



## DYNAMIC DATA-DRIVEN GRAY-BOX MODELS OF COMPONENTS OF HVAC SYSTEMS

Nabil Nassif      Radu Zmeureanu  
Centre for Building Studies, Department of Building,  
Civil and Environmental Engineering, Concordia University  
Montréal, Canada  
[n\\_nassif@encs.concordia.ca](mailto:n_nassif@encs.concordia.ca)      [zmeur@bcee.concordia.ca](mailto:zmeur@bcee.concordia.ca)

### ABSTRACT

Operating performance of HVAC systems could be improved through real-time control and optimization. Therefore, component models are required to estimate performance of HVAC systems. It is of practical importance to develop a simple, yet accurate and reliable dynamic model that will better match the dynamic behavior of local control process and overall system over the entire operating range. Dynamic data-driven models based on gray-box approach are proposed in this paper. Model parameters are tuned by using genetic algorithm in order to minimize the errors between measured and estimated performance data. Dynamic models such as the zone temperature model, the cooling coil model and the fan model are developed and validated against real data gathered from an existing HVAC system. The validation results show that the data-driven gray-box component models produce better accuracy compared with measured data. These models have a great potential for different applications especially for real-time control and fault detection.

### INTRODUCTION

The operation of heating, ventilating and air conditioning (HVAC) systems plays an essential role in the optimizations of energy use in buildings for providing thermal comfortable and healthy conditions for occupants. The performance of HVAC systems can be improved through better *local-loop* and *supervisory control* (ASHRAE 2003). Accurate and reliable component models that will better match the dynamic behavior of local control process and overall system over the entire operating range are then required.

Models can be developed by two distinct methods (ASHRAE 2005): forward models and data-driven models. Forward models may need detailed physical information that could not be always available. Moreover, it is almost impossible to develop a model based on physical knowledge that perfectly simulates the real system behavior. Data-driven models are the alternative approach to be used for existing systems. The accuracy of the data-driven models could be an issue when they work outside the training range. As a

result, the system should not be totally data-driven. A pure black box model, totally dependent on the experimental data, is not likely to be robust enough. Physical considerations must be included to make the system more robust (Ahmed et al 1996). Dynamic data-driven gray-box (DDGB) models of HVAC components are proposed in order to ensure more reliability and accuracy. First a physical model is selected to represent the system, and then its parameters are tuned using a genetic algorithm in such a way to reduce the error between measured and calculated data. This paper presents a few DDGB models such as the zone temperature model, the cooling coil model, and the fan model that are developed and validated against real data gathered from an existing HVAC system.

### OPTIMAL CONTROL STRATEGY

HVAC systems are typically controlled using two-level control structure. Lower-level local-loop control of a single set point is provided by an actuator. The upper control level, that is the supervisory control, specifies set points and operation strategies. The performance of HVAC systems can be improved by using a robust intelligent control strategy providing optimal PI parameters of controller (better lower-level local loop) and set points (better upper control level). This intelligent control strategy includes mainly dynamic models of system components and two genetic algorithms (GAs) that are used for two purposes: (i) the selection of optimal PI controller parameters and set points (using  $GA_1$ ) and (ii) the tuning of model parameters (using  $GA_2$ ). The optimal variables such as PI controller parameters and set points are determined through this control strategy integrated in the building energy management system (BEMS). As shown in Figure 1, the controller of real system (Real System Side) receives the optimal PI controller parameters and set points from the genetic algorithm  $GA_1$ , which are expected to produce the best or 'near-best' performance of real operating system. The performance depends on the accuracy of the model used for the estimation of energy performance. Data-driven gray-box (DDGB) component models proposed in this paper are used for this purpose. The parameters of proposed DDGB model are periodically modified during system operation.

The operation sequence of intelligent control strategy is summarized as follow. At each time interval (e.g. 10 min), the genetic algorithm  $GA_2$  is used for tuning the parameters of dynamic component models by reducing the error between measured sample data  $S$  (real data) taken from previous periods and predicted or estimated data. The genetic algorithm  $GA_1$  and the dynamic component models are then used for determining optimal set points and PI controller parameters for the next operating interval (e.g. 10 min), called optimization period  $J$ .

The optimization period  $J$  ( $\Delta t_p=10$  min) is divided into  $n$  (e.g.  $n=10$ ) simulation time steps  $i$  ( $\Delta t_{sim}=1$  min). For instance, as shown in Figure 2, at time equal to 30 min, the  $GA_1$  determines the optimal set point of zone air temperature  $T_{z,set}$  (23.1 °C) and the optimal set point of supply air temperature  $T_{s,set}$  (14.5 °C) for the optimization period  $J$ . Within this period  $J$ , models simulate the zone air temperature  $|Tz|_J$  and supply air temperature  $|Tsi|_J$  for ( $i=1,\dots,10$ ). In these calculations, the thermal sensible load ( $qs_J$ ), which is predicted from previous periods ( $qs_{J-1}$  and  $qs_{J-1}$ ), is assumed to be constant during the optimization period  $J$ .

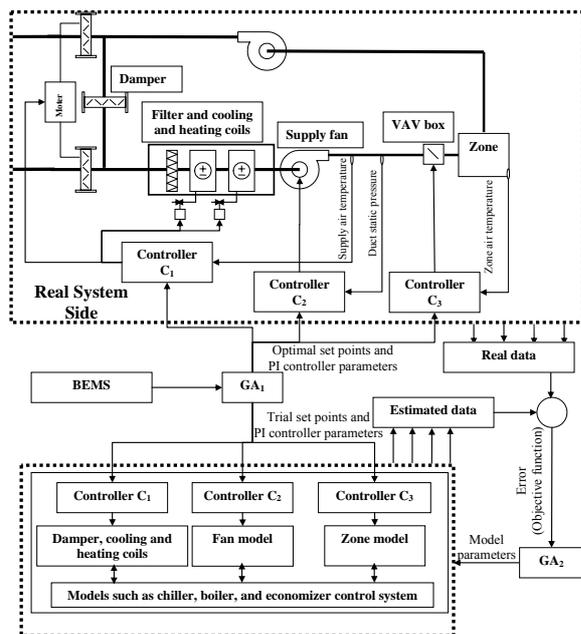


Figure 1. Schematic of optimal control strategy including the investigated HVAC system

Figure 3 shows the flowchart of genetic algorithm  $GA_1$  for optimal control.  $GA_1$  starts with a random generation of the initial population (initial solution). The problem variables (controller set points and PI controller parameters) are encoded to form a chromosome (a string of variables) that represents an individual (one solution) in the population. The performance or objective function of each individual of the first generation is estimated. The second generation is generated using operations on

individuals such as selection, crossover, and mutation, in which individuals with higher performance (fitness) have a greater chance to survive. The performance of each new individual is again evaluated. The process is repeated until the maximum number of generations ( $G_{max}$ ) is reached. Assuming that  $G_{max}=500$  and the population is composed of  $P=50$  individuals, the number of simulations is  $10*50*500 = 250,000$  for each optimization period  $J$ . It should be noted that measured data at the end of previous period is used as initial values for dynamic models.

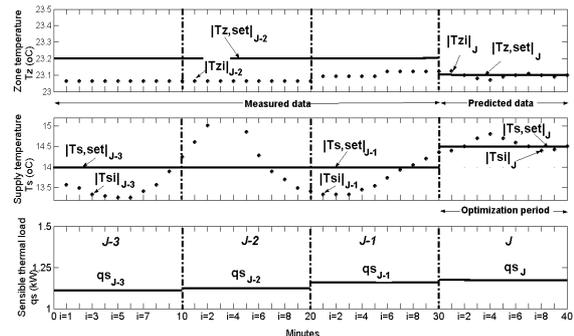


Figure 2. Data from previous measurements and predictions for optimization period  $J$ .

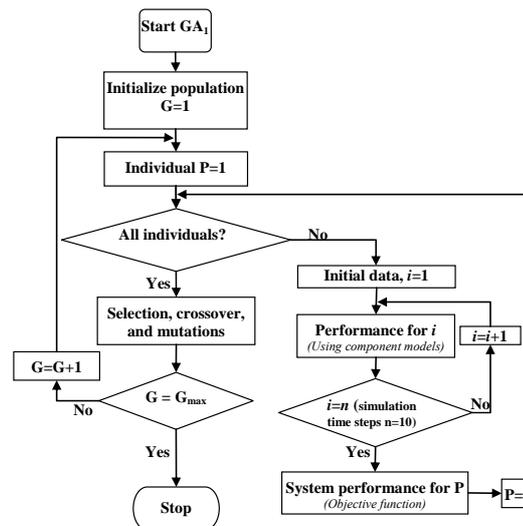


Figure 3. Flow chart of genetic algorithm  $GA_1$  for optimal control

## SYSTEM DESCRIPTION AND COMPONENT MODELS

The investigated VAV system is installed at the *École de technologie supérieure (ÉTS)* campus, in *Montréal* (see Figure 1, *Real System Side*). The system consists mainly of (i) the return and supply fans, (ii) the outdoor, discharge, and recirculation dampers, (iii) the air handling unit (AHU) with components such as filter and cooling and heating coils, (iv) the pressure-independent VAV terminal boxes, and (v) the local-control loops (i.e.  $C_1$ ,  $C_2$ , and

$C_3$ ). The supply air temperature is controlled by the common controller ( $C_1$ ). The duct static pressure is controlled by the controller ( $C_2$ ). The zone air temperature is controlled by the controller ( $C_3$ ).

As shown in Figure 3, the objective function of each solution is simulated using the component models developed in this study. The objective function includes the energy consumption, the thermal comfort, the indoor air quality, and stabilities of local-control loops over optimization period  $J$ . Several component models such as the zone temperature model, the cooling and heating coils model, the damper model, chiller model, and the fan model are developed for this purpose. Due to space limitations, only a few models are presented here (Figure 4):

1. Zone temperature model is used to calculate the zone air temperature as the controlled variable. This model includes also the estimation of sensible thermal load.
2. Fan model is used to compute the duct static pressure as the controlled variable as well as the fan energy use.
3. Cooling coil model is used to compute the supply air temperature as the controlled variable and other variables such as cooling coil load and relative humidity. The model includes a valve model that is used to determine the chilled water flow rate as a function of valve opening.

All inputs to models, as presented in Figure 4, are obtained through direct measurements, are recorded by the BEMS and therefore are available. The only exceptions are the zone thermal loads, the fan airflow rate, and the air conditions entering the cooling coil. The zone thermal load is estimated from measured data at previous periods, as discussed in next section (equations 4 and 5). The fan airflow rate is equal to the sum of measured zone airflow rates. The relative humidity and temperature of air entering the cooling coil are calculated by using the measured relative humidity and temperature of outdoor and return air flows as well as estimated fan and outdoor airflow rates. The outdoor airflow rate is estimated by using the damper model, as a function of measured damper position and mixing duct static pressure. The damper model was developed and validated by (Nassif et al 2004). The cooling coil load, which is an output of the cooling coil model, becomes an input to the chiller model. The cooling coil load is determined by using the estimated airflow rate, and the conditions of air entering the coil (estimated values) and leaving the coil (measured values).

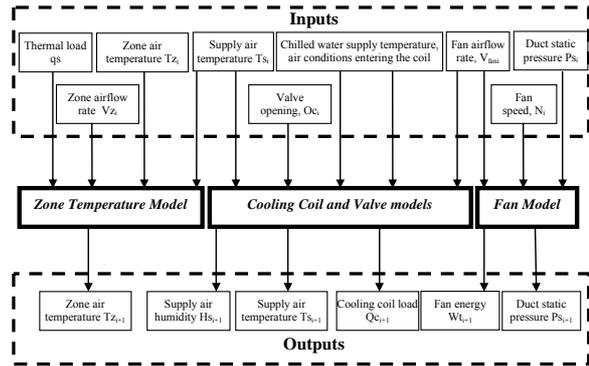


Figure 4. Inputs and outputs of selected component models

## TUNING OF MODEL PARAMETERS

In the proposed data-driven gray-box (DDGB) component models, their parameters are tuned in such a way to reduce the error between measured and estimated data. Model variables can be tuned with respect to the reference value  $Va_{ref}$  as follows:

$$Va_i \rightarrow Va_i + a_i \cdot Va_{ref,i} \quad (1)$$

where the term  $a_i$  is the tuning parameter included in the model. The reference value  $Va_{ref}$  could be the design value or any other value with a significant impact on the process. The tuning parameters of model  $a_i$  are determined by the genetic algorithm  $GA_2$  in such a way that predictions of these parameters lead to the least error ( $e_k$ ) in the estimation of selected variables, compared with sample data  $S$  that is selected from previous periods. The objective function  $f$ , which should be minimized, is written as:

$$f = \sum_{k=1}^S e_k \cdot \lambda^k \quad (2)$$

where  $\lambda$  ( $0 < \lambda > 1$ ) is a forgetting factor to give higher weight to more recent data ( $k=1$ ) than older data ( $k=S$ ). When the objective function of all individuals in a generation is calculated, genetic algorithm operators (Deb 2001) are used. The simulated binary crossover SBX is used to create two offsprings from two-parent solutions. The random simplest mutation operator is applied to create a new solution from the entire search space. This algorithm uses the elite-preserving operator, which favors elites of a population by giving them an opportunity to be directly carried over to the next generation. After two offsprings are created using the crossover and mutation operators, they are compared with both of their parents to select two best solutions among the four parent-offspring solutions. Stochastic universal sampling-SUS version of proportionate selection is used (Deb 2001). To control the rate of tuning, the genetic algorithm search is restricted to the range of  $[-1, +1]$ , for instance  $[-0.1, 0.1]$ . Since the design

value is used as the reference value, the tuning parameters should be in the range of [-1, +1].

The performance of a number of genetic algorithms with adjustments made to their control parameters has been investigated. The parameters setting of GA<sub>2</sub> that produces best performance for tuning of component models are the following: the crossover probability  $p_c=0.8$  with distribution index  $\eta_c=3$ , the mutation probability  $p_m=0.05$ , the population size  $P=50$ , and the maximum number of generations  $G_{max}=100$ .

## ZONE MODEL

Zone temperature model simulates variations of zone air temperature for  $n$  time steps within the optimization period  $J$  under the new set points of zone and supply air temperatures, as calculated by GA<sub>1</sub> (see Figure 2). Since the zone temperature variation depends on the sensible thermal load within this period, this model includes also the estimation of sensible thermal load during the period  $J$  from previous measured data.

Assuming the well mixed model of air in the zone, the temperature  $Tz$  can be calculated as follows, using the zone air heat balance equation:

$$M \cdot c_p \cdot \frac{dTz}{dt} = \dot{Vz} \cdot \rho \cdot c_p \cdot (Ts - Tz) + qs \quad (3)$$

where  $M$ ,  $\rho$ , and  $C_p$ , and  $\dot{Vz}$  are mass, density, specific heat, and volumetric flow rate of air in zone, respectively. The terms  $Ts$  and  $Tz$  are the supply and zone air temperatures. Sensible thermal load  $qs_J$  during the optimization period  $J$  is estimated from measured data at previous periods (Figure 2), and it is considered to be constant during the whole period  $J$ :

$$qs_J = qs_{J-1} + (qs_{J-1} - qs_{J-2}) \quad (4)$$

It is assumed a linear variation of zone sensible load between the three periods  $J$ ,  $J-1$ , and  $J-2$ . Zone sensible load during period  $J-1$  is calculated as follows:

$$qs_{J-1} = \frac{\left| \sum_{i=1}^n \dot{Vz}_i \cdot c_p \cdot \rho \cdot (Tz_i - Ts_i) \right|_{J-1}}{n} + \frac{M \cdot c_p \cdot \left| \sum_{i=1}^n Tz_i \right|_{J-2} - \left| \sum_{i=1}^n Tz_i \right|_{J-1}}{\Delta t_p \cdot n} \quad (5)$$

where  $\Delta t_p$  (i.e. 10 min) is the time interval between two optimization periods, in which the variation of zone temperature is considered;  $n$  is number of points ( $n=10$ ) within the optimization period.

The “steady state” zone air temperature  $Tz^{ss}$  could be calculated by assuming no change in the zone air temperature, that is  $dTz/dt=0$  in equation 3:

$$Tz^{ss} = \frac{qs}{c_p \cdot \rho \cdot \dot{Vz}} + Ts \quad (6)$$

Taking  $qs$  from equation 6 and inserting into equation 3, we obtain:

$$M \cdot c_p \cdot \frac{dTz}{dt} = \dot{Vz} \cdot \rho \cdot c_p \cdot (Tz^{ss} - Tz) \quad (7)$$

During simulation time step  $i$  ( $\Delta t_{sim}=1$  min), the zone airflow rate and supply air temperature are assumed to be constant and the solution of equation above becomes:

$$Tz_i = \left( 1 - e^{-\frac{\Delta t_{sim}}{\tau_z}} \right) \cdot Tz_i^{ss} + e^{-\frac{\Delta t_{sim}}{\tau_z}} \cdot Tz_{i-1} \quad (8)$$

where the zone time constant  $\tau_z$  is given by:

$$\tau_z = \frac{M}{\rho \cdot \dot{Vz}} \quad (9)$$

If the thermal zone load is overestimated, the simulated “steady state” (equation 6) and consequently the zone temperature (equation 8) will be overestimated. To increase the accuracy of models, the data-driven gray-box DDGB model is developed. In that case, by using tuning parameters, equations 6 and 9 become:

$$Tz^{ss} = \frac{qs + a_2 \cdot qs_{des}}{\left( \dot{Vz} + a_1 \cdot \dot{Vz}_{des} \right) \cdot \rho \cdot c_p} + Ts + a_3 \cdot 5 \quad (10)$$

$$\tau_z = \frac{M}{\rho \cdot \left( \dot{Vz} + a_1 \cdot \dot{Vz}_{des} \right)} + a_4 \cdot \tau_{z,des} \quad (11)$$

Parameter  $a_1$  considers the effect of inaccurate measured value of the airflow rate. Parameter  $a_2$  corrects errors in load estimations. Parameter  $a_3$  considers the error in supply air temperature, due for instance to heat transfer in ducts (5°C is used as a reference value). Parameter  $a_4$  corrects the impact of inaccurate estimated value of air mass  $M$  as well as the effect of some unknown parameters such as zone materials. Figure 5 shows the dynamic DDGB model of zone air temperature.

When the four tuning parameters ( $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$ ) are equal to zero (default values), the dynamic DDGB model (equations 8, 10 and 11) equal the simple model presented in equations (equations 6, 8, and 9).

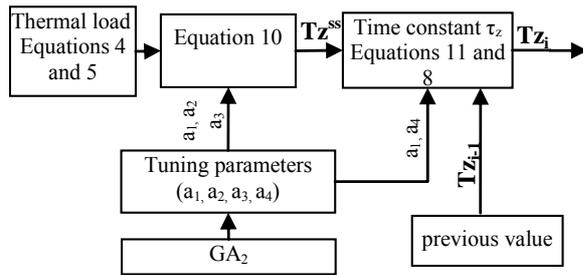


Figure 5. Dynamic DDGB model of zone temperature

Predictions of the zone air temperature calculated by the simple model (equations 6, 8, and 9) and by the data-driven gray-box DDGB model (equations 8, 10, and 11) are compared with measured data from the existing VAV system. Using a sample of measured data of two previous periods  $J-1$  and  $J-2$ , the model parameters are tuned by  $GA_2$  and the zone sensible load (equations 4 and 5) is estimated. This sample data consists of  $S=20$  points. The zone air temperature for period  $J$  is simulated for ten simulation time steps ( $i=1, \dots, 10$ ) and compared with measured data. The initial value of zone temperature ( $i=1$ ) is equal to the measured temperature at the end of previous period  $J-1$ . As zone and supply air temperature set points are constant during normal operation of the investigated existing VAV system, and the initial value of zone temperature ( $i=1$ ) is equal to measured one, the errors are small when both models are used for predicting the zone air temperature for next time steps ( $i=2, \dots, 10$ ).

In a second validation of the model, the zone air temperature is simulated for one full day ( $i=1, \dots, 1440$ ) using only one measured point ( $i=1$ ) as initial value. In this case, data from two previous days ( $S=2880$ ) are used to tune model parameters. Sensible load in zone is predicted by equations 4 and 5 for each interval of 10 minutes ( $n=10$  points) using previous measured data of 20 points, and they form the inputs of the simple and proposed models (equations 6 and 10). Other inputs such as zone air flow rates and supply air temperature are directly taken from measured data. Figure 6 shows the comparison of simulated and measured zone air temperatures for one full day. In this day, the set point values of zone temperature vary from 26°C to 23°C at 7 AM. This validation is repeated to cover three summer months (June, July and August 2003). The validation results for this period indicate that the accuracy of zone temperature model significantly improved by using tuning parameters. The average error decreases from 5% to 2% by using the DDGB model instead of the simple model.

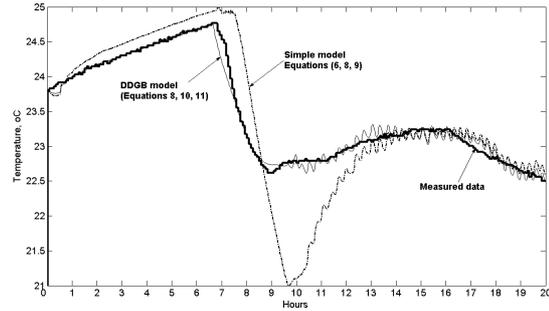


Figure 6. Measured zone air temperature compared with predictions of the simple and proposed DDGB models

### COOLING COIL MODEL

The cooling coil model simulates the supply air temperature, supply air humidity, and cooling coil loads required for estimations of overall plant energy consumptions (Figure 4). Chilled water flow rate is calculated as a function of valve opening by valve model (Valve and Actuator Manual 1994, Nassif et al 2004).

The steady state simple cooling coil model SIMCCM presented by (Brandemuehl et al 1993) is used. In this model, the internal and external heat transfer coefficients are determined from the performance of the coil at a single rating point, and are assumed to be constant. However, in the model proposed in this paper, both coefficients vary as functions of the liquid and airflow rates ( $\dot{V}_l$  and  $\dot{V}_{fan}$ ) (Morisot et al 2002):

$$UA_{int} = UA_{int,rate} \left( \frac{\left( \dot{V}_l + a_1 \cdot \dot{V}_{l,rate} \right)^{a_2}}{\dot{V}_{l,rate}} \right) \quad (12)$$

$$UA_{ext} = UA_{ext,rate} \left( \frac{\left( \dot{V}_{fan} + a_3 \cdot \dot{V}_{fan,des} \right)^{a_4}}{\dot{V}_{fan,des}} \right) \quad (13)$$

When the tuning parameters  $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$  are set to zero, the proposed steady state model equals the steady state SIMCCM model. Tuning parameters  $a_2$  and  $a_4$  considers the relations between the heat transfer coefficients and the liquid and airflow rates. The tuning parameter  $a_1$  corrects the error obtained from determining liquid flow rate as a function of valve opening using the valve model. However, since the air flow rate over cooling coil, that is equal to fan airflow rate, is not measured but determined as the sum of measured airflow rates for all zones, the parameters  $a_3$  is expected to correct errors due to air leakage and measurement errors.

Since the fan is mounted in the air stream, the heat transferred to the air is equal to motor electric input (function of  $\dot{V}_{fan}^2$ ):

$$T_s^{ss} = Tsc^{ss} + 5 \cdot \left( \frac{\dot{V}_{fan} + a_3 \cdot \dot{V}_{fan,des}}{\dot{V}_{fan,des}} + a_5 \right)^3 \quad (14)$$

where  $Tsc^{ss}$  is the outlet cooling coil temperature determined by the proposed steady state model (SIMCCM with equations 12 and 13);  $T_s^{ss}$  is the steady state supply air temperature; and the term  $a_5$  is tuning parameter.

The dynamic behavior of the cooling coil is modeled by filtering the steady state supply air temperature and using a single time constant  $\tau_c$  (Clark 1985, and Lebrum and Bourdouxhe 1996).

$$T_{s_i} = \left( 1 - e^{-\frac{\Delta t_{sim}}{\tau_c}} \right) \cdot T_{s_{i-1}}^{ss} + e^{-\frac{\Delta t_{sim}}{\tau_c}} T_{s_{i-1}} \quad (15)$$

The cooling coil time constant  $\tau_c$  is given by (Lebrum and Bourdouxhe 1996):

$$\tau_c = \frac{C_c}{UA_{tot}} + a_6 \cdot \tau_{c,des} \quad (16)$$

The design time constant  $\tau_{c,des}$  that equals the coil thermal capacitance  $C_c$  divided by the design total heat transfer coefficient  $UA_{tot,des}$  is used as the reference value. Figure 7 shows the proposed DDGB model of the cooling coil. When the tuning parameters  $a_i$  ( $i=1,..,5$ ) are set to zero, the proposed dynamic DDGB model equals the SIMCCM model with simple time constant (dynamic SIMCCM).

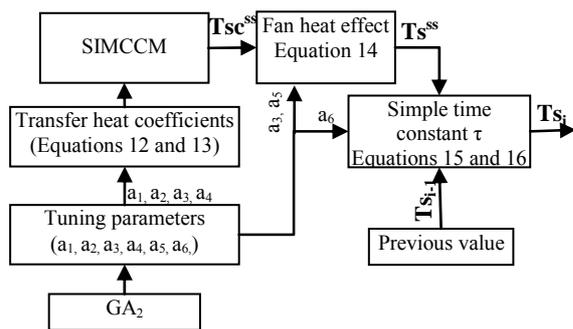


Figure 7 Dynamic DDGB model of cooling coil

The validation is made for one full day ( $i=1,..,1440$ ) using only one measured point ( $i=1$ ) as initial value and a time step of 1 minute. The supply air temperature obtained from measured data is compared with predictions from the dynamic SIMCCM model and proposed dynamic DDGB model (Figure 8). The validation for investigated period (three summer months) shows that the

accuracy of proposed model was significantly better than dynamic SIMCCM model (SIMCCM with simple time constant). The error decreases from 10% to less than 2% by using DDGB model compared with measured data.

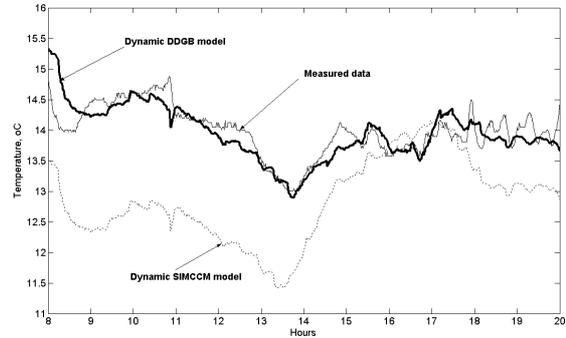


Figure 8. Measured supply air temperature compared with predictions of the dynamic SIMCCM model and the proposed dynamic DDGB model

## FAN MODEL

The fan model is used to calculate the electric demand and the duct static pressure over a wide range of operation conditions (Figure 4). The steady state fan model introduced by Clark (1985) is used. The proposed model uses the dimensionless coefficients of flow ( $\Phi$ ), pressure head ( $\Psi$ ), and shaft power ( $\eta_f$ ):

$$\Phi = \frac{(\dot{V}_{fan} + a_1 \cdot \dot{V}_{fan,des})}{(N + a_2 \cdot N_{des}) \cdot d^3} \quad (17)$$

$$\Psi = \beta_0 + \beta_1 \Phi + \beta_2 \Phi^2 + \beta_3 \Phi^3 + \beta_4 \Phi^4 \quad (18)$$

$$\eta_f = \delta_0 + \delta_1 \Phi + \delta_2 \Phi^2 + \delta_3 \Phi^3 + \delta_4 \Phi^4 \quad (19)$$

Equations 18 and 19 represent the performance of a fan by a polynomial regression of the manufacturer's data (Clark 1985). The coefficients  $\beta_i$  and  $\delta_i$  are determined from the manufacturer's data. The term  $d$  is the fan diameter and  $\rho$  is the air density. The fan static pressure  $P_{s,fan}$  is given by:

$$P_{s,fan} = (\Psi + b_1 \cdot \Psi_{des}) \cdot \rho \cdot (N + a_2 \cdot N_{des})^2 \cdot d^2 \quad (20)$$

When parameters  $a_1$ ,  $a_2$ , and  $b_1$  are set to zero, the fan static pressure  $P_{s,fan}$  calculated above equals that calculated by Clark's model. To determine the "steady state" duct static pressure ( $P_s^{ss}$ ) from the "steady state" fan static pressure calculated by equation 20, the static pressure drop in duct upstream of sensor should be considered:

$$P_s^{ss} = P_{s,fan} - P_{s,mix} + \Delta P_{comp} \quad (21)$$

where  $P_{S_{mix}}$  is the static pressure in mixing plenum box, and  $\Delta P_{com}$  is the pressure drop between plenum mixing box and sensor of duct static pressure. They are calculated as follows:

$$P_{S_{mix}} = C_m (1 + b_2) \cdot \left( \dot{V}_{fan} + a_1 \cdot \dot{V}_{fan,des} \right)^2 \quad (22)$$

$$\Delta P_{comp} = C_c (1 + b_3) \cdot \left( \dot{V}_{fan} + a_1 \cdot \dot{V}_{fan,des} \right)^2 \quad (23)$$

Coefficients,  $C_c$  and  $C_m$ , are determined from the known pressure drop through each component at rating conditions. Tuning parameters  $a_1$  and  $a_2$  that tunes the fan air flow rate and speed are used for determining the fan duct static and electric demand. Parameters ( $b_i$ ) are only for duct static pressure calculations and parameters ( $c_i$ ) are only for fan energy use calculations.

As in the cooling coil model, the dynamic behavior of the fan is modeled by filtering the steady state duct static pressure and using one single time constant ( $\tau_f$ ):

$$P_{S_i} = \left( 1 - e^{-\frac{\Delta t_{sim}}{\tau_f}} \right) \cdot P_{S^{ss}} + e^{-\frac{\Delta t_{sim}}{\tau_f}} \cdot P_{S_{i-1}} \quad (24)$$

For each simulation time step of 1 min, the tuning parameter  $b_4$ , which is restricted in the range [0, 1], can replace the term ( $e^{-\Delta t/\tau_f}$ ).

Fan electric demand is determined as follows:

$$\dot{W}_s = \frac{\left( \dot{V}_{fan} + a_1 \cdot \dot{V}_{fan,des} \right) \cdot \left( P_{S_{fan}} + c_1 \cdot P_{S_{fan,des}} \right)}{\eta_f + \eta_{f,des} \cdot c_2} \quad (25)$$

Fan power ( $\dot{W}_t$ ) is determined using the shaft power ( $\dot{W}_s$ ) and the motor efficiency ( $\eta_m$ ):

$$\dot{W}_t = \frac{\dot{W}_s}{\eta_m} \quad (26)$$

Figure 9 shows the dynamic DDGB fan model. When the parameters ( $a_1$ ,  $a_2$ ,  $c_1$ , and  $c_2$ ) are set to zero, the fan electric demand, calculated by the proposed model, equals the value calculated by Clark's model. There are six tuning parameters ( $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$ ,  $b_3$ , and  $b_4$ ) for duct static pressure calculations. These parameters are tuned by GA<sub>2</sub> in order to minimize the error between measured and calculated duct static pressures for previous sample data. In addition, the parameters ( $a_1$ ,  $a_2$ ,  $c_1$ , and  $c_2$ ) are tuned to minimize the error between measured and calculated fan electric demand.

The validation is made for one full day ( $i=1, \dots, 1440$ ) using only one measured point ( $i=1$ ) as initial value. The duct static pressure obtained from measured data is compared with predictions of the physical model and the proposed dynamic DDGB model (Figure 10). The accuracy of proposed model is significantly higher than that of the physical model, as the error decreases from 6.8% to 1.3% by using DDGB model, when compared with measured data for investigated period (three summer months).

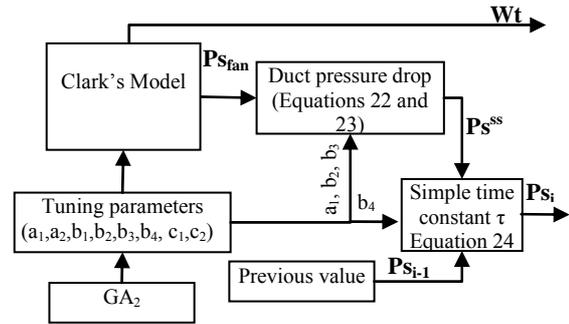


Figure 9. Dynamic DDGB fan model

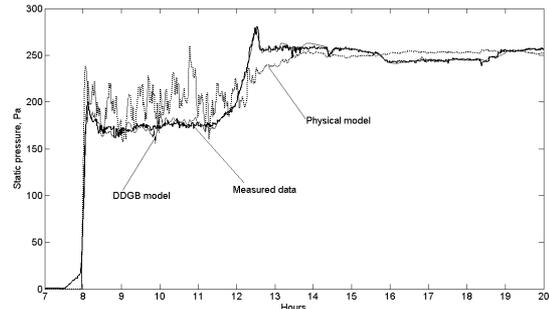


Figure 10. Measured duct static pressure compared with predictions of the physical model and the proposed dynamic DDGB model

## RESULT DISCUSSIONS

Proposed DDGB component models of the zone temperature, cooling coil, and fan (Figures 5, 7, and 9) are validated against measured data of three summer months (June, July and August 2003). Using a sample of measured data of two previous days ( $S=2880$ ), the tuning parameters of models are determined by genetic algorithm GA<sub>2</sub>. Validation results using two statistical indicators, the coefficient of variance (CV) and the maximum error (ME), between the predicted values and measured data are presented in Table 1.

DDGB models with tuning parameters produce higher accuracy compared to physical models without tuning parameters (pure physical model). The advantages of proposed DDGB models are summarized as follows:

1. The accuracy of models increases significantly by using data-driven physical model (DDGB)

than that for physical model; for instance, the coefficient of variance (CV) of the cooling coil model decreases from 14% to 2.6% and the maximum error decreases from 3.3°C to 0.5°C.

2. Proposed DDGB models maintain the structure of physical models. This is an important issue that gives advantage to the use of DDGB models against pure data-driven models such as neural networks, especially in the case of inaccurate results due to effects of dominating local training.
3. Proposed DDGB models do not required training efforts such as for pure data-driven model (e.g. neural networks).
4. Proposed DDGB models can also be used for fault detection. By comparing measured data with predictions of proposed DDGB models, it is possible to detect faults in the operation of HVAC components.

As a result, the DDGB models proposed in this paper could provide higher reliability and less training efforts than pure data-driven models, and higher accuracy than pure physical models.

*Table 1  
Validation results of proposed and physical models.*

	ZONE MODEL		COOLING COIL MODEL		FAN MODEL	
	DDGB	Phy.	DDGB	Phy.	DDGB	Phy.
ME	0.35 °C	2 °C	0.5 °C	3.3 °C	13 Pa	80 Pa
CV (%)	2.3	8	2.6	14	2.2	10

## CONCLUSION

Dynamic data-driven models based on gray-box approach, called DDGB models, are proposed. Model parameters are tuned by using genetic algorithm in order to minimize the errors between measured and estimated performance data. Dynamic models such as the zone temperature model, the cooling coil model, and the fan model are developed and validated against measured data from an existing HVAC system. The validation is made using measured data during three summer months (June, July and August). The results showed that the accuracy of proposed DDGB models is significantly better than that of pure physical models. The coefficient of variance (CV) is between 2.2% and 2.6%, when the predictions of DDGB models are compared with measured data, whereas the CV is between 8% and 14% in the case of pure physical models.

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