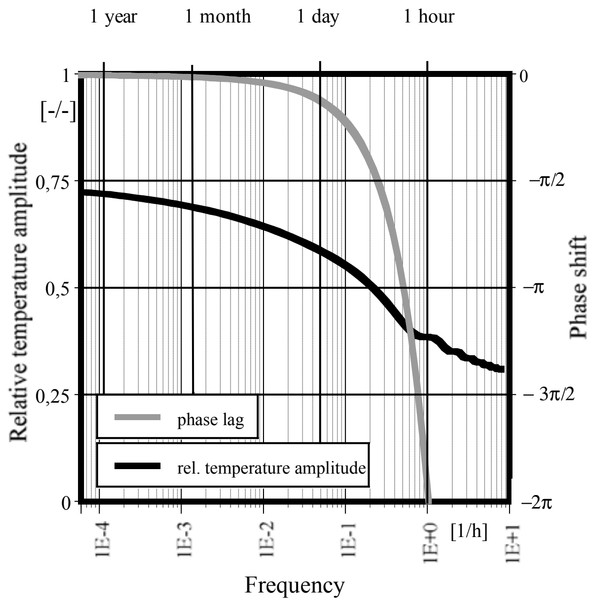

Figure 5: Temperature amplitude at the output of borehole vs. frequency of input temperature including heat exchange between the pipes $\tilde{q}_2 \neq 0$.

Transformation into an optimised RC network

When the frequency response has been established, an optimised RC network can be developed. To derive a model for a half-infinite body with good agreement for even high and low frequencies, a “half 5 node” network (Akander 2000) is chosen.

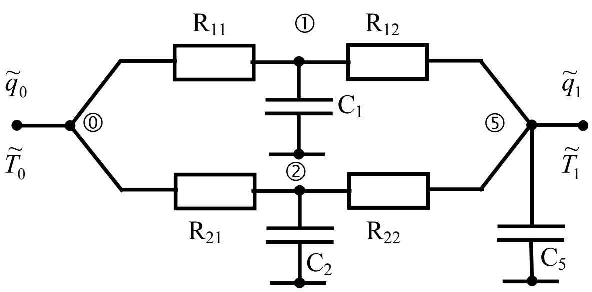

Figure 6: A half 5-node RC network modelling the thermal response at the surface 0 on a body with an adiabatic boundary in 1.

Starting from two main optimisation frequencies (period time of one year and one hour), each of the two parallel T-chains (two resistors and one capacitor) are estimated for a good agreement for the high frequency and the other one for the low frequency. C5 is given as the remaining part of the total heat capacity. The solution is then improved by a number of iteration steps. The deviation between the optimised RC network model and the analytical solution is estimated for a number of relevant frequencies.

The best suitable configuration or set of resistances and capacitances with the least calculated error according to equation 2 is chosen. For a configuration, a control is done in the frequency domain, as shown in Figure 7.

$$Err = \frac{\sum_{i}^{n} \left| \frac{Y_{analytic}(\omega_i) - Y_{RC\_configuration}(\omega_i)}{Y_{analytic}(\omega_i)} \right|}{n_{\omega_i}} \quad (3)$$

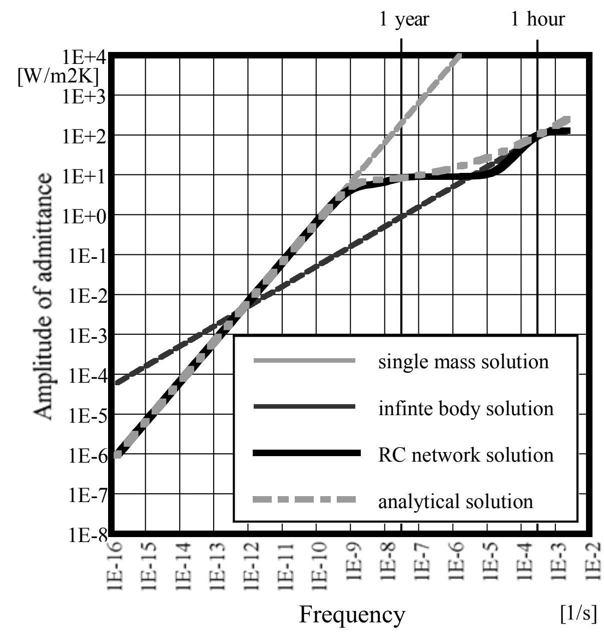

Figure 7: Amplitude of the admittance for a borehole wall in soil giving the analytical and the RC network solution and the asymptotes for single mass and an infinite external boundary.

With this set of values for R and C, calculations in the time domain are possible. As an example, the results of a temperature step at the inner surface of the pipe are show here in the time domain. The temperature response in the regarded half 5 node network, according to Figure 6, is shown. The temperatures at the surface and in node 0, in the chosen timeframe, are nearly equal. For the two parallel branches, there are differences shown for the high frequency T-chain (node 2) and the low frequency T-chain (node 1). The properties of the slow acting mass in the borehole are modelled in node 5.

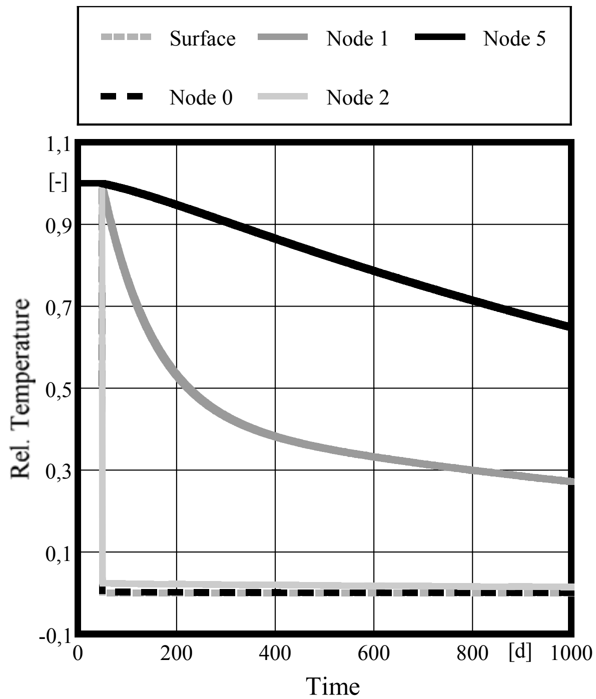

Figure 8: Calculation of the system response in time domain for the optimised RC network of a normalised relative temperature step of "1" at the surface.

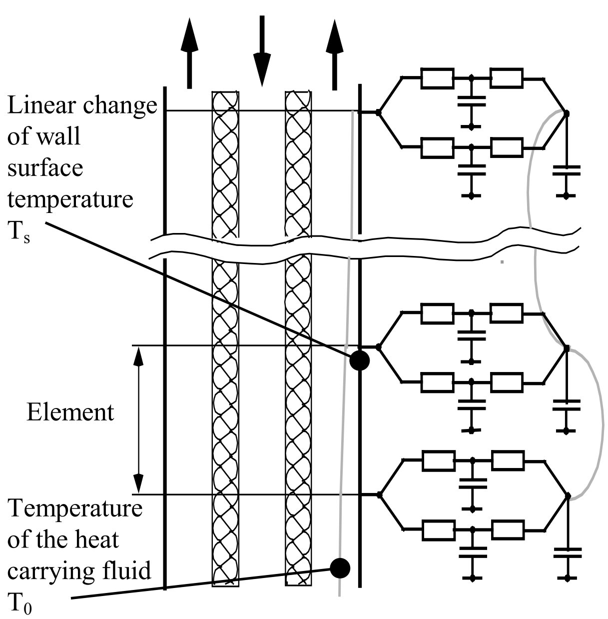

Figure 9 Possible Macro Element Method transformation model with half-5-node RC networks for a bore hole.

For the transformation of the entire borehole, it is divided into a finite number of elements. For each element, a heat transfer matrix is formulated, connecting the optimised RC networks to the mass flows in the pipes. Assuming that the pipe wall surface temperature changes linearly along the pipe, each element can cover a longer piece of borehole. The elements become "Macro-Elements", and the necessary number of elements for the model is reduced.

The modelling along the pipe is done via an energy balance for each element. For example, the heat transfer balance, assuming there is no heat exchange between the pipes, gives the following:

$$\frac{\partial T_0(x,t)}{\partial x} - \frac{O \cdot h}{u \cdot \rho \cdot c \cdot A} T_0(x,t) + \frac{1}{u}\frac{\partial T_0(x,t)}{\partial t} = -\frac{O \cdot h}{u \cdot \rho \cdot c \cdot A} T_s(x,t)$$

including: $T_s = a \cdot x + b$

$$T_0(x) = \left[T_{inlet} - \left(\frac{a}{\frac{O \cdot h}{u \cdot \rho \cdot c \cdot A}} + b \right) \right] \cdot e^{\frac{O \cdot h}{u \cdot \rho \cdot c \cdot A} \cdot x} + a \cdot x + \frac{a}{\frac{O \cdot h}{u \cdot \rho \cdot c \cdot A}} \cdot b$$

This transformation of a bore hole, a hybrid building construction, into a number of discrete RC configurations connected by a system of differential equations, is called the Macro-Element-Method. A solution for the simultaneous equations for heat transfer between the flows is carried out similarly to the solution for the double air gap construction.

CONCLUSIONS

This paper introduces a general method of modelling sections with two-dimensional surrounding bodies of a duct with mass flow that can have applications for ducts in the ground, as well as hybrid building components. It has been indicated that accurate results can be obtained with a limited number of necessary nodes. The limited number of nodes required for the MEM method clears the path for being able to effectively include such components in more general system simulations.

FURTHER WORK

In on-going work, the Macro-Element-Method will be tested and the double air gap construction will also be modelled with the methodology explained above (Schmidt, Jóhannesson 2001). Both construction examples will be implemented into the IDA simulation environment. The analysis of the interactions between a borehole storage, a double air gap wall construction and a solar collector are of major interest for further work. It will be a combined system for the heating and cooling of buildings utilising renewable energy sources.

LIST OF SYMBOLS

Symbol	Quantity	unit
ω	Cycle frequency	*1/s*
ρ	Density	*kg/m³*
a	Factor	
A	Area	m^2
b	Factor	
c	Specific heat capacity	*J/kgK*
C	Capacitance	*J/m³K*
Err	Error, model deviation	-
i	Imaginary part	-
L	Length, Borehole depth	*m*
m	Mass	*kg*
O	Perimeter	*m*
q	Specific heat flow	*W/m²*
Q	Heat flow	*W*
r	Radius	*m*
R	Resistance	*m²K/W*
t	Time	*s*
T	Temperature	*K*
u	Velocity	*m/s*
x	Coordinate of place	*m*
Y	Admittance	*WK/m²*

Indices

Notation	Meaning
e	External surrounding
i	Inside surrounding
inlet	Inlet
s	Surface
tot	Total

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