eSim 2018
«BUILDING SIMULATION TO SUPPORT BUILDING SUSTAINABILITY»
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A pathway for the derivation of control-oriented models for radiant floor heating applications

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Abstract: *Radiant floors have significant potential for load management in buildings on account of their substantial energy storage capacity. However, the long time constants associated with these systems make their control more challenging. Predictive control can be used to take into consideration the time delay of the system and ensure that thermal comfort is maintained. To implement effective and practical predictive control strategies, simple yet accurate models are needed. This paper focuses on the development of a detailed 3-D model of a radiant floor heating slab. The model is validated with data collected from a concrete slab in an experimental facility under controlled conditions. The experimental results and the detailed model are then used in the derivation of a control-oriented, low-order 1-D model. Parametric studies based on the detailed model will provide guidelines for the rapid generation of low-order models of radiant floor systems.*

Keywords: *radiant floor heating, control-oriented modelling, energy storage*

INTRODUCTION

Radiant floors are an effective option to provide heating to a room while maintaining or improving thermal comfort (Olesen 2002). Although these systems offer distinct advantages, there is room for improvement regarding design approaches and control practices, particularly when radiant floors are used as thermally active building systems (TABS). The term TABS designates building components that can be used to store and release thermal energy, and thus enable peak load shifting (Gwerder et al. 2008) and predictive control (Candanedo et al., 2011). Predictive control requires models with an appropriate level of complexity. These models must be accurate enough to provide reliable information, yet flexible and nimble enough for rapid decision-making (Joe et al. 2016).

As part of a larger project on the interaction between radiant floor heating and displacement ventilation, this paper investigates the development of control-oriented models for radiant floor heating based on a detailed model carried out in COMSOL™, a finite-element multi-physics simulation tool (Comsol 2016). The outputs of the detailed COMSOL model are then compared with experimental results. Experimental data –along with the information obtained from the detailed model– is used to calibrate a low-order resistance-capacitance (RC) circuit thermal model. This study will provide the basis for a parametric study targeting the development of simplified control-oriented models from basic geometric parameters.

Recent studies have used COMSOL as a tool to study the performance of radiant floor and ventilation in a new terminal unit (Hernández et al. 2016), and in the study of the dynamic performance of radiant slabs (Wang et al. 2014). The main focus of the current study is the fine-tuning of low-order control-oriented models.

METHODOLOGY

The methodology used in this study can be summarized in the following steps. First, a step input response test was carried out in an experimental facility under a controlled environment. Then, a detailed model (attempting a reasonable reproduction of the experimental conditions) was created in COMSOL. The results of the experiment were then used to validate the results of the detailed COMSOL model. Finally, a 2nd order RC model was created to represent the dynamic behaviour of the radiant floor and calibrated to match the experimental results.

Experimental setup

The experiment was carried out with a perimeter zone test hut (PTH). The PTH was built inside the *Paul Fazio Solar Simulator/Environmental Chamber* (SSEC) facility at Concordia University (Figure 1). The temperature of the SSEC can be adjusted within a wide range of temperatures (between -40 °C and +50 °C), thus allowing a great deal of flexibility in terms of testing conditions.

ISBN 978-2-921145-88-6

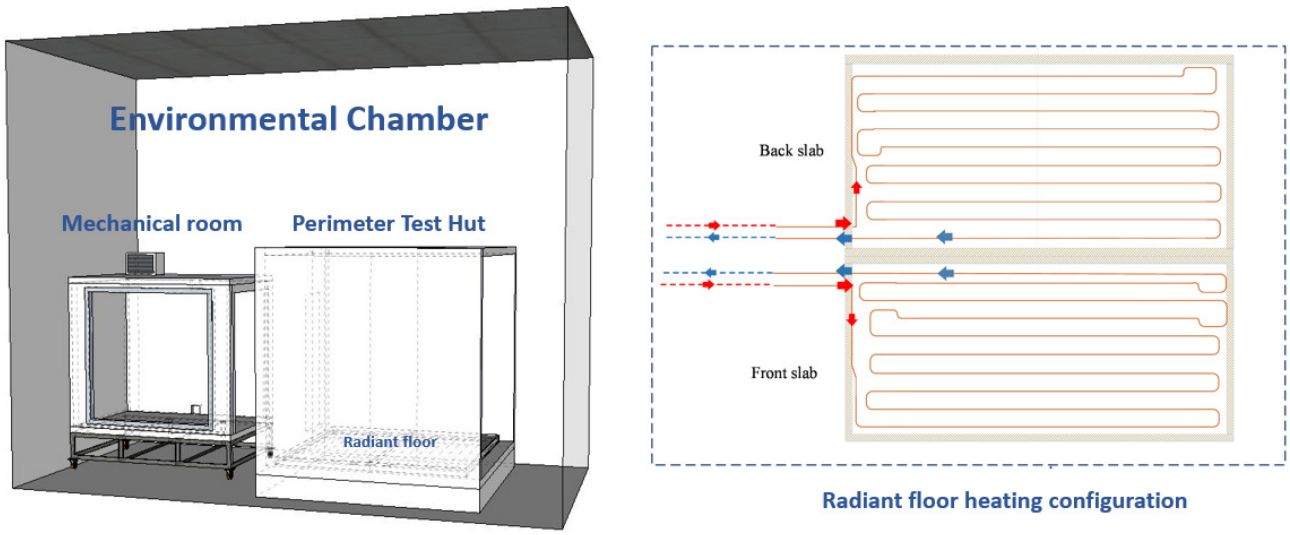

Figure 2. Environmental chamber with test hut (left); detail of radiant floor heating (right). Image courtesy of Jiwu Rao.

Figure 1. Front and back sections of concrete slab.

The perimeter test hut, which intends to mimic an office space near a window, is a room with a nearly cubic shape (about 3 m x 3 m x 3 m). The front façade of the test hut was open at the time of the experiment; the temperature of the air inside the test hut was thus essentially the same as the whole environmental chamber. The other three walls have a plywood inner finishing. The floor of the PTH is divided into two nearly identical sections, "front" and "back" (Figure 2 and Figure 1). Both sections contain radiant floor piping which can be controlled independently to simulate the conditions at different distances from a window. The radiant floor system pipes are made of conventional cross-linked polyethylene (PEX). These pipes a have an internal diameter of ½" (12.7 mm) and a wall thickness of 1/16" (1.59 mm). The pipes are installed in a "foam matrix" of insulating material which facilities their installation and keeps them in place. The pipes have an approximate separation of 15 cm. An 8-cm layer of concrete was poured within a wooden frame on top of the pipes (Figure 4).

Figure 3. Insulation foam matrix

Figure 4. Pouring of concrete slabs.

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Experimental procedure

The experiment carried out in the environmental chamber can be summarized as follows:

- The air temperature of the environmental chamber (and the test hut) was set to about -5 °C. A period of 24 h was allowed in order to approach steady state conditions for the room and the floor. It was assumed that steady state conditions were achieved when the concrete slab temperatures remained almost constant. This rather low temperature range below zero was used because this exercise was part of the preliminary steps of a study on an ice-melting application for highways.
- Seven T-type thermocouples, with an accuracy of 0.5 °C, were installed on the surface of each of the slabs.
- At the initial time ($t = 0$), flow to the radiant floor heating system was activated. An ethylene glycol water solution (50% / 50%) was used. A flow rate of approximately 0.4 gpm in the front slab and 0.5 gpm in the back slab (Figure 5) was supplied. This difference is due to some minor glitches with the fine-tuning of the flow rate, although the flow measurements were measured with an ultrasonic flow meter having an accuracy of ±2% for a 100:1 turndown ratio.

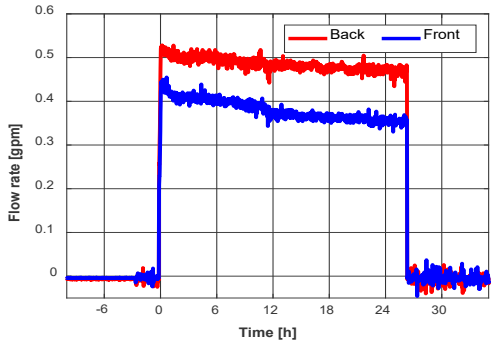

Figure 5. Flow rates into both slabs.

- The supply temperature into the slabs was kept between 30 and 35 °C (Figure 6).

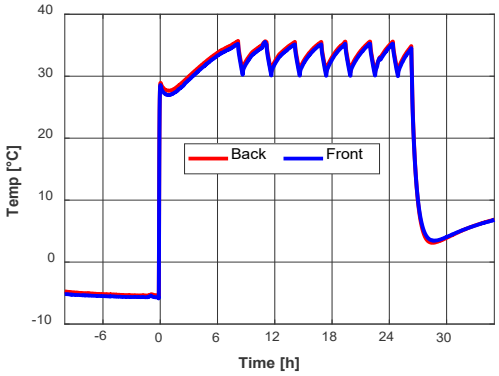

Figure 6. Supply temperatures into the slabs.

- The ambient air inside the perimeter test hut followed the curve show in Figure 7. The "glitches" correspond to defrost cycles in the cooling system.

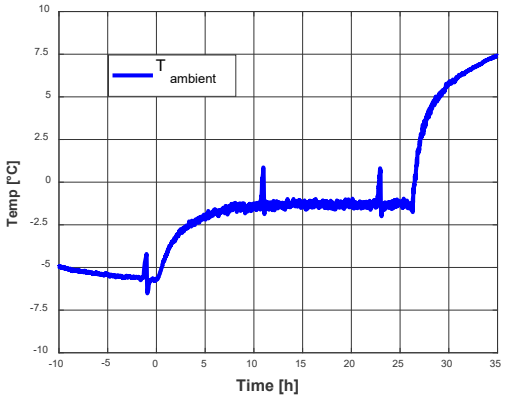

Figure 7. Air temperature measurements inside the PTH.

- Figure 8 shows the average surface temperature of the slabs, as measured by a set of seven thermocouples distributed in the surface of each slab.

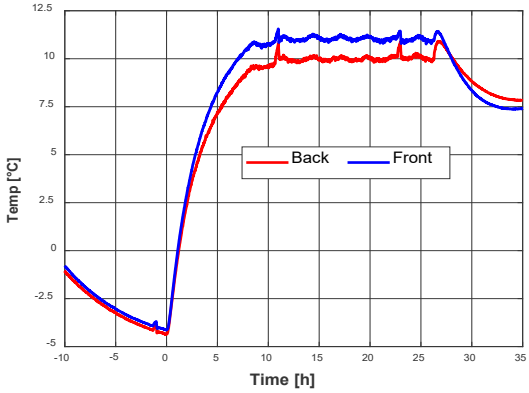

Figure 8. Experimental average surface temperatures.

COMSOL™ Model

A detailed model of the two slabs was built in COMSOL™, an environment for multi-physics simulations, i.e. simulations treating multiple physical phenomena that occur simultaneously. This study attempted to approximate the geometry of the foam matrix underneath the concrete slabs and the piping installed within its grooves (Figure 9).

The COMSOL model was used to determine the evolution of the temperature field over time within the concrete slab over time using a finite element scheme. The heat transfer conduction analysis focused only on the concrete section of each slab. The solution was found for time steps of 300 seconds (5 min) for the time period from t = -4 h to t = 44 h, using a MUMPS solver, a solver for finite element applications (Amestoy, Duff, and L'excellent 2000).

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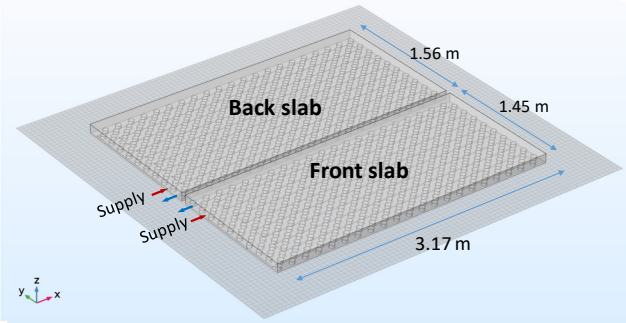

Figure 9. Perspective view of the COMSOL model

The foam base and its "bumps" were modelled with a rectangular block with an array of cylindrical objects on top of it (Figure 10). The cylinders had a diameter of 6.25 cm and were about 3 cm high. As mentioned before, the thickness of the concrete slab poured on top was 8 cm. A geometric "subtraction" operation was then performed in COMSOL, so that each of the slabs had a set of cylindrical holes underneath it.

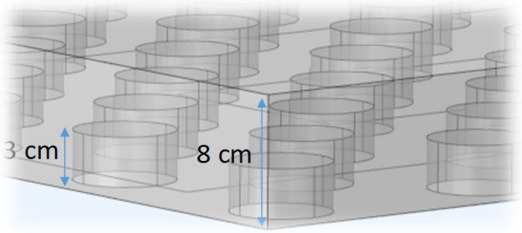

Figure 10. Cylindrical "bumps" in the model.

A relatively "coarse" mesh was used, with nearly 72,000 elements in it, with an average volume 9.8 cm^3.

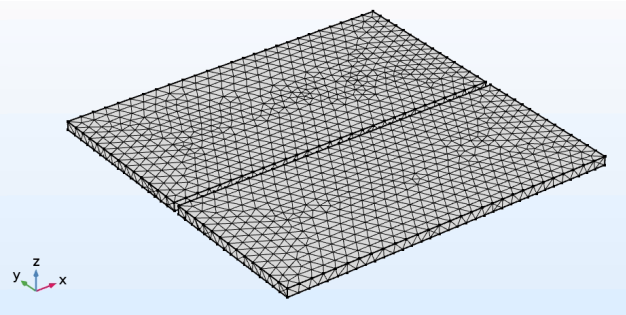

Figure 11. Meshing of the domain.

COMSOL

Assumptions used in the detailed model include:

- The properties of a 50/50 water-glycol were used.
- The bottom surfaces of both slabs are assumed to be perfectly connected to the insulation.
- The bottom of the insulation is assumed to be exposed to the temperature of the room. This assumption is made because there is an air space underneath the insulation (the test hut is a mobile room on wheels).
- It is assumed that these bottom surfaces lose heat to the air with an "effective" heat transfer coefficient of 0.5 Wm^{-2}K^{-1}.

$$q_{ins} = (0.50\ \mathrm{Wm^{-2}K^{-1}})(T_{bottom} - T_{room}) \quad (1)$$

  This value was chosen considering an R-10 insulation, and neglecting the heat transfer coefficient between the insulation and the air.
- The lateral surfaces facing outwards are assumed to be connected to the wooden frame, with no insulation. Again, it is assumed that this wooden frame is exposed to the room air temperature. The boundary condition is:

$$q_{wood,sides} = (8\ \mathrm{Wm^{-2}K^{-1}})(T_{lateral\ surfaces} - T_{room}) \quad (2)$$

  This heat transfer coefficient is calculated based on a typical thermal conductivity of wood.
- The lateral surfaces facing inwards (i.e., the small gap between the slabs connected to the wooden frame) is assumed to be adiabatic.
- A uniform initial temperature of -5 °C was assumed for the entire domain of both concrete slabs.
- COMSOL employs by default the correlation by Lloyd and Moran (1974), as cited by Incropera and DeWitt (2002), to evaluate natural convection between the floor surface and the air.

$$Nu = \begin{cases} 0.54Ra^{1/4} & \text{if } 10^4 \le Ra \le 10^7 \\ 0.15Ra^{1/3} & \text{if } 10^7 \le Ra \le 10^{11} \end{cases} \quad (3)$$

  The Rayleigh number was calculated using the length scale proposed by Goldstein et al. (1973), where $L = A/P$.
- Radiative heat transfer is assumed for the top surface of the concrete slabs, assuming grey-body radiation from a diffuse surface with an emissivity of 0.95, and an ambient temperature ("sky temperature") equal to the room air temperature shown in Figure 7.
- For the purpose of conduction heat transfer in the slabs, the PEX pipes are treated as linear heat sources.
- The simulation flow rates in each slab are based on the measured flow rates, as shown in Figure 5.
- The temperature at the inlets used in the simulation are the measured inlet temperatures (Figure 6).
- The properties used for the concrete are as follows:
  - ρ = 2010 kg/m^3,
  - c_p = 800 J/(kg·K);
  - and k = 1.7 W/(m·K).

Results of Detailed Model

Figure 12 and Figure 13 show the average temperature at the surfaces of the back and front slabs respectively. The temperatures predicted by the model for the back slab are higher (by about 1 °C) than the ones measured in the

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experiment. Given that the flow rate is higher in the back slab, it would be reasonable to expect higher temperatures there. The discrepancy might be due to the location of the sensors near the slab surface, which may not be representative of the average temperature. The temperatures predicted for the front slab match quite well the experimental results. The simulation predicts similar temperatures for both slabs.

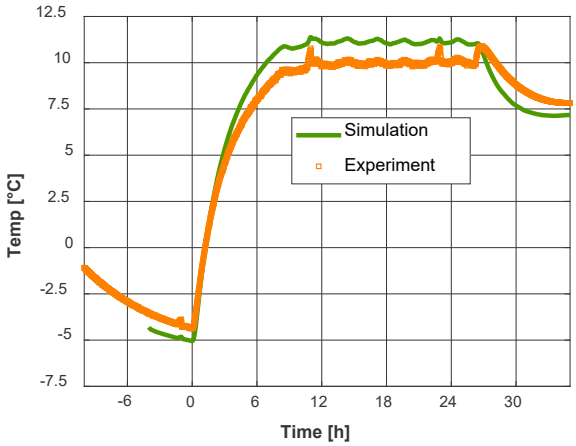

Figure 12. Average surface temps (back slab).

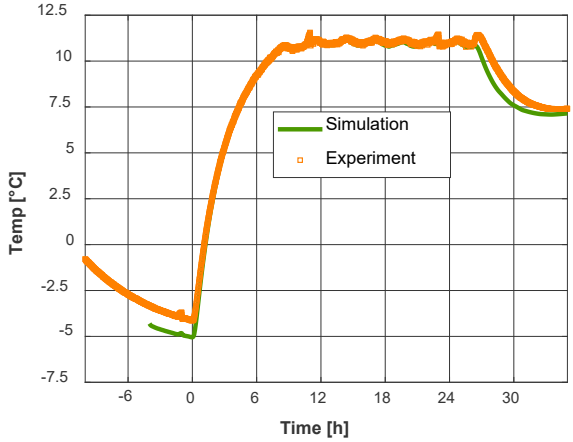

Figure 13. Average surface temps (front slab).

The results of the temperature distribution at the surface of the slab (Figure 14) are quite informative. In both slabs, the temperatures at $t = 23.75$ h are between 10 and 14 °C in the central area of the concrete slabs. The path of the fluid can nevertheless be clearly seen, and it is clear that the concrete temperature is lower near the edges of the slabs and near the points where the fluid exits the slab.

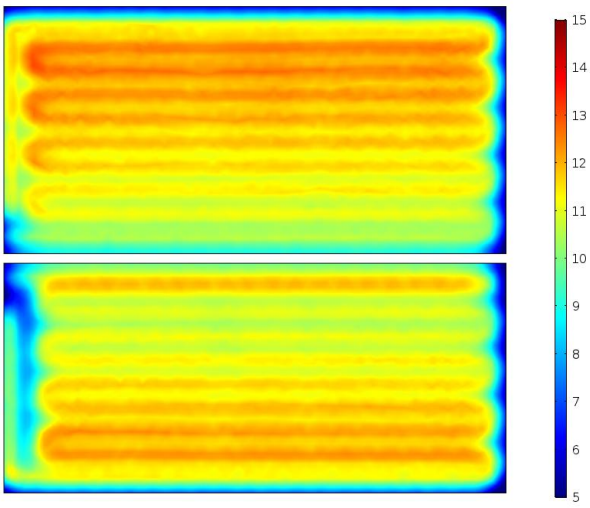

Figure 14. Surface temp. distribution (°C), t = 23.75 h.

Histograms from simulation results are also quite informative. About 90% of the back slab surface is between 10 and 13 °C (Figure 15).

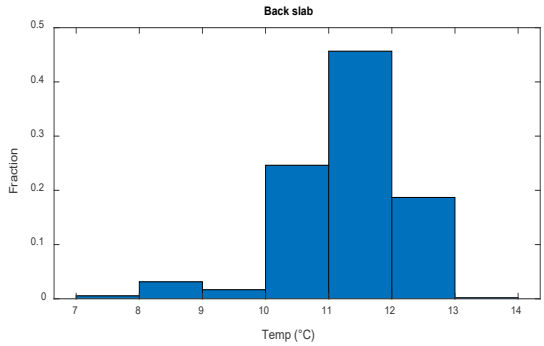

Figure 15. Surface temperatures (back slab), t = 23.75 h.

Regarding the front slab, about 75% of temperatures are between 10 and 12 °C (Figure 16).

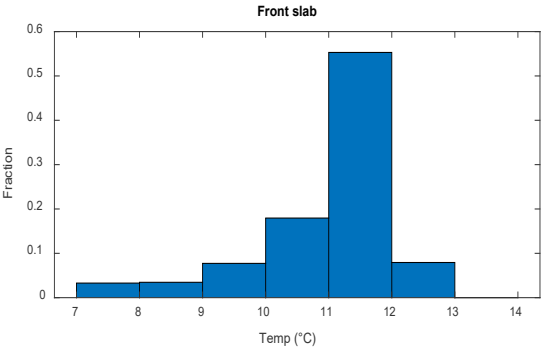

Figure 16. Surface temperatures (front slab), t = 23.75 h.

Further insight can be obtained by analyzing the heat transfer rate from the pipes to the slab and then comparing it with the heat transfer from the floor surface to the room.

Figure 17 shows the heat transfer rate from the pipes to the concrete. Its peak value occurs at $t = 0$ (i.e., the moment

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when the temperature differences between the water and the concrete are the largest). The heat transfer rate declines steadily until about t = 8 h. At this point, the fluctuations in the supply water temperatures cause similar oscillations in the heat transfer rates.

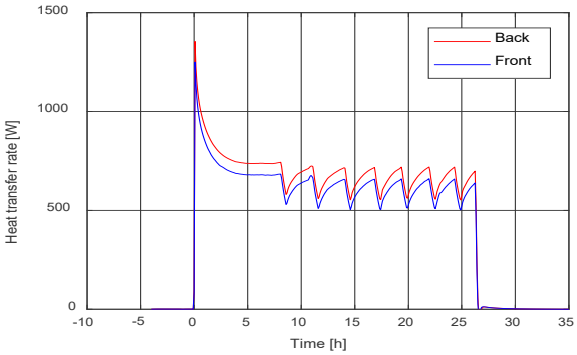

Figure 17. Heat transfer rates from pipes to slabs.

Figure 18 shows the heat flux from the surface of both slabs to the room. As expected, there is a delay of several hours when compared to the heat transfer rate from the pipes to the concret.

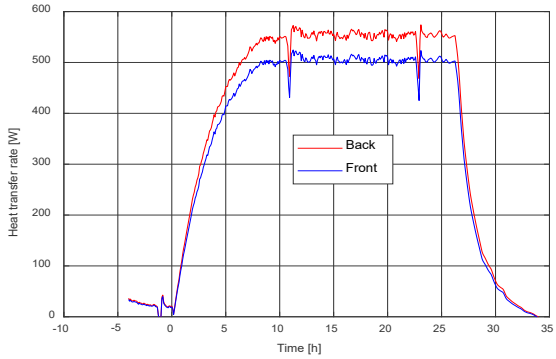

Figure 18. Heat flux from the floor to the room.

Simplified RC Model

A second order (5R2C), one-dimensional model was developed to represent each of the slabs (Figure 19). The concrete slab was divided in two identical blocks, and only 1-D heat conduction was considered (i.e., no edge effects). The room air temperature and the heat input from the pipe (T_{air} and Q_{aux}), obtained from the experimental results, were used as inputs for the simulation. As before, it is assumed that the bottom of the insulation is exposed to the room air. A time step of 60 seconds (Δt = 60 s) was used.

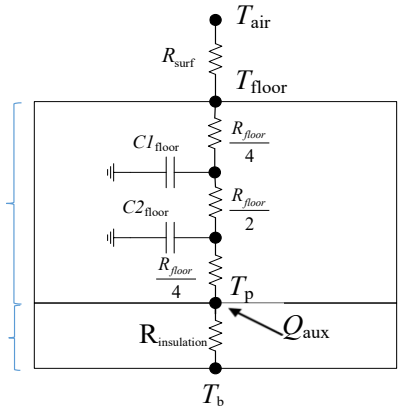

Figure 19. Second order model of radiant floor.

The simulation was carried out by applying a simple fully explicit finite difference scheme, as shown in the following equations.

$$T_{floor}^{i+1} = \frac{\frac{T_{air}^{i}}{R_{surf}} + \frac{T_{C1}^{i}}{0.25R_{conc}}}{\frac{1}{R_{surf}} + \frac{1}{0.25R_{conc}}} \quad (4)$$

$$T_{C1}^{i+1} = \frac{\Delta t}{C_1}\left(\frac{T_{floor}^{i} - T_{C1}^{i}}{0.25R_{conc}} + \frac{T_{C2}^{i} - T_{C1}^{i}}{0.50R_{conc}}\right) + T_{C1}^{i} \quad (5)$$

$$T_{C2}^{i+1} = \frac{\Delta t}{C_2}\left(\frac{T_{C1}^{i} - T_{C2}^{i}}{0.50R_{conc}} + \frac{T_{p}^{i} - T_{C2}^{i}}{0.25R_{conc}}\right) + T_{C2}^{i} \quad (6)$$

$$T_{p}^{i+1} = \frac{Q_{aux}^{i} + \frac{T_{C2}^{i}}{0.25R_{conc}} + \frac{T_{air}^{i}}{R_{ins}}}{\frac{1}{0.25R_{conc}} + \frac{1}{R_{ins}}} \quad (7)$$

Given the complex geometry of the foam base, the following simplifying assumption was made: the actual volume of the concrete slab (volume of the "block" minus the volume of the foam cylinders and the pipes) was divided by the area exposed to the surface to obtain an "equivalent" thickness

$$L_{eq} = \frac{V_{conc}}{A_{slab}} = \frac{V_{block} - V_{cyl} - V_{pipes}}{A_{slab}} \quad (8)$$

In this case, the equivalent thickness was 7.31 cm. It was further assumed that this thickness is divided in two layers with identical thermal capacitance. The properties of the concrete used in the RC model were the same as described above for the COMSOL model.

For one slab, the RC parameters are as follows (Table 1):

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Table 1. RC parameter values

Parameter	Value
C_1	265.9×10^3 J/K
C_2	265.9×10^3 J/K
R_{conc}	9.42×10^{-3} K/W
R_{ins}	0.386 K/W
R_{surf}	$1/(h_f * A_{slab})$

RC model calibration

The resistance between the air in the room and the surface of the slab depends on the film heat transfer coefficient employed, which represents the combined effect of convection and radiation. The following value, based on a trial and error calibration, provided the best results:

$$h_f = 9.5 \text{ W/(m}^2\text{K)} \quad (9)$$

Figure 20 compares the experimental data with the results of the COMSOL model, and the results of the RC network with three different heat transfer coefficients.

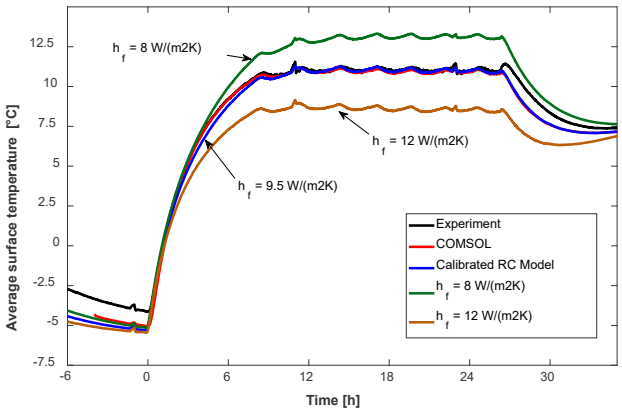

Figure 20. Comparison of experiment, COMSOL model and calibrated RC model.

A value of 8 W/(m^{2}K) overestimates the surface temperature; 12 W/(m^{2}K) underestimates it. A calibrated value using the experimental results (9.5) gives a good match. While a calibration is likely to be necessary, the study by Cholewa et al. (2013) provides guidelines for the combined film coefficients in radiant floor system.

The results of both the COMSOL detailed model and the calibrated RC network match quite well the experimental data. However, a detailed model has the advantages of providing good *a priori* information. The results of the RC model are very sensitive to the film heat transfer coefficient.

CONCLUSION

This study has shown the potential of a detailed simulation to investigate the influence of parameters such as pipe spacing, pipe depth, concrete thickness and different geometries on control-oriented models. Moreover, detailed simulations enable to conduct studies in unconventional setups. Our research team intends to investigate:

- Effect of geometric and material parameters.
- Effect of displacement ventilation
- Effect of solar gains.
- Different heat transfer rates in both halves, or in as many sections as required.

A detailed simulation also provides valuable information on the temperature distribution in the floor, and makes it possible to study small effects at the edge of the domains.

On the other hand, this investigation also shows that a very simple 2nd order finite difference model, while less informative in general, can be quite adequate for the purpose of calculating a particular variable of interest (in this case, the temperature of the floor surface). The information obtained by performing parametric runs with a detailed model will enable the rapid derivation of reasonably accurate low-order models. This being said, a low-order model will almost always require some calibration, especially considering the complexity of heat transfer at the surface of the floor. A low-order RC model is also useful since it provides significant insight in the physics of the problem (time constants, delays, rule-based control, etc.). This RC model could be readily incorporated into advanced control algorithms, either in its finite difference formulation or a state-space equivalent.

Finally, this study illustrates the fact that creating a model requires keeping in mind the specific purpose of the model (design, simulation, parametric studies, control and operation, etc.).

ACKNOWLEDGEMENTS

The authors would like to thank the Office of Energy Research and Development of NRCan for the financial support of this project (PERD-BE1-01). We would like to thank our colleagues Michel Tardif, Kamel Haddad and Sébastien Brideau for their inputs. The authors would like to thank Jiwu Rao, from Concordia University, for technical clarifications and for his detailed drawing of the test hut. The comments of Justin Tamasauskas and Étienne Saloux are gratefully acknowledged.

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