In-situ energy performance analysis and prediction of a hybrid heating system

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Abstract

Domestic hot water and space heating accounts for 14% and 56% of the total housing energy consumption in Canada, respectively. Conventional electric heat pumps can provide energy-efficient space and domestic hot water heating; however, they have lower energy efficiency in wintertime due to low outdoor temperatures. To enhance the efficiency and economic benefits of the heating system, also reduce the greenhouse gas emission, a hybrid energy source system that integrates a conventional air-source electric heat pump and a natural gas boiler is proposed as a potential solution. This research aims to monitor and analyze the in-situ energy performance of a hybrid heating system installed in a net-zero energy house located in a cold-climate city, Edmonton, Canada, and based on the in-situ performance, the energy consumption model of the hybrid system will be developed using a data-driven approach. The findings from this study will help advance research and development in reducing the energy consumption in water and space heating.

Introduction

Facing global warming and the energy crisis, renewable energy has attracted the interest of researchers in the past few decades. Housing consumes approximately 17% of secondary energy in Canada ("Energy Efficiency Trends in Canada," 2016), household water and space heating accounts for a large portion of the total housing energy consumption ("HVAC & Energy Systems ", 2020). Use of renewable energy and energy efficient heating system is needed to save the energy and the spending on the utilities and reduce the greenhouse gas emission. Conventional electric heat pump has been widely used with advantages in energy efficiency, but their performance can be much weakened by low ambient temperatures. A hybrid energy source system that integrates a conventional air-source heat pump (ASHP) and a natural gas boiler can be a potential solution to enhance the efficiency of the heating system in cold wintertime. This system can be used for space and water heating and can alternate its operation between the ASHP (electricity) and the boiler (natural gas) based on system efficiency and energy cost. Other than savings in the capital (as compared to cold-climate heat pumps) and operational costs, the hybrid system can be used in retrofit in conjunction with existing natural-gas furnace or boiler systems.

Many studies shows the advantages of the hybrid heat pump and natural gas heating system in energy and cost savings. Park, Nam, Jang, and Kim (2014) introduced a hybrid heating system which could perform space heating combined with domestic hot water heating, and analyzed its performance based on the weather condition in Korea. The heat pump system and gas heater system were able to operate together or independently even with the shared water line. The researchers concluded that the energy savings of this system are highly dependent on the operation method and ambient temperature, the efficiency of the system is about 2% to 30% higher than the conventional water heater, the annual savings on cost is about 8.9% in U.S and it also can have much more saving potential in lower electricity and gas cost countries. F. Li, Zheng, and Tian (2013) modelled an operation strategy of the hybrid gas boilers-centrifugal heat pumps heating system and suggested constant water flow rate in the system, resulted that the hybrid heating system consumes much lower energy than the conventional coal-fired boiler heating system and has huge energy saving potential. Klein, Huchtemann, and Müller (2014) also researched on hybrid heat pump systems comprised of electrically driven air-to-water compression heat pump and a condensing gas boiler in full-year dynamic numerical simulations, resulting in appreciable primary energy savings (12%-26%) compared with the conventional boiler systems. Wang et, al. (2020) conclude that the hybrid gas boiler and electric heating system was able to provide a more economical heating solution for the smart home, led about 22.6% and 21.9% operating energy cost savings compared to the pure electricity and pure gas solutions, respectively. With the cold ambient temperature challenge, G. Li and Du (2018) conducted the experiment of the heat pump gas water heater and space heating hybrid system under two control strategies with different operation modes, resulted that the hybrid system could save about 20% to 65% of energy cost than the gas water heater from -5 ˚C to 20 ˚C ambient temperatures, it also could save about 6% to 70% of hourly energy cost than gas heater from -15 ˚C to 20 ˚C ambient temperature in space heating application. Furthermore, G. Li (2018) proposed a parallel loop configuration for the hybrid heating system as an economic-based new control strategy that able to have a 10% to 60% system operating economic benefits for -12 ˚C to 20 ˚C ambient temperature. He also indicated that raising water flow rates could increase the system efficiency and the fuel price ratio could strongly affect the saving potential.

Although the above-mentioned studies confirmed the benefits of the hybrid heat pump and natural gas heating system, there are limited studies on the actual performance of the system and effective models to assist energy consumption prediction of hybrid heating systems. Among various data-driven building energy consumption prediction methods, the artificial neural network (ANN) based model has become one of the most commonly used model (Amasyali & El-Gohary, 2018). Ahmad et al. (2014) summarized the advantages and disadvantages of ANN, and mentioned due to the complexity of building energy systems, the ability of ANN in executing non-linear analysis is an advantage in buildings energy consumption. Zhao and Magoulès (2012) compared ANN with SVM and other building energy consumption forecasting methods, resulted the ANN model has relatively high running speed and accuracy. Z. Li, Han, and Xu (2014) reviewed twelve methods for benchmarking building energy consumption and suggested using ANN for the prediction of space heating load and total energy consumption.

Hence, this study aims to monitor and analyse the in-situ energy performance of a hybrid heating system, consisting of an electric heat pump and a natural gas boiler, implemented in a net-zero energy house located in a cold-climate city, Edmonton, Canada, and propose a method to find the operation switch point between the heat pump and natural gas boiler. Furthermore, based on the in-situ performance of the hybrid system, an ANN model has been developed to predict heating energy consumption.

Method

The methodology used in this study can be summarized as follows: Install the monitor system to collect the data, analysis the in-situ energy performance and housing heating energy demand, after that, conduct a cost-efficient analysis to determine the operation switch point between the electric heat pump and natural gas boiler, moreover, with the monitored data, develop an ANN model that can predict the heating energy (i.e., Electricity and natural gas).

Hybrid system description

Figure 1 shows the structure of the studied system. The system is composed of an air-source heat pump, and a natural gas tankless water heater that can provide both space heating and domestic hot water for users. The return air from the house will enter the air handling unit (AHU) and be heated by the heat pump or by the natural gas tankless water heater (TWH) independently. The control unit inside AHU will sense the thermostat of the house and exterior temperature to manage the air flowrate and control the operation of the system. In wintertime, when the difference of set point temperature and exterior temperature becomes larger, the COP of the heat pump decreases, then, the space heating will be processed by the natural gas TWH to increase the efficiency of the system. Also, when the efficiency of heat pump and TWH are the same. the cost of natural gas for operating TWH for space heating is much cheaper than the cost of electricity for heat pump operation.

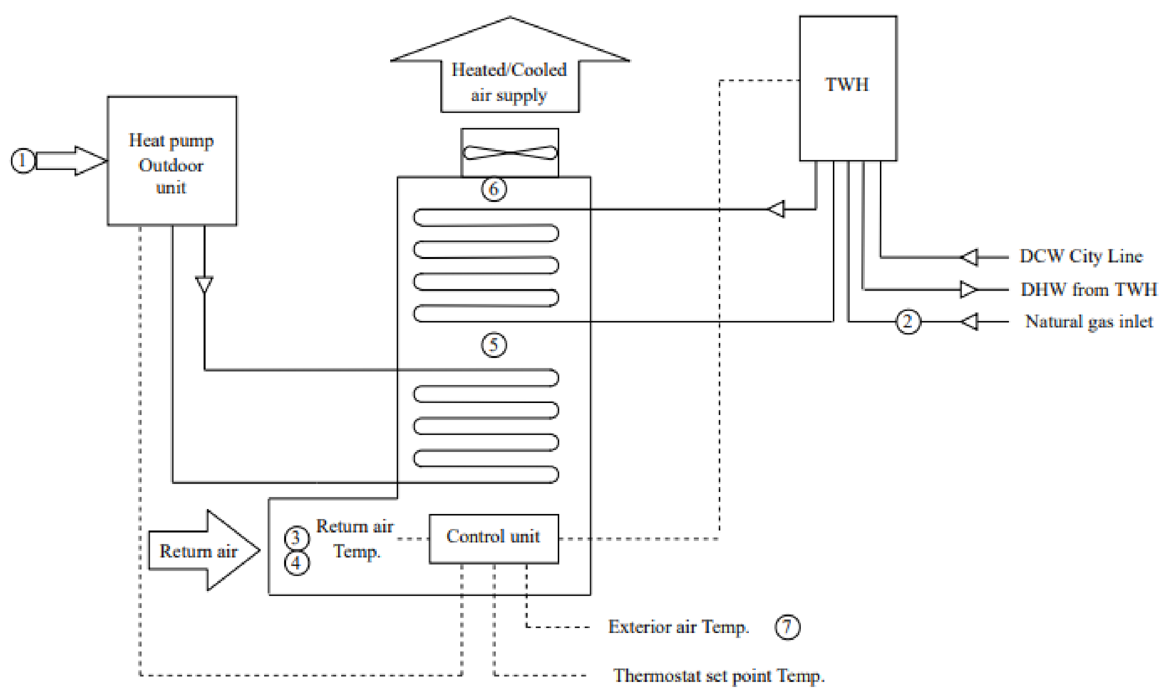


Figure 1. Schematics of the hybrid heating system and monitor measurements.

The system is implemented in a residential duplex house with the main floor area of 1898 sq ft and the basement area of 1494 sq ft, located in Edmonton, Canada. The average ambient temperature in the wintertime is around –14 ˚C. To meet the required heating load, the heat pump with a heating capacity of 11.2 kW and TWH with a space heating capacity of 32.8 kW are used.

Monitoring system measurements

Table 1 shows the monitoring parameters, and their corresponding locations are shown in Figure 1. The data were obtained from June 1st,2021 to December 15th,2021, in minutely, hourly, and daily intervals during the monitoring.

Table 1. Monitor system measurements.

1	Electricity consumption of heat pump.
2	Natural gas consumption of TWH.
3	Return air flow rate.
4	Temperature of return air and relative humidity.
5	Temperature of air across heat pump system.
6	Temperature of air across TWH heating system.
7	Ambient air temperature.

Evaluation of in-situ performance

The system performance while using electricity heat pump for space heating, $COP_{sys,HP,heat}$, is calculated by:

$$COP_{sys,HP,heat} = \frac{\dot{Q}_{hl}}{\dot{W}_{HP}} \quad (1)$$

where $\dot{W}_{HP}$ is the work input to the heat pump in $Watts$, and the heating load of the house, $\dot{Q}_{hl}$, can be determined by:

$$\dot{Q}_{hl} = \dot{v}_{air} A_d \rho_{air} c_{p,air} (T_2 - T_1) \quad (2)$$

where $\dot{v}_{air}$ is the return air flow rate in m/s, A_d is the cross-sectional area of the return air duct in m^2, ρ_{air} is the density of air in kg/m^3, $c_{p,air}$ is the specific heat of air in kJ/kgK, T_1 is the temperature of return air, and T_2 is the temperature of air across heat pump system.

And the system cooling efficiency is:

$$COP_{sys,HP,cool} = \frac{\dot{Q}_{cl}}{\dot{W}_{HP}} \quad (3)$$

where $\dot{Q}_{cl}$ is the heat dispelled from the house:

$$\dot{Q}_{cl} = \dot{v}_{air} A_d \rho_{air} c_{p,air} (T_1 - T_2) \quad (4)$$

The system efficiency while using natural gas TWH for space heating, $\eta_{sys,TWH}$, is calculated by:

$$\eta_{sys,TWH} = \frac{Q_{hl}}{E_{NG}} \quad (5)$$

where E_{NG} is the amount of energy of natural gas in kWh, and Q_{hl} is the corresponding amount of energy output for space heating.

To evaluate the “switch point” between the electricity heat pump and natural gas TWH during space heating, the energy cost of system is calculated. The fuel price of electricity (C_{elec}) and natural gas (C_{NG}) of Edmonton can be seen in Table 2, The prices are fixed and obtained from EPCOR utility service ("Encor Home Energy Plans and Rates," 2022). The energy cost, $Cost_{HP}$ of using heat pump can be determine as:

$$Cost_{HP} = C_{elec} \frac{Q_{hl}}{COP_{sys,HP,heat}} \quad (6)$$

The energy cost while using TWH is:

$$Cost_{TWH} = C_{NG} \frac{Q_{hl}}{\eta_{sys,TWH}} \quad (7)$$

The heating performance of the system, $COP_{sys,HP,heat}$ and $\eta_{sys,TWH}$ can be estimated with previous calculations in Eqs. (1) and (5). By using the relationship between the COP of the system and ambient air temperature, the energy cost of using heat pump and TWH can be obtained. Hence, if $Cost_{HP} \geq Cost_{TWH}$, then the system operates with TWH only, otherwise, if $Cost_{HP} < Cost_{TWH}$, the system operates with heat pump only.

Table 2. Fuel price.

Electricity price (C_{elec})	0.0759 CAD/kWh
Natural gas price (C_{ng})	0.0158 CAD/kWh (4.39 CAD/GJ)

Prediction model

Multi-Layer Perceptron (MLP)

ANN is a computational method inspired by the biological neural networks. It was first developed by McCulloch and Pitts (1943), and has been widely used in many fields of studies due to its advantages in building non-linear relationship between the input and output variables. Multi-Layer perceptron is one of the most commonly used ANN models in the case of predicting building energy consumption. The general structure of the fully connected, feed forward multi-layer perceptron network is shown in Figure 2, the network is composed of an input layer, a hidden layer and an output layer, each layer contains a number of neurons. During the training process of the neural network, the synaptic weights and bias will be adjusted to generalize the targeting outputs (Da Silva, Spatti, Flauzino, Liboni, & dos Reis Alves, 2017). The relationship between input and output k-th neurons are as follows:

$$Y_k = G\left(\sum_{m=1}^{z} X_m W_{km} + b_k\right) \quad (8)$$

where Y_k is the output of the k-th neuron, G is the activation function, X_m is the input of the m-th neuron, W_{km} is the weight of the connection between the m-th neuron of the input layer and k-th neuron of the hidden layer, b_k is the bias for the k-th neuron of the hidden layer, and G is the activation function, in this case, the activation function should be sigmoid:

$$G(x) = \frac{1}{1 + \exp(-x)} \quad (9)$$

The training algorithm is selected as Levenberg-Marquardt algorithm, since it appears to be the fastest method for training MLP networks (Hagan & Menhaj, 1994).

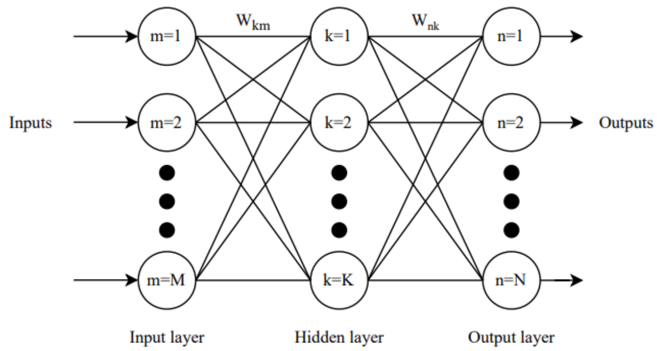


Figure 2. The structure artificial neural network (ANN).

Data selection

Hourly data from 0:00 November 1, 2021 – 23:00 November 30, 2021, was collected to set up the baseline model. The data set includes heating/cooling electricity consumption (heat pump) and natural gas consumption (TWH), return air flow rate, temperature across the heat exchanger, and weather data. The statistics of the collected data are shown in Table 3. The total number of data points is 718.

Data preparation

By using the equation below, the collected data except for heat pump electricity consumption and TWH natural gas consumption were normalized within the range of 0-1.

$$X^{*} = \frac{X_i - x_{min}}{x_{max} - x_{min}} \quad (10)$$

Heat pump electricity consumption and TWH natural gas consumption were normalized using logarithm to have the data distributed normally:

$$y^{*} = \log(1 + y) \quad (11)$$

Model Inputs and Outputs

The ANN model was conducted using a built-in neural network toolbox in MATLAB R2020b. Return air flow rate, relative humidity, the temperature of return air, the temperature of air across heat pump system, the temperature of air across TWH heating system, and ambient air temperature were selected as inputs. Electricity consumption of HP for heating, and natural gas consumption of TWH for space heating were set as output. Thus, there are six input neurons, and two output neurons. The ANN model has designed with 16 hidden neurons, which was defined by the sensitivity analysis in the next section. Within the collected data set, 70% of randomly selected data were used for training the baseline ANN model, and the rest of 30% of data were used to validate the accuracy of the model.

Model performance evaluation

To select the best ANN structure, a sensitivity analysis was carried out. Figure 3 shows the MSE of the training set and validation set along with different numbers of hidden neurons. MSE can be estimated by:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2 \quad (12)$$

As the number of hidden neurons increases, the MSE of the training set decreases, and the MSE of the validation set increases, which means the model becomes overfitted with more numbers of hidden neurons. The best performance of the network was selected with the least MSE of the validation set and was obtained by 16 hidden neurons, and the convergence criteria were set to 10^{-5}.

Table 3. Statistics of the collected data of hybrid heating system.

Variables	Unit	Mean	Median	Minimum	Maximum	Standard Error
Return air flow rate	m^3/s	0.12	0.12	0.038	0.25	0.0012
Indoor air relative humidity	-	37.95	37.87	31.07	43.53	0.07
Temperature of return air	$^{\circ}C$	22.72	22.82	18.61	25.58	0.04
Temperature of air across heat pump system	$^{\circ}C$	26.29	26.43	19.81	39.38	0.10
Temperature of air across TWH heating system	$^{\circ}C$	38.79	40.09	20.39	51.13	0.26
Ambient air temperature	$^{\circ}C$	-1.62	-1.1	-16.7	10.6	0.19
Electricity consumption of heat pump for heating	kWh	0.49	0.46	0	2.46	0.01
Natural gas consumption for heating (TWH)	kWh	1.30	1.16	0	4.06	0.02

The performance of the model was evaluated with: Mean Absolute Error (MSE), Root Mean Square Error (RMSE), R-squared (R^2), and Coefficient of Variation of the Root Mean Square Error (CVRMSE). These measures are the most commonly used evaluation measures of energy prediction models, and can be determined as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2} \qquad (13)$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \qquad (14)$$

$$CVRMSE(\%) = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}}{\bar{y}_i} \times 100 \qquad (15)$$

Where $\hat{y}_i$ is the predicted energy consumption value of the model, y_i is the measured energy consumption value, $\bar{y}_i$ is the average energy consumption value, and n is the total number of observations.

Results

Hybrid system in-situ performance

The in-situ performance of the system while using an 11.2 kW electric heat pump is illustrated in Figure 4, the COP of the system can be seen under different ambient temperatures. When the system is in cooling mode, the COP changes from 15 to 2 in a temperature range of 5 to 37 ˚C. The COP decreases when ambient temperature increases, since the cooling capacity of the heat pump decreases as the ambient temperature becomes higher. When the system is in heating mode, the COP changes from 1.5 to 6.5 in a temperature range of -24 to 20 ˚C, it can be observed that the low ambient temperature can weaken the COP. This is because the heat capacity of the heat pump decreases when the ambient temperature is lower. Thus, while operating a heat pump, the system will consume more energy for space heating during cold weather. By calculating the in-situ performance of the system operating natural gas TWH. The efficiency of the system has an average value of 0.7929. The performance of a natural gas heater can be slightly affected by the ambient temperature, since the inlet temperature of the working fluid is considered as a critical factor affecting the performance of natural gas heater; the efficiency becomes lower when the inlet temperature of working fluid increases; and inlet temperature of the working fluid changes along with the ambient temperature. Hence, as the weather gets warmer, there will be a minor reduction in efficiency. However, the performance is barely affected compared with heat pump operation, the efficiency of the natural gas TWH is assumed to be constant when calculating the operation switch point.

The monitored heating load of the studied house at different ambient temperatures is shown in Figure 5. The studied house has heating demand under 20˚C, there is a linear relationship between the hourly heating load and ambient air temperature.

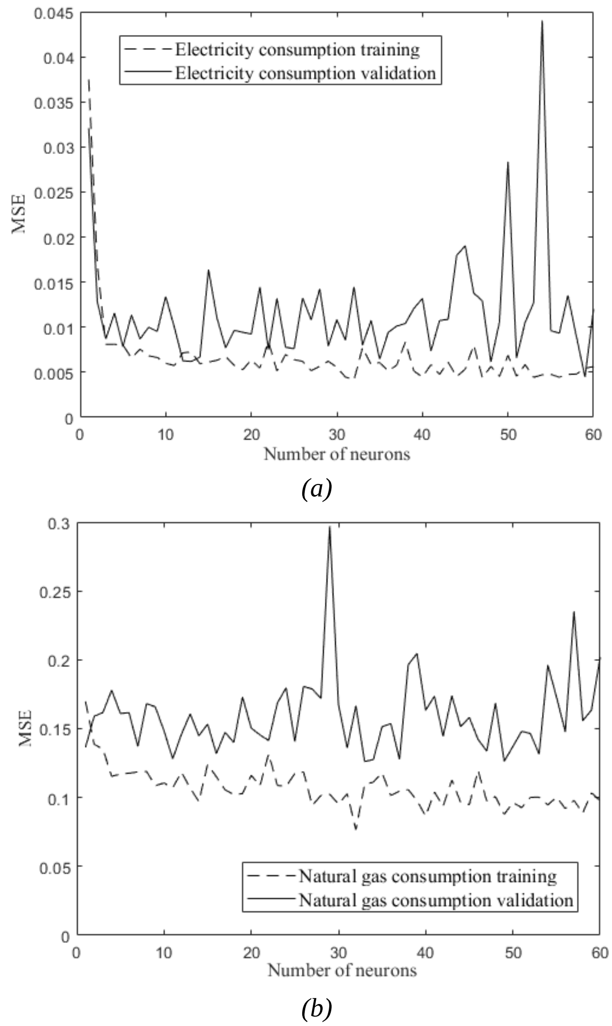


Figure 3. Results of sensitivity analysis of ANN model. (a) Electricity and (b) Natural gas consumption.

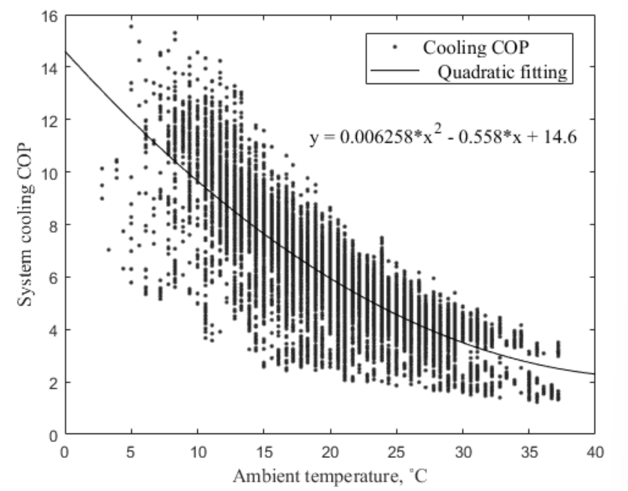
With the heating demand of the house and the performance of the system, the hourly energy cost of the system operating electricity heat pump and natural gas TWH are calculated using Eqs. (6) and (7) and shown in Figure 6. The energy cost of operating natural gas TWH is much cheaper than the electricity heat pump in low ambient temperature. Since as the ambient temperature gets lower, the system heating COP of the operating heat pump falls off, however, the efficiency of natural gas TWH almost remains unchanged. Moreover, as mentioned in Table 2, the energy cost of natural gas is more economical than electricity. The energy cost of the system operating heat pump becomes lower as the ambient temperature gets higher, it can be observed that the energy cost of the system operating heat pump becomes cheaper than natural gas TWH when the temperature is higher than 7.2 ˚C. Hence, to have the system operate with a cost-efficient strategy, the hybrid system should operate with the natural gas TWH only when the ambient temperature below 7.2 ˚C, and operates with the electrical heat pump only when ambient temperature becomes higher than 7.2 ˚C. If regardless of economic benefits, it is better to operate heat pump all the times since the system COP of using heat pump is always higher than the system efficiency of using TWH.

Energy prediction

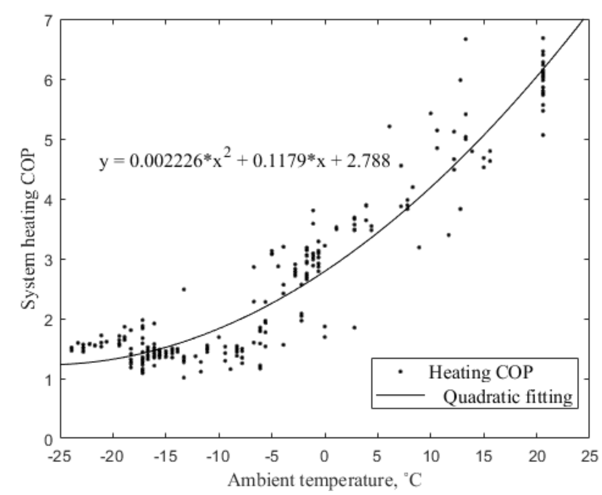
The results of the ANN model with the optimal model structures are summarized in Table 4.The regression of the actual and predicted values are shown in Figure 7 and Figure 8, respectively. The value of R^2 indicates the goodness of prediction, as it gets closer to 1, the better the model fits the targets, and RMSE expresses the accuracy of the networks. It can be seen in the table that the validation CV of electricity consumption and natural gas consumption are 24.62% and 31.37%, respectively. Compare with existing research in ANN models for building energy prediction, Edwards, New, and Parker (2012) use hourly real data to train the model and results in the CV of 24.32%-37.15% on heating energy consumption, Yun, Luck, Mago, and Cho (2012) have resulted in 23.0% - 23.3% CV in modelling heating energy consumption for the residential building based on hourly data. Thus, the ANN model in this study has reliable performance. The developed ANN model allows a rapid energy profile prediction of such hybrid system and can be facilitated in future building heating energy demand prediction.

Conclusion

In this study, the in-situ performance of a hybrid space heating system that integrates an air source heat pump and natural gas water heater is presented, and an approach to develop an energy prediction model for the studied hybrid system is proposed. Regarding the monitored data, while system operating heating pump, the system cooling COP changes from 15 to 2 within the ambient temperature range of 5 ˚C to 37 ˚C, and system heating COP changes from 1.5 to 6.5 in the ambient temperature range of -24 ˚C to 20 ˚C. When the system operates natural gas water heater for space heating, the average system efficiency achieves 0.7929. To determine the system operation switch point between the air source heat pump and natural gas water heater, an energy cost analysis has been conducted. The system should operate the air source heat pump for space heating while the ambient temperature is greater than 7.2 ˚C, and switch to the natural gas water heater while the ambient temperature is less than 7.2 ˚C. To develop the heating energy prediction model of the hybrid system, the multilayer-perceptron neural network (ANN) was selected as the learning algorithm. By using one month of monitored data, the model shows a relatively good performance with the validation coefficient of variation of 24.62% and 31.37% for electricity and natural gas consumption, respectively. This study presents an approach to help with the in-situ performance analysis and hourly energy consumption prediction of the similar hybrid system, and for future research, other parameters such as solar radiation and occupancy schedule can be considered as inputs of the model to refine the accuracy of the results.



(a)



(b)

Figure 4. System (a)cooling COP and (b)heating COP operating heat pump under various ambient temperature.

Table 4. Results of the artificial neural networks (ANN).

	Training			Validation		
	R^2	RMSE	CV	R^2	RMSE	CV
Electricity consumption	0.9386	0.0822	16.37%	0.9296	0.0790	24.62%
Natural gas consumption	0.7355	0.3249	17.04%	0.6231	0.3944	31.37%

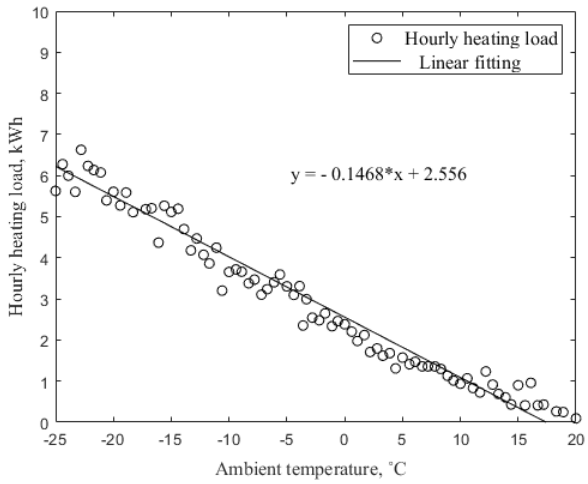


Figure 5. Hourly heating load of the studied house.

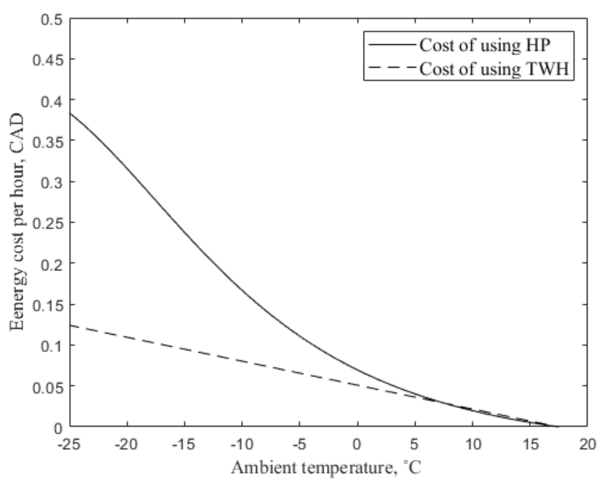
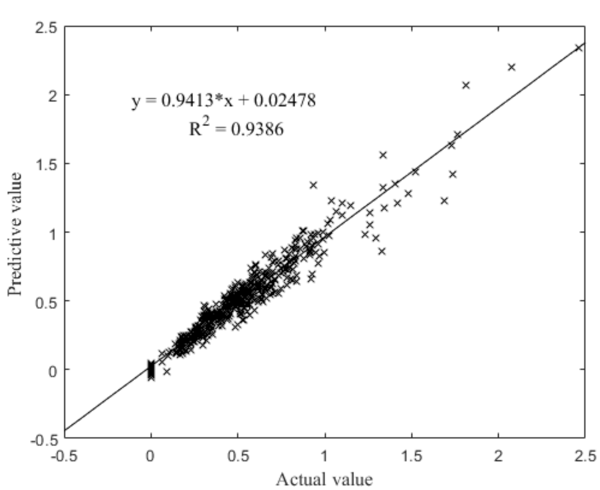


Figure 6. Energy cost of hybrid system.



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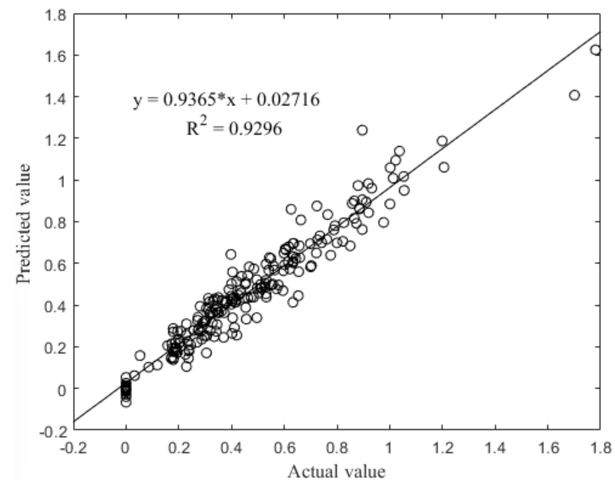
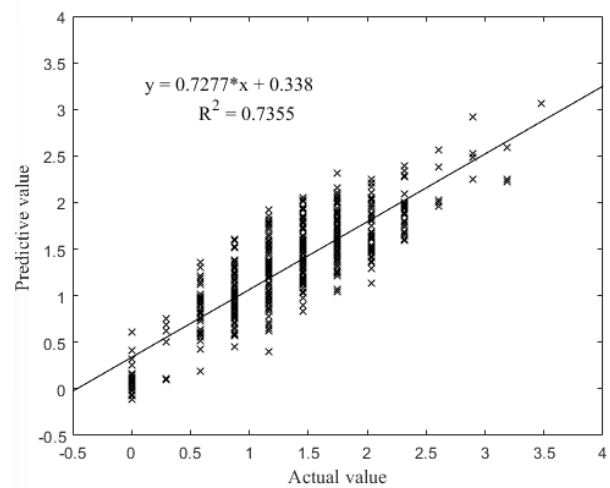


(b)

Figure 7. Regression plots of electricity consumption of heat pump, (a) Training, (b) Validation



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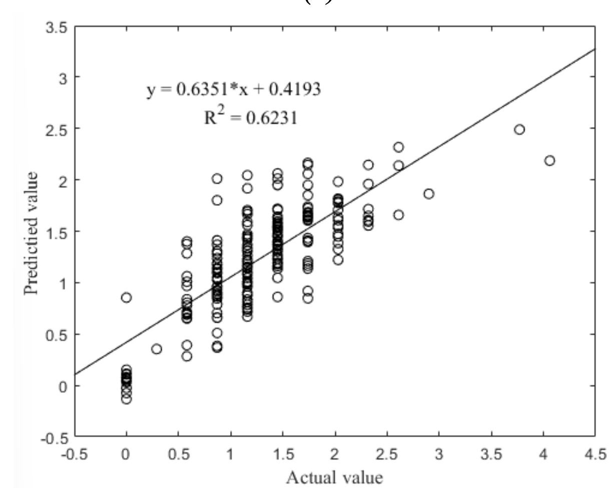

(b)

Figure 8. Regression plots of Natural gas consumption of TWH, (a) Training, (b) Validation.

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