

# A SIMPLIFIED THERMAL RESISTANCE NETWORK MODEL FOR BUILDING THERMAL SIMULATION

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## ABSTRACT

This paper describes a simplified single zone thermal resistance network model (TR), which has been coded within LT Europe, an integrated energy design tool. This model is driven by an annual climate file giving hourly values of temperature and overheating risk. To validate this model, comparisons were made between simulations of TR and the Esp-r program, which itself has been rigorously validated. Simulation for over 150 cases relating to different orientations, building mass types and ventilation rates have been performed.

## INTRODUCTION

Interest in low energy building design continues to make demand for design tools, which can assist architects when evaluating energy use in buildings. Better use of daylight, the use of natural ventilation, and the adoption of passive building design with regard to orientation, window size, and use of shading devices, have won wide acceptance by architects and engineers. Computer programs for architectural design have developed to become very sophisticated and precise, but in the process require very steep learning curve and large amounts of data and time to produce useful results. Furthermore, detailed simulation models require a description of the building and its systems that is often unavailable at early design stage. It is increasingly recognised however, that the ultimate performance of the building is largely pre-determined. For example, decisions about plan depth, orientation, and fenestration, will have an influence on the potential for the use of daylight, natural ventilation, heating and cooling demand. This in turn has knock-on effects on plant and equipment, ultimately having a major impact on performance.

It is important to develop simple and fast running design tools that architects can use in the early phases of the design process. This kind of design tool should

require minimum data input, give reliable indication of trends and sensitivities, and be user friendly. The architect can then generate with little effort positive inputs to his/her design. The LT method is a widely used energy design tool that enables users to evaluate the energy performance of a number of options and to make comparisons. The LT 'curves' were generated using a quasi-steady state thermal model and an hourly daylight/artificial light model (figure 1), but no free-floating temperatures were then available [Baker].

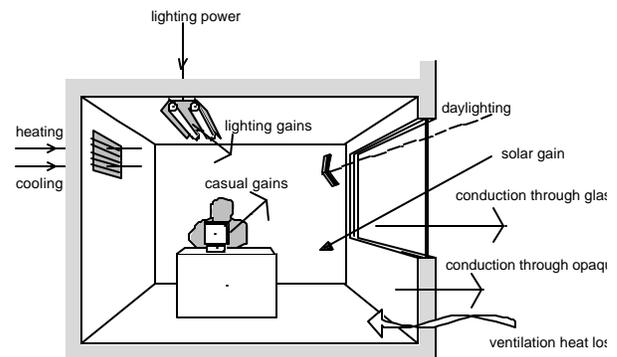


Figure 1: The basic energy flows modelled by the LT Method.

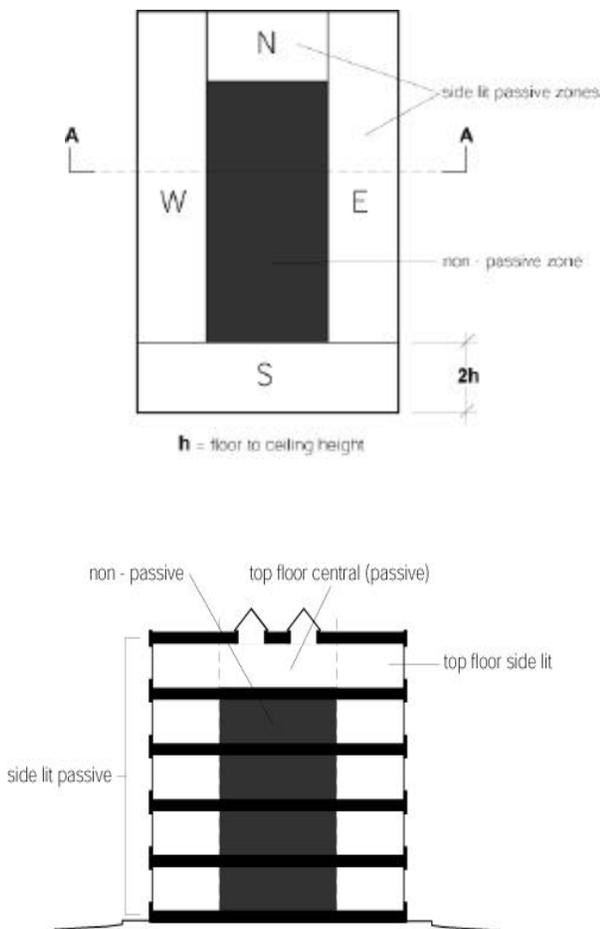
This paper describes a simplified thermal resistance network model, which has been developed and integrated within LT, providing predictions of overheating and the effects of varying key parameters such as glazing ratio, orientation, thermal mass, ventilation and night ventilation.

## METHODOLOGY

### The Passive Zone Concept

A basic premise of the LT Method is that the building can be divided into two types of space – a *passive zone* at the perimeter, both benefiting and dis-benefiting from interaction with the outside climate, and a *non-passive zone* which, with the exception of the energy needed to pre-heat fresh air, has no interaction with outside conditions. Because of the strong influence of solar radiation, the passive zones are further categorised by orientation (figure 2).

Figure 2: Passive and non passive zones in plan and section.



### The Thermal Model (TR)

The performance criterion for the TR solver is its ability to distinguish accurately between realistic room response types – lightweight, medium and heavyweight, and that it can model the build-up of

heat in the deep structure of the building in response to the occurrence of successive hot days. This is particularly important when assessing designs for northern Europe, since overheating there is rarely present on average days or isolated exceptional days. The solver also has to model the effect of night ventilation, requiring a ventilation conductance that is coupled to the internal surface of the room.

The solver does not simulate the whole building. Instead it simulates a standard module with parameters (such as glazing ratio, fabric U-value, internal gains, lighting datum level, thermostat setting, occupancy profile) to replicate those chosen for the proposed design. The performance of the whole building is then assembled from an appropriate number of LT module performance data, corresponding to parameters such as orientation, glazing ratio and overshadowing. This approach, was originated in the earlier manual LT Methods, using pre-computed graphical data.

This allows a greatly simplified resistance network to be used, which can be easily integrated with the lighting model, although models of this type, are not new in themselves [Crabb et al] [Tyndale].

The heat flows of the module is represented as a four-node simplified thermal resistance network. The model consists of four nodes: 1, a room node, 2, a surface node, 3, a deep mass node and A, an outdoor ambient node (fig 3). C1 corresponds to the room air and contents having a time constant much less than 1 time step. C2 represents the surface plus a mass layer having a time constant much less than 24 hours, and C3 represents mass at a depth sufficient to have a time constant greater than 24 hours.

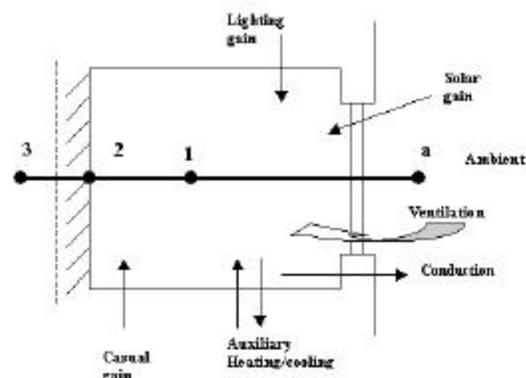


Figure 3: The TR solver, a four node resistance/capacitance network representing the LT module, driven by hourly inputs of solar irradiance, outdoor temperature, casual gains, lighting gains, auxiliary heating and cooling.

The heat flow through each node in the room can be described as follows:

$$\Delta Q_1 = (t_a - t_1) g_{1a} + (t_2 - t_1) g_{12} + Solar_1 + Cas_1 + Light_1 + aux_1 \quad (1)$$

$$\Delta Q_2 = (t_2 - t_1) g_{12} + (t_2 - t_3) g_{23} + Solar_2 + Cas_2 + Light_2 + aux_2 \quad (2)$$

$$\Delta Q_3 = (t_2 - t_3) g_{23} \quad (3)$$

$$\Delta Q_1 = C_1 \Delta t_1 \quad (4)$$

$$\Delta Q_2 = C_2 \Delta t_2 \quad (5)$$

$$\Delta Q_3 = C_3 \Delta t_3 \quad (6)$$

In the formulas,

$Q_1, Q_2, Q_3$  (W): represents the heat balance at nodes 1, 2, 3 respectively.  $g_{1a}$  (W/°C) is the conductance between N1 and node A (infinite capacity). This is partly conductive, via the glass and the opaque wall, and partly convective due to ventilation and infiltration. It can be written as the following formula:

$$g_{1a} = GR \cdot U_{glaze} A_{wall} + (1-GR)U_{wall} A_{wall} + 0.3V \cdot n \quad (7)$$

where, GR is the ratio of the glazed area to total external wall;  $U_{glaze}$  (W/m<sup>2</sup> °C) is the U value of the window;  $U_{wall}$  (W/m<sup>2</sup> °C) is the U value of the wall;  $A_{wall}$  (m<sup>2</sup>): is the area of the external wall; V (m<sup>3</sup>) is the volume of the simulation room; n (ac/h) is the air change rate;  $g_2$  (W/°C) is the conductance-partly radiative and partly convective;  $g_3$  (W/°C) is the conductance of the massive material ;  $Solar_1$  (W): is the solar gain delivered to node 1;  $Solar_2$  (W) is the solar gain delivered to node 2;  $Cas_1$  (W): is the casual gain (equipment and people) delivered to node 1;  $Cas_2$  (W): is the casual gain (equipment and people) delivered to node 2;  $C_1$  (W/°C) is the thermal capacity of the room, which includes content;  $C_2$  (W/°C) is the thermal capacity of the shallow surface material of the room;  $C_3$  (W/°C) is the thermal capacity of deep material of the room;  $Light_1$  (W): is artificial lighting gain delivered to node 1 and  $Light_2$  (W): is artificial lighting gain delivered to node 2.  $aux_1$  (W): is artificial lighting gain delivered to node

1 and  $aux_2$  (W): is auxiliary heating or cooling delivered to node 2.

### Solar Gain

The solar heat gain entering the room through the window can be calculated as:

$$Solar = IT_s \cdot F_{shad} A \quad (8)$$

$Solar$  (W/m<sup>2</sup>) is the total solar gain entering the room;  $I$  (W/m<sup>2</sup>) is the total solar irradiance falling on the plane;  $T_s$  is the solar transmittance,  $F_{shad}$  is the factor of shading device; A (m<sup>2</sup>) is the area of the glazing.

For the horizontal surface,

$$I_{bh} = I_{bn} \sin H \quad (9)$$

$$I_{gh} = I_{bh} + I_{dh} \quad (10)$$

$$I = I_{gh} \quad (11)$$

For the vertical surface,

$$I_{bv} = I_{bn} \cos q \quad (12)$$

$$I_{dv} = 0.5I_{dh} + 0.5rI_{gh} \quad (13)$$

$$I_{gv} = I_{bv} + I_{dv} \quad (14)$$

$$I = I_{gv} \quad (15)$$

where  $I_{bh}$  (W/m<sup>2</sup>) and  $I_{bv}$  (W/m<sup>2</sup>) are the direct beam radiation on the plane of the horizontal and vertical glazing respectively ;  $I_{gh}$  (W/m<sup>2</sup>) is the total radiation on the horizontal surface;  $I_{dv}$  (W/m<sup>2</sup>) is the diffuse radiation on the plane of the vertical glazing ;  $I_{bn}$  (W/m<sup>2</sup>) is the normal beam radiation ;  $?, H$  and  $?$  are incidence angle, solar altitude and ground reflectivity, respectively.

It is assumed that a vertical window receives half as much diffuse radiation as a horizontal surface and half of the total radiation reflected from the ground [Lam].

### Lighting gain

For the energy and overheating analysis the TR model requires hour-by-hour lighting gains as a result of photo-sensitive switching. Internal average daylight factors of front and back halves of the room module (fig 4) are calculated based on the window geometry [Hopkinson][Lynes].

Daylight Factor is given by

$$DF = SC + ERC + IRC \quad (16)$$

SC is the sky component; ERC is the external reflected component; IRC is the internal reflected component.

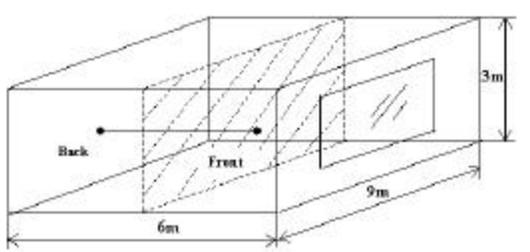


Fig.4 Front and back half zones for the daylight calculation.

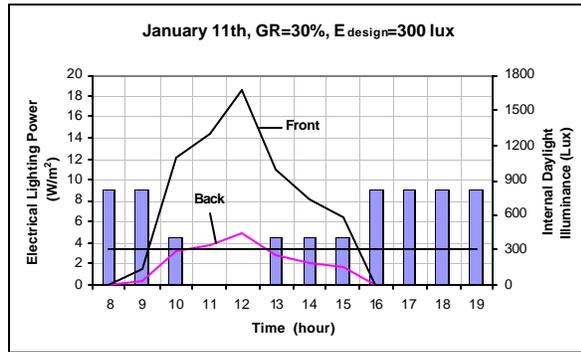


Fig. 5 Hourly electrical power profile

The internal illuminance in each zone can be given by:

$$E_{Back} = \overline{DF}_{Back} E_{OV} \quad (17)$$

$$E_{Front} = \overline{DF}_{Front} E_{OV} \quad (18)$$

$$E_{BOV} = 0.396 E_{OH} \quad (19)$$

$E_{OV}$  and  $E_{OH}$  (lux) are vertical and horizontal outdoor illuminances;  $\overline{DF}_{Front}$  and  $\overline{DF}_{Back}$  are the daylight factors of the front and back zones respectively;  $E_{Front}$ ,  $E_{Back}$  are indoor illuminances at the room front and back halves.

If  $E_{design}$  is the indoor design illuminance then if  $E_{Front}/E_{Back}$  is less than  $E_{design}$ , the artificial lighting will be switched on. In this way the hourly artificial lighting energy profile was produced.

Fig.5 shows one day of results for the hourly power switching on/off profile within an office building simulation cell. The internal design illuminance is

300 lux. The window is south facing with a glazing ratio of 30%. The working plane luminous efficacy is 33lm/W, which includes the luminaire Utilisation Factor and Maintenance Factor. From the figure we can see that from 0800 to 0900, the front and back illuminances are both less than the design illuminance therefore the electrical power is turned on; at 1000, the front illuminance is greater than the design illuminance, therefore the front electrical power is turned off; from 1100 to 1200, both the front and back illuminance are greater than design illuminance, therefore both the front and back electrical power are off. This electrical power profile provides the hourly artificial lighting gain for the thermal model.

### Thermal Mass

The room can be defined by three building construction types: Heavy, Medium and Light. Table 1 is the definition of the building type, as specified for the Esp-r simulation. In using the TR solver, rooms do not have to be of this precise construction, only to have the same overall thermo-dynamic characteristics.

### PROGRAMME DEVELOPMENT & VALIDATION

The computer program is developed using Visual Basic version 5.0. The internal free floating air temperature and the overheating days were calculated hourly for the simulation room throughout the whole year. The validation for the programme was carried out by comparing the results from the TR programme with Esp-r, a well validated thermal simulation software. The climate data used was UK kew67 climate data. Over 150 cases, which related to parameters such as building type, orientation, internal illumination level, internal gain and ventilation rate, were simulated. Setting parameter values

### Parameter values

The reduced resistance network approach is, of course a simplification of the physical process, but remains a model of the actual physical processes. This is in contrast to a 'black box' empirical model where mathematical functions are derived to fit large amounts of data – usually real measured, but possibly generated by a well validated detailed model. However although the structure of the model is based on the physics, parameter values can be derived in order to match predicted data with the reference data, in our case the ESP-r output.

The procedure was to propose initial values based loosely on 'physics values', and then refine them until the best match was made. After having obtained good

agreement in the 150 cases tested, including extreme cases, the un-provable hypothesis is that it will predict all other cases with acceptable accuracy.

In particular, parameters such as the room node capacity, solar split between the surface node and the room node, and the partition of surface and deep mass nodes, were fine-tuned in this way. This approach was also used to achieve stability of the model; the four parameters – time step, room mass and room-to-surface conductance interacted to control stability. We found that quite small changes in these relative values would effect stability, but would have very little effect on the free-floating temperature.

The time step of one hour was chosen as the largest for which adequate dynamic response could be modeled, also being compatible with readily available climate data and overheating criteria. Shorter time steps would have increased computation time.

Fig. 6 shows an example of the simulation results of internal free floating air temperature for a room with a light mass and south-facing glazed, and 3 air changes per hour in July. The TR and Esp-r simulation results show very good agreement. A particular concern was that the number of overheating days simulated by TR showed, has good agreement with Esp-r simulation results. Since this is a threshold effect it was thought to be a sensitive test. Table 2 shows a summary of values. In the table H, M and L refer to heavy, medium and light building types respectively. 1, 3 and 12 represent air change rates. The relative error is less than 10% to 20%, for the most part, showing good agreement.

#### CONCLUSION

Our approach in deriving the model lies somewhere between explicit physics based modeling and ‘black-box’ empirical modeling. Having a physics basis, the modeler can use reasoning and experience to refine parameter values and obtain good agreement with reference data, quite quickly.

By this means we have been able to derive a very fast solving model which convincingly predicts the effect

of night-ventilation and deep mass heat build-up, and gives acceptable agreement with the sensitive threshold test of overheating frequency. Bearing in mind that the solver is to provide feedback on trends and sensitivities early in design development, and is not a precision simulator, we feel that its performance is fully adequate.

The simplicity of the model is partly facilitated by its application to a generic module rather than a representation of the whole building.

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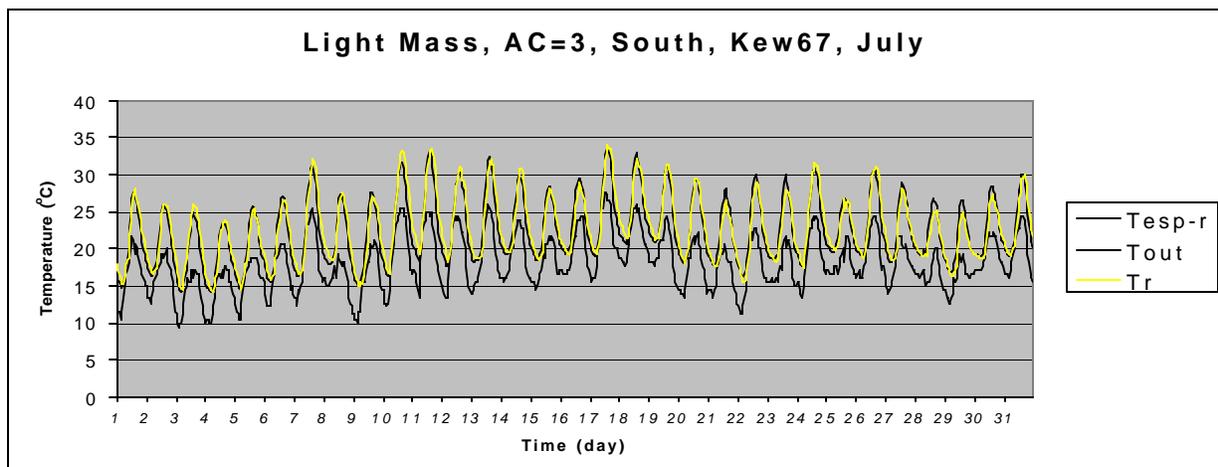
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**Table 1 Thermal mass definition**

	<b>Heavy</b>	<b>Medium</b>	<b>Light</b>
<b>Floor</b>	<ul style="list-style-type: none"> <li>• dense concrete</li> <li>• ceramic tiles</li> </ul>	<ul style="list-style-type: none"> <li>• dense concrete</li> <li>• carpet tiles</li> </ul>	<ul style="list-style-type: none"> <li>• dense concrete</li> <li>• air gap</li> <li>• chip board</li> <li>• carpet</li> </ul>
<b>Walls</b>	<ul style="list-style-type: none"> <li>• density concrete</li> <li>• gypsum plaster</li> </ul>	<ul style="list-style-type: none"> <li>• lightweight concrete block</li> <li>• gypsum plaster</li> </ul>	<ul style="list-style-type: none"> <li>• gypsum plaster</li> <li>• air gap</li> <li>• gypsum plaster</li> </ul>
<b>Ceiling</b>	<ul style="list-style-type: none"> <li>• dense concrete</li> <li>• gypsum plaster</li> </ul>	<ul style="list-style-type: none"> <li>• dense concrete</li> <li>• lightweight plaster</li> </ul>	<ul style="list-style-type: none"> <li>• dense concrete</li> <li>• air gap</li> <li>• mineral acoustic tiles</li> </ul>

**Table 2 Overheating days comparison by TR and Esp-r simulation**

	<b>Orien tation</b>	<b>H1</b>	<b>H3</b>	<b>H12</b>	<b>M1</b>	<b>M3</b>	<b>M12</b>	<b>L1</b>	<b>L3</b>	<b>L12</b>
<b>TR</b>	South	33	2	0	58	2	0	79	14	0
<b>Esp-r</b>		27	1	0	53	2	0	81	21	0
<b>TR</b>	East	38	1	0	38	1	0	58	9	0
<b>Esp-r</b>		31	1	0	31	1	0	45	7	0
<b>TR</b>	West	59	3	0	40	8	0	62	10	0
<b>Esp-r</b>		55	4	0	55	7	0	64	12	0
<b>TR</b>	North	5	0	1	0	0	0	10	3	0
<b>Esp-r</b>		4	0	0	0	0	0	13	1	0
<b>TR</b>	Horizo ntal	70	21	1	76	43	2	90	64	2
<b>Esp-r</b>		59	20	1	63	23	2	70	56	1



*Figure 6: Comparison of free-floating temperatures predicted by TR1 and ESPr.*