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Applying multi-attribute decision making to identify robust multi-objective design solutions in building design application

Yin Li[1], Bruno Lee[2]

1 Department of Building, Civil & Environmental Engineering, Concordia University, Montreal, Canada
2 Department of Building, Civil & Environmental Engineering, Concordia University, Montreal, Canada

Abstract: *The building design industry has become geared towards performance-based design that considers multiple design aspects. Consequently, multi-objective optimization plays an important role in modern building design. However, results from multi-objective optimization usually offer too many solutions which makes the decision-making process impractical. The purpose of this paper is to identify a multi-attribute decision making (MADM) method to assist the process of building design application. Different MADM methods are compared by applying them to the results of a multi-objective optimization, which is based on a case study of an industrial building in Amsterdam. The comparison is summarized according to the advantages and limitations of the different methods and concluded with a method that yields the most robust solutions.*

Keywords: *Building performance simulation, Multi-objective optimization, Robust decision making, Design support*

INTRODUCTION

The construction industry consumes 40% of the total energy used in North America which presents a huge potential for energy saving and the reduction of carbon dioxide emission. Net-Zero Energy Buildings can reduce operational costs, save the power back to grid, and put less stress on the environment compared to more traditional buildings. The advantages of Net-Zero Energy Buildings attract great interest from policymakers and industry. In 2015, there were a total of 191 certified net zero energy construction projects across the United States, and by 2016 this number soared to 332. To achieve the goal of Net-Zero Energy Building design, many researchers have developed different methods to integrate the simulation engines and optimization algorithms. Nguyen (2014) conducted a comprehensive literature review about building performance simulation-based optimization.

Depends on the number of optimization objectives, the literature could be divided into single objective optimization or multi objective optimization. Multi objective optimization is more relevant to reality than single objective optimization since the designers often must make decisions to satisfy conflicting design objectives simultaneously. However, multi objective optimizations often yield more solutions. It is difficult for decision makers to pick one compromising solution from optimization results. Other than making decision based on expert's experience or decision maker's preference, researches have conducted about applying multi attribute decision making in building design. Delgarm (2016) combined the optimization of building energy performance with the Simple Additive Weighted Method (SAW) in the early phase of building design. Delgarm (2016) applied Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to select the optimum solution on the pareto front with two objectives: building energy efficiency and creating indoor thermal comfort. Mela (2012) applied six methods to the multi objective optimization to building design and discussed their performance. These methods which included what are most applied to decision making like the weighted sum method, the weighted product method, TOPSIS. All these methods aggregate multi objectives to a single objective by way of utility function. The expression or modeling of preferences by means of aggregate functions is a broad field of study called "Multi-attribute decision making" (Emmerich 2016). The process includes three steps: determination of weight, normalization, aggregation.

There are various methods to be applied in each step. For the step of aggregation, Mela (2012) discussed the performance of different aggregation methods. For the step of normalization, Chakraborty (2007) discussed the performance of different normalization methods for the weighted sum method. Chakraborty (2009) also evaluated four normalization methods for the TOPSIS method and suggested that vector normalization generated the most consistent results for a case study of the deposit banking

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market. Although the normalization and aggregation methods have been discussed separately before, there is no research has considered their combined effect. Also, few comparison researches have been done in the building design field.

This paper first selects the three most used multi attribute decision making methods, as well as four normalization methods. The different combination of these two steps are applied to a multi objective optimization of an industrial hall design. Kendall's tau coefficient and Spearman's rank correlation coefficient are calculated for the results of each of the two combinations. The performance and applicability of these methods are also discussed later in this paper.

METHODOLOGY

Multi attribute decision making

Multi attribute decision making is a well-developed field. The main two families within the multi attribute decision making methods are the Multi attribute utility theory and the outranking methods. When the attribute and the corresponding weighting factors are determined the multi attribute utility theory should be applied. In this paper three methods were selected to be discussed.

Simple Additive Weighted (SAW)

The Simple Additive Weighted (SAW) or Weighted Sum method is the most simple and used method. Generally, this method first determines the attributes and weighting factors. It is common that different attributes are measured by different units. The performance value of different attributes should be transformed into a compatible form by normalization. The final decision should be made by the rank of the linear aggregation of different weighted normalization values. The steps are:

Step 1: Determine the weighting factor for different attributes which can be defined as a vector;

$$\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T, \omega_i \epsilon [0,1], \sum_{i=1}^{n} \omega_i = 1; \quad (1)$$

Where the ω is the weighting factor

Step 2: Normalize the attribute rating value;

Step 3: Determine the linear weighted aggregation;

$$U_i = \sum_{i=1}^{n} \omega_i f_i \quad (2)$$

Step 4: Make the final decision based on the ranking of the aggregation value.

SAW is easy for decision makers to understand and apply because of its simple idea and calculation. However, there are some limitations to use this method which will be discussed in the last part of this paper.

Weighted product (WP)

The weighted product method is like SAW but different in the final step. In stead of using the linear aggregation value as ranking criteria, WP uses the following equation.

$$U_k = \prod_{i=1}^{n} {f_{ik}}^{\omega_i} \quad (3)$$

The purpose of this method is to heavily penalize poorer attribute values.

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

This method is a geometric approach which is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS). The positive and negative ideal solutions are the best and worst value of all attributes for all corresponding alternatives. The distance from alternative to PIS and NIS are compared to determine the priority. The condition of the final decision supposes to be closest to the PIS and furthest to NIS. However, during analysis an alternative might be closest to PIS but not furthest to NIS. A similarity function which integrates the two indicators is often applied to solve this problem. The steps are:

Step 1: Determine the weighting factor for different attributes which can be defined as a vector using equation (1);

Step 2: Normalize the attribute value;

Step 3: Determine the linear weighted aggregation.

$$v_{ij} = \sum_{i=1}^{n} \omega_j f_{ij} \quad (4)$$

Step 4: Determine the PIS and NIS;

$$A^+ = \left\{(\max V_{ij} \mid i \epsilon I), (\min V_{ij} \mid i \epsilon I') \mid j = 1,2,\dots,m\right\}$$
$$= \{v_1^+, v_2^+, v_1^+ \dots . v_1^+\} \quad (5)$$
$$A^- = \left\{(\max V_{ij} \mid i \epsilon I), (\min V_{ij} \mid i \epsilon I') \mid j = 1,2,\dots,m\right\}$$
$$= \{v_1^-, v_2^-, v_1^- \dots . v_1^-\} \quad (6)$$

Where "I" is the benefit attribute. "I-" is the cost attribute.

Step 5: Calculate the distance between the target alternative and the worst condition and the distance between the alternative and the best condition

$$S_i^+ = \sqrt{\sum_{i=1}^{n} (v_{ij} - v_j^+)^2} \quad i = 1,2,\dots . m \quad (7)$$

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$$S_i^- = \sqrt{\sum_{i=1}^{n}(v_{ij} - v_j^-)^2} \quad i = 1,2,\ldots..m \qquad (8)$$

where S_i^+ and S_i^- are L2-norm distances from the target alternative to the worst and best conditions respectively.

Step 6: Calculate the similarity to the worst condition:

$$U_i = \frac{S_i^-}{S_i^+ + S_i^-} \qquad (9)$$

Step 7: Rank the alternatives according to the results of step 6.

Normalization method

The MADM methods usually require a step of normalization to transform different attribute values to a compatible format in order to aggregate them. In this paper, four normalization methods were selected to discuss as shown in Table 1.

Table 1: Normalization methods

Normalization method	Equation
Max-Min method	$r_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}}$ (10)
Max method	$r_{ij} = \frac{x_{ij}}{x_j^{\max}}$ (11)
Vector method	$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$ (12)
Sum method	$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^2}$ (13)

Where

r_{ij} is the j[th] attribute value of i[th] design after normalization

x_{ij} is the j[th] attribute value of i[th] design before normalization

$x_j^{\max}$ is the maximum j[th] attribute value for all design before normalization

$x_j^{\min}$ is the minimum j[th] attribute value for all design before normalization

The attributes can be classified into monotonic and nonmonotonic types. The Monotonic attributes are either a benefit attribute or cost attribute. For benefit attributes the decision maker prefers to maximize the value: the greater the attribute value the more it is preferred, for example income and energy generation. For cost attributes the goal is to minimizes the attribute value, the smaller the attribute value the more it is preferred like initial investment and life cycle cost of a building. For nonmonotonic attributes the preferred value is the one closest to the ideal level such as comfortable room temperature or humidity. Cost attributes and benefit attribute can be interchangeable by taking inverse values or negative values. (Karami 2011, Emmerich 2006).

Comparison between different normalization and aggregation methods

Three MADM methods combined with four normalization methods make 12 combinations. Each of the combinations are applied to the result of multi objective optimization of building design in order to generate a compromising solution. To discuss the consistency between different combinations the Kendall's tau-b and Spearman's (rho) rank correlation coefficients are selected. The Spearman's (rho) rank correlation coefficients is calculated by equation (14).

$$\boldsymbol{\rho} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - y)^2}} \qquad (14)$$

The Kendall's Tau-b correlation coefficient is calculated by equation (15).

$$\tau_B = \frac{n_c - n_d}{\sqrt{(n_0 - n_1)(n_0 - n_2)}} \qquad (15)$$

Where $n_0 = n(n-1)/2$

$$n_1 = \sum_i t_i(t_i - 1)/2$$

$$n_2 = \sum_j u_j(u_j - 1)/2$$

$$n_c = Number\ of\ concordant\ pairs$$

$$n_d = Number\ of\ disconcordant\ pairs$$

$$t_i = Number\ of\ tied\ value\ in\ the\ i^{th}\ group\ of\ ties\ for\ the\ first\ quantity$$

$$t_i = Number\ of\ tied\ value\ in\ the\ j^{th}\ group\ of\ ties\ for\ the\ second\ quantity$$

The values of these two coefficients vary from -1 to 1. A positive correlation coefficient indicates a positive relationship between the two variables, while a negative correlation coefficient expresses a negative relationship. A correlation coefficient of 0 indicates that no relationship between the variables exists at all. A correlation coefficient of 1 or -1 indicates strong relationship between the variables.

Firstly, the results of one MADM method with four different normalization methods will be compared. This comparison is to used to evaluate the impact of selecting different normalization methods.

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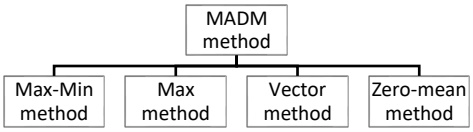

Figure 1: One MADM method with four different normalization methods

Secondly, the result of the three MADM with the same normalization methods are compared. Since all the data in this comparison are normalized with the same method, the uncertainties originating from different normalization methods can be avoided. The result of this type of comparison can reveal how different the final decision varies for different MADM methods.

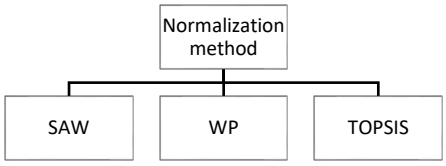

Figure 2: Comparison of different MADM methods with same normalization method

CASE STUDY

In this section, we consider the multi objective optimization of an industry hall as presented in Lee (2016). This is a warehouse in Amsterdam (logistics is one of the main Dutch industries) has a process load of 5W/m2 (arbitrary value based on energy benchmarks cited by Chartered Institution of Building Services Engineers). A rectangular shaped hall with an arbitrary dimension of 100 m(W) × 40 m (D) × 6 m (H) is adopted in this paper. Six design parameters have been identified as the most influential ones. They are the insulation values of the roof and walls, the construction type (steel or concrete) of the roof and walls, the skylight coverage and the transpired solar collector coverage. Table 2 presents the studied design parameters with their respective ranges and resolutions of the investigation. Steel sheets are assumed to have no thickness and no thermal resistance. Concrete is assumed to have a thickness of 0.2 m with a density of 2400 kg/m3, a thermal conductivity of 2.1W/m K, and a thermal capacity of 1 kJ/kg K. For newly built industrial halls, steel and concrete constructions can be considered as quite airtight with infiltration mainly comes as a result of opening doors, which is more of an operation issue. A constant infiltration rate of 0.2 ACH is assumed. More details of the energy model are outlined by Lee (2014).

Table 2: List of design parameters

Parameters	Design range	Levels of investigation
Insulation (thermal resistance, roof)	1.5–4.5 m^2 K/W	7
Insulation (thermal resistance, wall)	1.5–4.5 m2 K/W	7
Construction types (roof)	Steel or concrete	2
Construction types (wall)	Steel or concrete	2
Skylights (as % of roof area	0–15%	4
Transpired solar collector (as % of south wall)	0–100%	6

The building energy simulation program TRNSYS was used to perform the energy analysis for heating and cooling demand. The energy analysis process was integrated in multi objectives optimization with two objectives: annual energy consumption and net carbon emission. The optimization which is conducted by MODEFRONTIER resulted in 32 alternatives which are depicted in Figure 3.

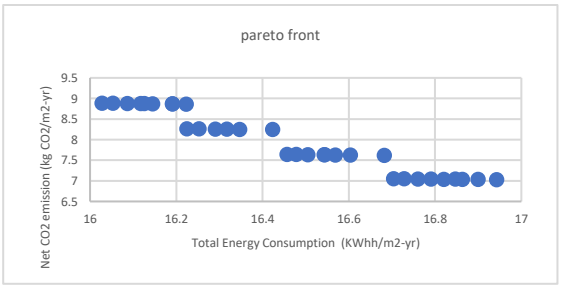

Figure 3: Results of multi objectives optimization

Comparison of methods

Comparison of normalization methods

(1) Different normalization methods for SAW method

It can be observed from Table 3 that for both coefficients the results of different normalization methods are highly related. It can also be observed from Figure 4 that the other methods are totally consistent with each other except the Max Min method.

Table 3: Comparison of normalization methods for SAW Method

		MAX Min	Vector	Max	Sum
Kendall's tau_b	MAX-Min	1.000	0.749	0.749	0.749
	Vector	0.749	1.000	1.000	1.000
	Max	0.749	1.000	1.000	1.000

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	Sum	0.749	1.000	1.000	1.000
Spearman's rho	MAX-Min	1.000	0.872	0.872	0.872
	Vector	0.872	1.000	1.000	1.000
	Max	0.872	1.000	1.000	1.000
	Sum	0.872	1.000	1.000	1.000

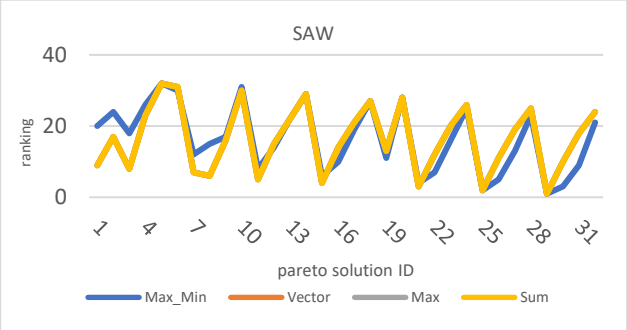

Figure 4: Comparison of the rank of different normalization for SAW Method

(2) Different normalization methods for the TOPSIS Method

It can be observed from Table 4 that for both coefficients the results of different normalization methods applied in TOPSIS are similar but less consistent compared with SAW. It can also be observed from Figure 5 that the other methods are basically consistent with each other except the Max Min method.

Table 4: Comparison of normalization methods for TOPSIS Method

		MAX Min	Vector	Max	Sum
Kendall's tau_b	MAX-Min	1.000	0.652	0.668	0.648
	Vector	0.652	1.000	0.984	0.996
	Max	0.668	0.984	1.000	0.980
	Sum	0.648	0.996	0.980	1.000
Spearman's rho	MAX-Min	1.000	0.787	0.801	0.785
	Vector	0.787	1.000	0.997	1.000
	Max	0.801	0.997	1.000	0.996
	Sum	0.785	1.000	0.996	1.000

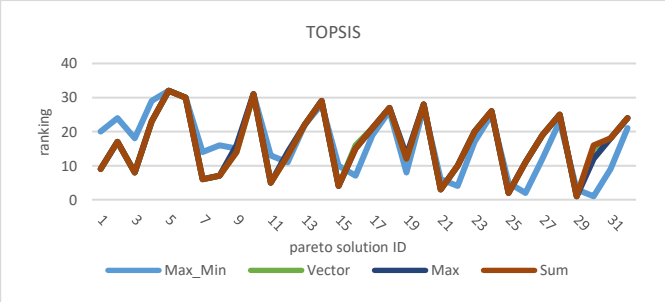

Figure 5: Comparison of the rank of different normalization for TOPSIS Method

(3) Different normalization methods for the WP Method

It can be observed from Table 5 that for both coefficients the results of the Vector, Max and Sum method are consistent with each other. The Max Min method is not consistent with the other methods. It can also be observed from Figure 6 that the ranking results of other methods are almost the same between each other except the Max Min method.

Table 5: Comparison of normalization methods for WP Method

		MAX Min	Vector	Max	Sum
Kendall's tau_b	MAX-Min	1.000	0.339	0.339	0.339
	Vector	0.339	1.000	1.000	1.000
	Max	0.339	1.000	1.000	1.000
	Sum	0.339	1.000	1.000	1.000
Spearman's rho	MAX-Min	1.000	0.470	0.470	0.470
	Vector	0.470	1.000	1.000	1.000
	Max	0.470	1.000	1.000	1.000
	Sum	0.470	1.000	1.000	1.000

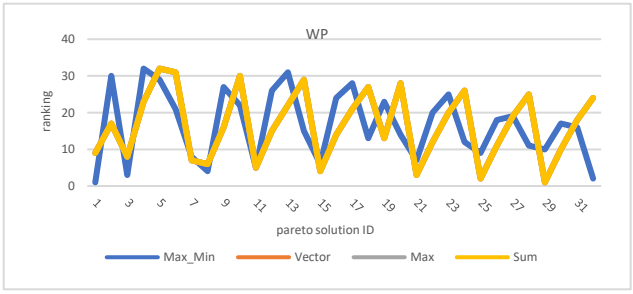

Figure 6: Comparison of the rank of different normalization for WP Method

Comparison of MADM methods

(1) Different MADM methods applying the Max Min Method

It can be observed from Table 6 that for both coefficients the results of SAW and TOPSOS are highly related. The results of WP are not related to SAW and TOPSIS. It can also be observed from Figure 7 that the decision made by the WP methods is not consistent with the other two methods.

Table 6: Correlations coefficients for MADM comparison of Max Min method

		SAW	TOPSIS	WP
Kendall's tau_b	SAW	1.000	0.895	0.232
	TOPSIS	0.895	1.000	0.151
	WP	0.232	0.151	1.000

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Spearman 's rho	SAW	1.000	0.976	0.299
	TOPSIS	0.976	1.000	0.215
	WP	0.299	0.215	1.000

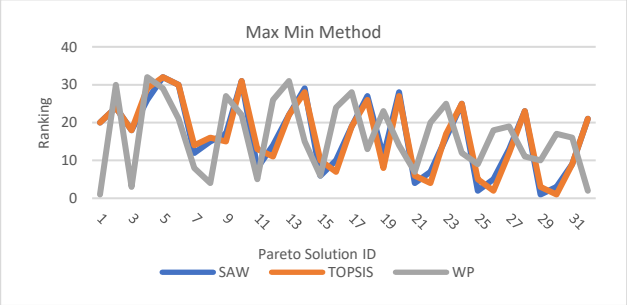

Figure 7: Comparison of the rank of different MADM for Max Min methods

(2) Different MADM method applying Vector Method

It can be observed from Table 7 that for both coefficients the results of the three methods are highly related. It can also be observed from Figure 8 that the results of TOPSIS are slightly different from the results of SAW and WP which are consistent with each other.

(3) Different MADM method applying Max Method

It can be observed from Table 8 that for both coefficients the results of the three methods are highly related. Similar to the Vector Method the results of the WP method are totally consistent with the SAW method. It can be observed from Figure 9 that that the results of the TOPSIS method are slightly different from the other two.

Table 7: Correlations coefficients for MADM comparison of Vector method

		SAW	TOPSIS	WP
Kendall's tau_b	SAW	1.000	0.960	1.000
	TOPSIS	0.960	1.000	0.960
	WP	1.000	0.960	1.000
Spearman 's rho	SAW	1.000	0.992	1.000
	TOPSIS	0.992	1.000	0.992
	WP	1.000	0.992	1.000

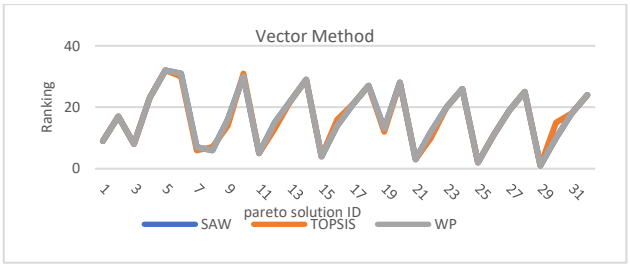

Figure 8: Comparison of the rank of different MADM for Vector methods

		SAW	TOPSIS	WP
Kendall's tau_b	SAW	1.000	0.976	1.000
	TOPSIS	0.976	1.000	0.976
	WP	1.000	0.976	1.000
Spearman 's rho	SAW	1.000	0.997	1.000
	TOPSIS	0.997	1.000	0.997
	WP	1.000	0.997	1.000

Table 8: Correlations coefficients for MADM comparison of Max method

(4) Sum Method

It can be observed that the results of Sum method are consistent between different MADM methods which is similar to the Max Method and the Vector Method.

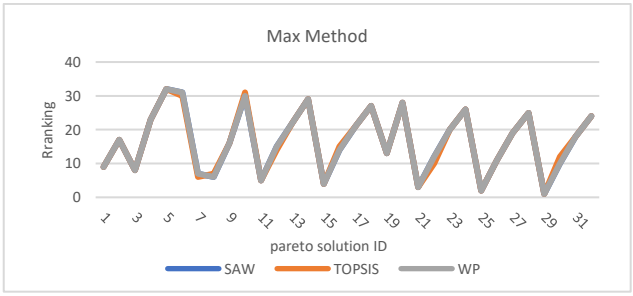

Figure 9: Comparison of the rank of different MADM for Max methods

Table 9: Correlations coefficients for MADM comparison of Vector method

		SAW	TOPSIS	WP
Kendall's tau_b	SAW	1.000	0.956	1.000
	TOPSIS	0.956	1.000	0.956

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	WP	1.000	0.956	1.000
Spearman 's rho	SAW	1.000	0.990	1.000
	TOPSIS	0.990	1.000	0.990
	WP	1.000	0.990	1.000

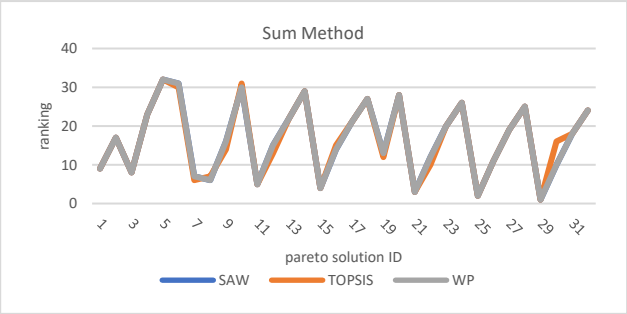

Figure 10: Comparison of the rank of different MADM for Sum methods

Robust decision

In literature, the Simple Additive Weighted method is noted to be a simple and easy procedure for most of the multi attribute decision making problems. When it comes to an important and complicated decisions, the TOPSIS could be applied. However, most of the research conducted in comparing the different methods to a case study, did not result in a general conclusion. In this case study, the preference for methods is uncertain and one of the ways to treat the uncertainty is by seeking a robust solution which could be adapted to most of the scenarios. In table 10 it can be observed the No.29 solution is the most robust one for different combinations of methods.

Table 10: Top ranked solution for different methods

	Max Min	Vector	Max	Sum
SAW	29	29	29	29
TOPSIS	30	29	29	29
WP	1	29	29	29

DISCUSSION AND RESULT ANALYSIS

The first part of the discussion is about normalization. Normalization is an important step for multi attribute decision making. It can transfer attributes with different units and scale into compatible formats. Normalization itself is a popular topic in data processing and many methods have been developed to solve problems like image processing or fast gradient descent. However, not all normalization is applicable in the MADM field or the building design field. The methods covered in this paper are all linear normalizations which can maintain the information since the judgement is based on the difference between two alternatives. For example, the increasing of the rating of energy consumption from 10 KWH to 11KWH and from 17KWH to 18KWH are the same. In this case, the linear normalization works well. But when the attribute is nonlinear, the linear normalization is inappropriate.

From the case study of this paper, it can be observed that the results of all the normalization methods are consistent with each other except the Max Min method. The reason for this is illustrated in Figure 11 and 12 which are the graphic interpretation for the normalization methods in this paper. The Vector Method, Max Method and Sum Method simply scale the original data. The range of results could vary from -1 to 1. The Max Min Method is a relative scaling method as shown on the right of Figure 11. The base for the relative value is the minimum which makes the range of results vary from 0 to 1 and the minimum value after normalization always at 0. Normalization in decision making has another role other than just to transform the data into compatible format: the results of normalization are the primary rating of decision making. The Vector, Max and Sum Methods emphasize on the proportion of each alternative in total, but the Max Min method emphasis is on the difference of each alternative compared with the minimum one like as shown in Figure 11 and 12.

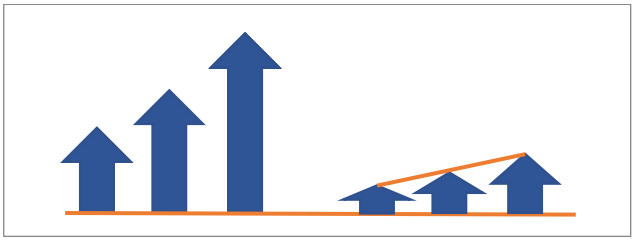

Figure 11: Illustration for Vector, Max and Sum Method

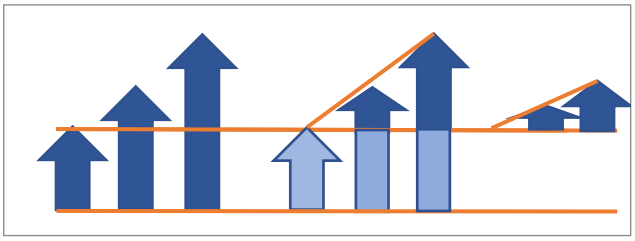

Figure:12 Illustration for Max Min Method

The second part of the discussion is about the comparison of different MADM methods. It can be found the SAW method and TOPSIS method are always consistent with each other when they apply the same normalization method. The WP method is consistent with the other two methods in most combinations except the MAX Min normalization. This is because the aggregation format of the WP method is nonlinear, and when the Max Min method is applied, it strengthens the reward and penalty.

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The next discussion is focused on the compatibility between the normalization method and MADM method. In the case study, energy consumption and carbon dioxide are chosen as performance indicators. Both only have a positive value and the objective trending is minimization which makes them compatible to all the methods.

However, the three MADM methods covered in this paper could not be applied to some other scenarios. For instance, the normalization results must be positive for WP method due to the exponential format. In the building design phase, the decision might be made based on the relative cash flow which has both negative and positive value. In this case, the Vector, Max and Sum normalization can not be applied if the decision maker decides to use the WP method since these normalization methods result in both negative and positive values. Also, indicators like indoor temperature are nonlinear indicators which has a rating that is intensive in a certain range but loose in the other range. In this case, the linear method like SAW is no longer adaptive. This discussion could be extended in future research.

CONCLUSION

Selection of a Multi attribute decision making method itself is a MADM problem (Yoon 1995). Instead of seeking the best method, Yoon (1995) suggested that the final decision can be made by integrating results from several methods. This integration could be done by a weight sum method which treats different methods as an attribute and makes the decision based on the final weighted summary or a more heuristic method such as the Borda Method. From the comparison of this research there is no significant difference between different methods when they are applicable for the case. But still MADM is not popular in building design. This is because of the difficulty of determining attributes and weight. The value of weight probably has more influence on the final decision than the uncertainty in the methodology of decision making. A sensitivity analysis of the weight's influence on building design could cast some light on this problem.

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