# A SYSTEM LEVEL SIMULATION MODEL OF SOFC SYSTEMS FOR BUILDING APPLICATIONS

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## ABSTRACT

A thermal and electrochemical model for the simulation of Solid Oxide Fuel Cell (SOFC) cogeneration system is developed in this study. The basic operating principle of SOFCs is briefly introduced, and the reforming process converting the fuel source into hydrogen-rich gas to be utilized for the generation of electricity is discussed as well. In addition, the modeling algorithms of electrochemical and thermal models for the simulation of SOFCs are described. Since the fuel cell stack itself is only a single component within the whole SOFC system, the modeling of the balance-of-plant (BOP) components is also performed to assess the system-level performance. Using the newly developed model, a parametric analysis is carried out to investigate the effect of specific cell operating parameters on the overall system performance. Results and discussions on the parametric analysis are also provided in this paper.

## INTRODUCTION

According to the U.S. Energy Information Administration, buildings accounts for 40% of the total energy consumption in this country. Therefore, the significance of reducing the energy consumption in the buildings has received increasing attention during the last years and led to the great interest in the utilization of cogeneration systems due to their potential for significantly reducing the heating and electrical energy usage in buildings. However, the development and application of building cogeneration systems are still in the early stages and the potential for energy saving in buildings are not thoroughly understood to date.

Among various technologies available for building applications, fuel cells have been considered to be one of the most promising technologies in terms of power generation and thermal energy production for building applications. A fuel cell is a device that directly converts the chemical energy of fuels into the electrical energy using hydrogen and oxygen through electrochemical process. Direct conversion of the chemical energy to electrical energy enables the fuel cell to have the potential to produce electricity at a higher efficiency than conventional power plants. The use of fuel cell technology also results in much lower emission of greenhouse gases and other air pollutants.
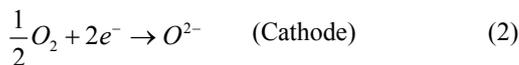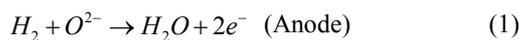
Among several fuel cell types available currently, SOFC is chosen for this study because of its ability to be utilized as a cogeneration system and its wide range of capacity covering all types of buildings from small scale residential buildings to large scale commercial buildings. In addition the operating temperature of SOFC fuel cells is high enough to provide sufficient heat for the fuel reforming process. This allows SOFCs to reform fuel internally, which removes the need for the expensive external reformers used to produce hydrogen in SOFCs. The high operating temperature also enables SOFCs to use the residual heat created as a byproduct of the power generation process for space and water heating, allowing SOFC to be used as cogeneration system.

Predicting the performance of SOFC cogeneration systems before they are actually manufactured and installed inside buildings is an important issue for optimal design and the resultant energy saving. Due to a variety of physical and chemical mechanisms occurring within SOFCs and the relevant parameters involved, computational modeling is typically chosen and a variety of researches have been done by many researchers on the computational model development such as the studies by E. Achenbach (1994), Costamagna et al. (1998), Bessette et al. (1995), Massardo et al. (2000), Xue et al. (2005) and Bove et al. (2006). However, one of the biggest limitations of every study discussed above is that they are all cell-level based and thus such models do not have a potential to specifically assess phenomena and processes taking place outside the fuel cell stack when simulating the overall system. In other words, balance-of-plant (BOP) components other than fuel cell stack itself are not taken into account in those models. Even though several system-level studies do exist, those studies are not as prevalent when compared to stack-level studies.

Since the fuel cell stack is only a single component within the cogeneration system, a computational model should be developed not only for the fuel cell stack itself but also for the other balance-of-plant (BOP) components as an entire cogeneration system level approach. Those BOP components include: fuel compressor, water pump, air blower, air and fuel pre-heater and burner, etc. All the fuel and electricity consumption by the BOP components are also considered as part of the whole system level simulation in this study. Using the newly developed model, a parametric analysis is carried out to investigate the effect of specific cell operating parameters on the overall system performance. Also, several available configurations for SOFC systems such as the basic configuration and those employing anode gas recycling, cathode gas recycling and both anode and cathode gas recycling configurations will be compared to each other.

## OVERVIEW OF SOFC SYSTEM

A fuel cell stack consists of many fuel cells and a single fuel cell consists of an electrolyte, an anode, a cathode and an interconnect which connects the cathode to the anode of the next cell. The principle of SOFC operation is presented in Fig. 1. At the anode side, oxygen ions coming from the cathode via oxygen ion-conducting electrolyte combine with hydrogen to form $H_2O$ and release electrons. Released electrons travel from the anode to the cathode through the external circuit, and react with the oxygen supplied to the cathode side to form oxygen ions. Those oxygen ions then travel from the cathode to anode via electrolyte. Those electrochemical reactions taking place in anode and cathode of the fuel cell are:

$$H_2 + O^{2-} \rightarrow H_2O + 2e^- \quad \text{(Anode)} \qquad (1)$$

$$\frac{1}{2}O_2 + 2e^- \rightarrow O^{2-} \qquad \text{(Cathode)} \qquad (2)$$

As a result, the overall reaction in the fuel stack can be expressed as:

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O \qquad (3)$$

Since hydrogen combines easily with other elements and for this reason it does not exist by itself. Fuel reforming is the conversion process of fuel sources into the hydrogen-rich gas which is then used to directly generate the electricity in the fuel cell stack. Although renewable energy sources such as wind, solar, biomass and geothermal energy should be used to produce hydrogen in the long term due to the depletion and the

resultant increase in the price of fossil fuel, fossil fuels, especially methane, are still the most widely used source to generate hydrogen, since methane reforming is already a mature technology and the infrastructure such as the delivery pipeline is well-established. Therefore, only methane ($CH_4$) will be considered as the fuel for SOFC system in this study. Methane can be reformed into hydrogen-rich gas via catalytic steam reforming (CSR) as follows:

$$CH_4 + H_2O \rightarrow CO + 3H_2 \qquad (4)$$
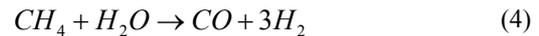
Newly generated carbon monoxide has an adverse effect on the fuel cell anode catalyst. This carbon monoxide is reduced and at the same time additional hydrogen can also be generated by an exothermic water-gas shift reaction as follows:

$$CO + H_2O \rightarrow CO_2 + H_2 \qquad (5)$$
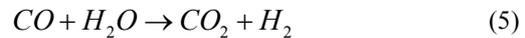
Because the overall reaction is highly endothermic, sufficient heat should be supplied in order for the reaction to take place. Those heat needed to activate the catalytic steam reforming can be extracted from the exhaust gas coming from the cell stack in the high operating temperature.
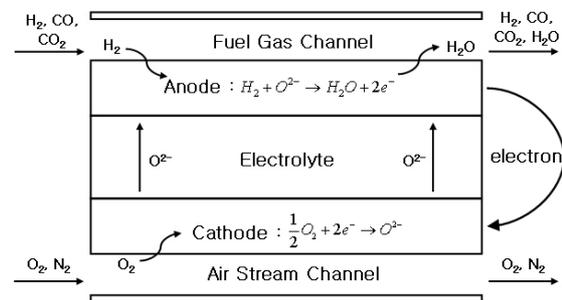


*Fig. 1 Operating principle of SOFC*

## SOFC MODEL DESCRIPTION

Due to the complex mechanisms occuring around the SOFC system, several simplyfing assumptions were made and are described below:

- All the gases inside the system are ideal.
- The electrical voltage output from each cell is constant.
- The temperature distribution in the axial direction is uniform.
- All the gases inside the stack are perfectly mixed so that the exhaust temperatures of the air and fuel from the stack are the same as the stack operating temperature.

- Incoming fuel and air are uniformly distributed to each individual cell in the stack.
- The fuel supplied to the reformer is methane.
- Inlet gas entering the anode is composed of methane, steam, hydrogen, carbon monoxide and carbon dioxide.
- The air supplied to the cathode is composed of oxygen and nitrogen.
- The water-gas shifting reaction, which is further described later, takes place at equilibrium, since the reaction reaches the equilibrium condition quickly.
- Ohmic resistance of the fuel cell is temperature-dependent.
- Temperature distribution in the perpendicular direction within solid cell components consisting of anode, electrolyte and cathode is uniform because of the small thickness of anode and cathode.

**Electrochemical model**

Electricity is generated through the electrochemical oxidation of hydrogen in the fuel cell. For the computation of total power output from the cell stack, it is necessary to determine the actual voltage produced by each fuel cell. The purpose of electrochemical model is to compute the actual cell voltage and the resultant power output from the fuel cell stack by taking into account chemical reactions taking place within the stack. The Nernst potential is related to the Gibbs potential, partial pressure of each gas and temperature as follows:

$$E_N = E^o + \frac{RT}{2F} \ln \frac{p_{H2}(p_{O2})^{1/2}}{p_{H2O}} \qquad (6)$$

$E^o$ is the Gibbs potential which is a function of the temperature (Karvountzi 2005):

$$E^o = -\frac{\Delta G^o}{nF} = 1.273 - 2.7645 \times 10^{-4} T \qquad (7)$$

Practically, the actual cell voltage is lower than the Nernst potential due to the irreversibilities related to ohmic losses, activation losses and concentration losses. Ohmic losses are due to the resistance to the conduction of ionic and electronic currents through the electrolyte and electrodes and due to the contact resistance between cell components. Ohmic resistance is known to depend only on the materials used. Activation polarization is related to the gas composition and the current density. It is the extra-potential necessary to make the electrode reactions take place at the desired rates (Achenbach

1994) and is known to become significant at low current densities. Concentration losses are due to the mass transport effect, occurring when the electrochemical reaction is faster than the diffusion of reactants through electrodes (Bove et al. 2005). It depends on the temperature and the current density and is known to become significant at low concentrations of reactants and at high current densities (Achenbach 1994). Therefore, the actual cell voltage and the corresponding electricity power output from the fuel cell can be determined as follows:

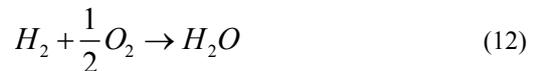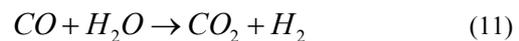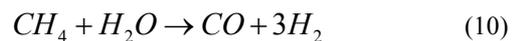$$V_{cell} = E_N - (\Omega_{ohm} + \Omega_{pol})i - R_{con} \qquad (8)$$

$$P_{cell} = V_{cell}i \qquad (9)$$

The detailed information regarding the computation of those voltage losses will not be provided in this paper.

**Thermal model**

The purpose of the thermal model is to determine the temperatures of the outlet gases and the stack solid structures so that the thermal energy that can be extracted for space and water heating can be computed. Thermal model can be divided into mass balance and energy balance.

In order to perform the mass balance inside each fuel cell, three chemical reactions occurring in fuel cells should be taken into account. Hydrogen is produced from the steam methane reforming process and the water-gas shifting reaction and is then consumed through the electrochemical oxidation to generate electrical power. These three reactions are expressed below, respectively:

$$CH_4 + H_2O \rightarrow CO + 3H_2 \qquad (10)$$

$$CO + H_2O \rightarrow CO_2 + H_2 \qquad (11)$$

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O \qquad (12)$$

For the determination of the fuel consumption and the resultant electrical power output, the reaction rate of each reaction should be computed. The reaction rate of the electrochemical oxidation of hydrogen is calculated from the fuel utilization factor, which is defined as the ratio of the consumed hydrogen to the supplied hydrogen into the fuel cell. The reaction rate of the reforming process can be determined using the model developed by Achenbach (1994):

$$r_{CH_4} = k_{CH_4} p_{CH_4} \exp(-\frac{E_{CH_4}}{RT}) A_{ra} \qquad (13)$$

As can be seen in Eq. (13), the reaction rate varies according to the partial pressure of methane and the cell temperature.

Since the water-gas shifting reaction is fast and reaches equilibrium quickly, the reaction rate of the water-gas shifting reaction can be calculated using the equilibrium constant which is a function of temperature as follows (Bove et al. 2005):

$$\ln(K_2) = \frac{5693.5}{T} + 1.077\ln(T) + 5.44 \times 10^{-4}T - 1.125 \times 10^{-7}T^2 - \frac{49170}{T^2} - 13.148$$

(14)

Where, $K_2$ is the equilibrium constant.

From the material balance of each reactant and the products of steam reforming, the water-gas shifting reaction and the electrochemical oxidation process, the outlet molar fractions as a result of those three reactions are evaluated:

$$n_{CH_4}^{out} = n_{CH_4}^{in} - r_{CH_4}$$ (15)

$$n_{CO}^{out} = r_{CH_4} - r_{ws}$$ (16)

$$n_{CO_2}^{out} = r_{ws}$$ (17)

$$n_{H_2O}^{out} = n_{H_2O}^{in} - r_{CH_4} - r_{ws} + r_E$$ (18)

$$n_{H_2}^{out} = 3r_{CH_4} + r_{ws} - r_E$$ (19)

Since all the other variables except $r_{ws}$ are already known including equilibrium constant determined from Eq. (14), the reaction rate of water-gas shifting process, $r_{ws}$, can be finally calculated by solving the following equation:

$$K_s = \frac{r_{ws}(3r_{CH_4} + r_{ws} - r_E)}{(r_{CH_4} - r_{ws})(n_{H_2O}^{in} - r_{CH_4} - r_{ws} + r_E)}$$ (20)

The energy balances are carried out separately around the fuel channel, the air channel, the solid cell component and the cell interconnects. Heat transfer mechanisms among adjacent cell components are illustrated in Fig. 2.

Although the solid cell component consists of anode, cathode and electrolyte, the temperature can be assumed constant within those components owing to the small thickness of anode and cathode (Murshed et al. 2007). The energy balance of the solid cell component can be written as:

$$\rho_S V_S C_{p,S} \frac{dT_S}{dt} = (Q_{oxid}^{in} + Q_{reform}^{in} + Q_{ws}^{in}) - (Q_{oxid}^{out} + Q_{reform}^{out} + Q_{ws}^{out})$$
$$- (Q_{rad,A} + Q_{rad,F} + Q_{conv,A} + Q_{conv,F}) - r_E\Delta H_E - r_{CH_4}\Delta H_{CH_4} - r_{ws}\Delta H_{ws} - P_{cell}$$

(21)

The terms inside the first two parentheses on the right hand side are the energy change by the mass transfer of reactants and products of the three reactions.

The energy balance in the fuel channel can be written as:

$$\rho_F V_F C_{p,F} \frac{dT_F}{dt} = (Q_F^{in} - Q_F^{out}) + (Q_{reform}^{out} + Q_{ws}^{out}) - (Q_{reform}^{in} + Q_{ws}^{in}) + \int_{T_{ref}}^{T_S} C_{p,H_2O}(T)dT$$
$$- \int_{T_{ref}}^{T_F} C_{p,H_2}(T)dT + Q_{conv,F} - Q_{conv,F-1}$$

(22)

In the similar way to the fuel channel, the energy balance around the air channel can be written as:

$$\rho_A V_A C_{p,A} \frac{dT_A}{dt} = (Q_A^{in} - Q_A^{out}) - \frac{1}{2}r_E \int_{T_{ref}}^{T_A} C_{p,O_2}(T)dT + Q_{conv,A} - Q_{conv,A-1}$$

(23)

Finally, the energy balance of the interconnect can be written as:

$$\rho_I V_I C_{p,I} \frac{dT_I}{dt} = Q_{conv,F-1} + Q_{conv,A-1} + Q_{rad,F} + Q_{rad,A}$$ (24)

By performing the electrochemical model, mass balance and energy balance discussed thus far, the actual cell operating temperature, the cell voltage, the electricity power output, the exhaust gas composition and their outlet temperatures can be finally determined.

Regarding the balance-of-plant (BOP) components modeling, well-known equations such as standard effectiveness model for heat exchangers are used and thus they are not discussed in this paper. In addition, FORTRAN 95 computer language is used to solve electrochemical and thermal models discussed thus far.

## PARAMETRIC ANALYSIS

Using the newly developed SOFC model, the effect of cell operating parameters on the overall system performance will be investigated using the new model for a comprehensive understanding of how fuel cell stack, BOP components and their performance are interconnected and how the fuel cell requirements should be met to get the desired power output and the system efficiency by the proper control of those operating parameters. Based on the literature, those key operating parameters that should be taken into account include:

·   Inlet fuel flow rate
·   Cell voltage
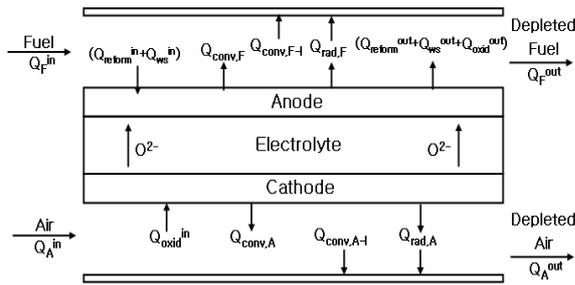·   Extent of fuel pre-reforming process
·   Fuel utilization

*Fig. 2 Energy balances in a single cell*

*Table 1 Description of simulation conditions*

|  | CONDITIONS |
|---|---|
| Electro-active area of each cell | $120 \times 120$ mm$^2$ |
| Air and fuel inlet temperature | 1,073 K |
| Operating pressure | 1 bar |
| Variables | Inlet fuel flow rate (mol/h) - 0.2, 0.3, 0.4, 0.5, 0.6, 0.8 Extent of fuel pre-reforming (%) - 0, 25, 50, 75, 100 Fuel utilization factor (%) - 40, 50, 60, 80, 100 Cell voltage (V) - 0.5, 0.6, 0.7, 0.8, 0.9 |

Parametric studies were performed on five or six different values of each parameter while the other parameters were maintained at the same values. The standard values of each variable were set at: 0.4 mol/h for inlet fuel flow rate, 50 % for recycling rate of cathode gas in the system configurations where cathode gas recycling concept is employed, 50 % for extent of fuel pre-reforming process and 80 % for fuel utilization factor. In other words, when changing only one variable at every simulation process for parametric studies, the other variables were kept at those values. Table 1 shows the details of simulation conditions for parametric analysis.

The analysis will not be carried out only for a particular configuration, i.e. several possible configurations will be considered since even the same parameters can have the different effects on the system performance under the different configurations. Based on the literature, many configuration designs can be considered. Among them, Fig. 3 illustrates the basic configuration design. More sophisticated SOFC system than the basic concept shown in Fig. 3 can be obtained by employing advanced gas processing idea such as cathode gas recycling and anode gas recycling (Riensche 1998). The main point of cathode gas recycling is to preheat the incoming air by mixing it with the exhaust gas leaving the cathode at

high temperature. On the other hand, the anode gas recycling is to use a part of hot exhaust steam leaving the anode for fuel reforming process instead of the steam preheated in the boiler (Riensche 1998). Both anode and cathode gas recycling concept is illustrated in Fig. 4.
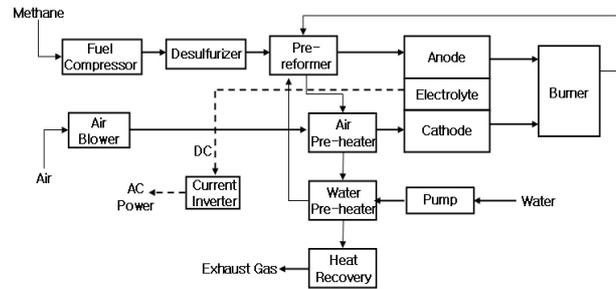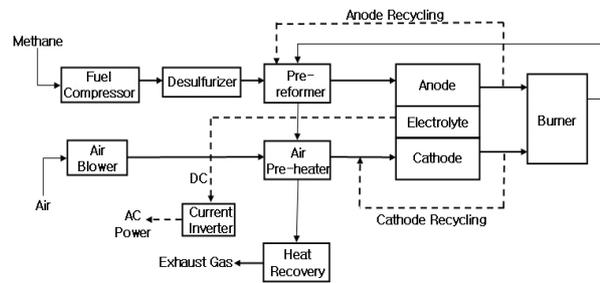


*Fig. 3 Basic design of SOFC system (Riensche 1998)*



*Fig. 4 Air and fuel preheating by cathode and anode gas recycling (Riensche 1998)*

## DISCUSSION AND RESULT ANALYSIS

### Influence of inlet fuel flow rate

Fig. 5 illustrates the net power output as a function of inlet fuel flow rate under four different configurations. Hereafter, basic design, anode recycling, cathode recycling and both anode and cathode recycling configurations will be signified as basic, anode, cathode and both, respectively.

Since the cell current is proportional to the electrochemical reaction rate which rises with the inlet fuel flow rate, the power output from the stack is positively affected by increased fuel flow rates. (Average increase rate of 230%)

However, there is a significant difference in the net cell power outputs among different configurations as can be seen in Fig. 5. The configurations employing the cathode gas recirculation, i.e. cathode and both recycling cases, have higher net power supply than the basic configuration. It should be noted that the power consumption by air blower accounts for approximately as much as 95% of the whole BOP power consumptions. The important point to consider is the concept of required amount of excess air for the air cooling of

SOFC stack, since the thermal stress on the solid structure should be controlled by the allowable temperature rise of the air passing through the stack. Typically, the air temperature rise that should be maintained for the prevention of thermal stability of stack components is 100 K. Therefore, inlet air flow rate into the system is adjusted to maintain the temperature rise of 100 K.

As mentioned earlier, both cathode and both recycling cases employ the cathode gas recycling concept. As expected, one of the key advantages of cathode gas recycling is decreased fresh air flow, reducing the air blower capacity and the resultant power consumption. Therefore, the power consumptions by BOP components in basic case were higher than those of cathode and both recycling cases as illustrated in Fig 5. The more the power consumption by the air blower, the less the net power is supplied from the system.

In addition, the net power outputs of anode and both recycling cases employing anode gas recirculation were clearly higher than that of basic case. Since the available amount of fuel gases that can be consumed for the electrochemical oxidation process such as methane, carbon monoxide and hydrogen gas increases when anode recycling concept is utilized, the power output significantly rises due to the anode gas recirculation given that the same amount of new methane is fed into the system.

Fig. 6 illustrates the system electrical efficiency based on the lower heating value (LHV) of methane gas. Cell efficiencie slightly decreases as the fuel flow rate rises. This is because of the decreased cell voltage with regard to the increased fuel flow. Cell power is a product of cell current and the cell voltage. Although the cell current is almost proportional to the fuel flow rate, the cell voltage does not remain at the same value and drops as the fuel flow rate rises, lowering the cell efficiency.
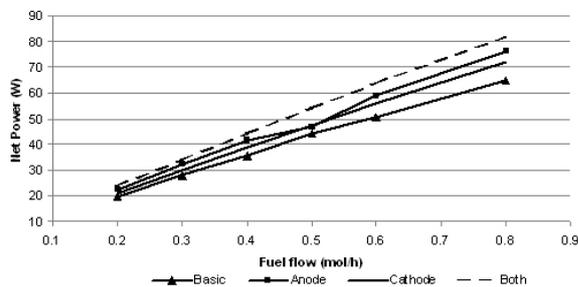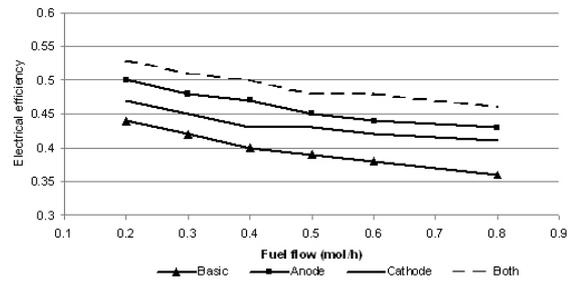


*Fig. 5 Fuel flow vs. net power output*



*Fig. 6 Fuel flow vs. cell electrical efficiency*

## Influence of methane pre-reforming extent

Net power supply vs. methane pre-reforming extent is represented in Fig. 7. The same variation pattern can be observed in four different configurations, i.e. the net power output declines as the extent of pre-reforming rises. (Average decrease rate of 22%) As the extent of methane pre-reforming increases, the highly endothermic methane reforming process taking place inside the stack becomes less active, raising the stack temperature. Increased operating temperature, in turn, raises the cooling air requirement to satisfy the air temperature rise of 100K across the stack, increasing the power consumption by air blowers. Therefore, both cooling air flows and BOP component power consumptions significantly increases with the extent of methane pre-reforming. Reduced BOP power consumption and enhanced net power supply is the main advantage of internal reforming over external reforming concept.
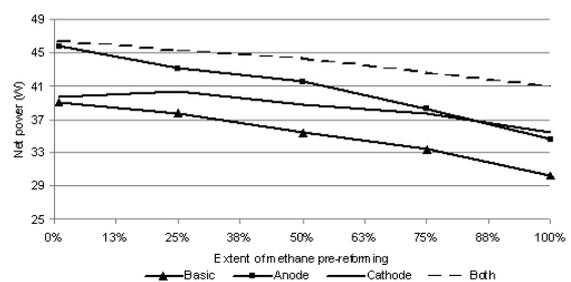


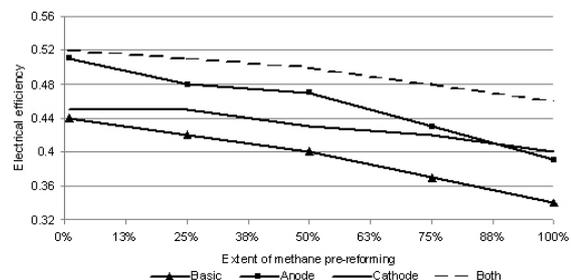*Fig. 7 Methane pre-reforming extent vs. net power*



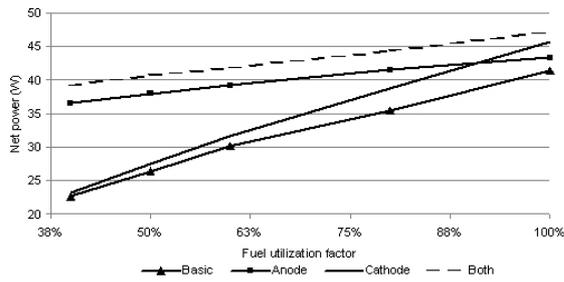*Fig. 8 Methane pre-reforming extent vs. cell efficiency*
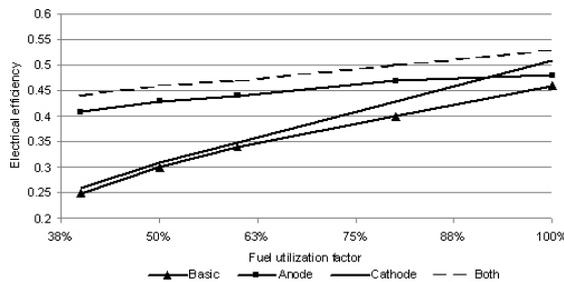
*Fig. 9 Fuel utilization vs. net power*



*Fig. 10 Fuel utilization vs. cell efficiency*

**Influence of fuel utilization factor**

Fuel utilization can be defined as the ratio of the amount of fuel gases that is consumed in the electrochemical oxidation process to the total amount of fuel gases that is injected into the anode compartment. As the fuel utilization increases, the cell current significantly rises since it is proportional to the electrochemical oxidation reaction rate, rasing the total power output. In addition as the fuel utilization increases, the reaction rate of highly exothermic electrochemical oxidation taking place inside the stack also increases, raising the operating temperatures of the fuel cell stack. The increased stack temperature requires larger amount of cooling air supply, raising the air blower capacity. Interestingly, the net power output also increases with the fuel utilization factor as illustrated in Fig. 9. (Average increase rate of 55%) The reason for the increased net power output even under the increased BOP component power consumption is that the increased amount of total power output from the stack is greater than the increased amount of power consumptions by BOP components, and thus the net power supply eventually becomes higher as the fuel utilization rises.

**Influence of cell voltage**

Total power supply increases with the cell voltage due to the fact that stack total power is a product of cell voltage and cell current. Since cell current is only related to the reaction rate of electrochemical oxidation,

it remains constant regardless of cell voltage. As a result, total power is proportional to the cell voltage. Regarding power consumptions by BOP components, it decreases as the cell voltage goes up. As the total power generation is enhanced due to the increased cell voltage, the amount of thermal energy that can be supplied within the stack is reduced, lowering the cell operating temperature. The reduced stack operating temperature, in turn, drops the cooling air flow requirement and accordingly the power consumption of air blower is eventually reduced. As a result of increased total power output and reduced BOP component power consumption, net power supply and cell efficiency significantly increase with the cell voltage as can be observed in Fig. 11 and Fig. 12. (Average increase rate of 165%)
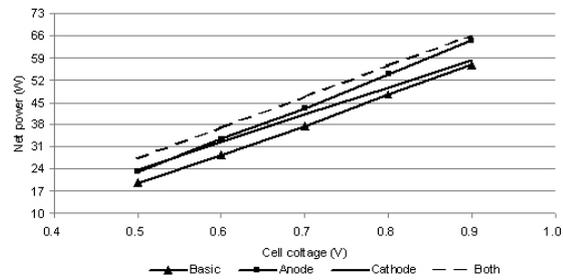


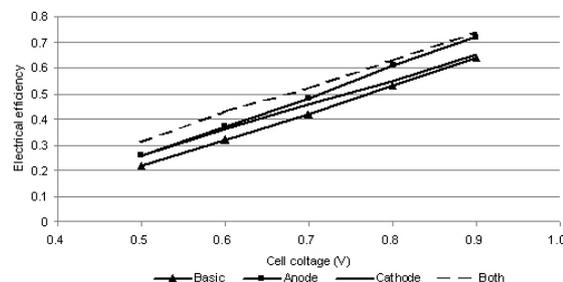*Fig. 11 Cell voltage vs. net power*



*Fig. 12 Cell voltage vs. cell efficiency*

## CONCLUSION

In this paper, the background, theoretical overview and the brief modeling algorithm for the simulation of SOFC systems are provided. The computational model is developed and the effects of important control parameters on the system performance are investigated using the new model. The following conclusions were drawn.

As the inlet fuel flow rate increases, cell current rises due to the increased reaction rate of electrochemical oxidation process and thus total power and net power supply from the system increases, while actual cell voltage and the system efficiency decrease. As the

extent of methane pre-reforming increases, net power output is significantly reduced because of the increased cooling air requirement and the corresponding air blower capacity, showing that internal reforming has an advantage over external reforming in terms of system power supply. As the fuel utilization factor rises, both total power and net power supply are enhanced. As the cell voltage rises from 0.5V to 0.9V, the system net power supply and the corresponding electrical efficiency rapidly increase due to the enhanced total power output and the reduced cooling air requirement.

In addition, it should also be noted that different system configurations yield different variation patterns of system performance. More advanced system configurations than basic design such as anode or cathode gas re-circulation generate higher power supply and thus each configuration concept might not be applied in the same way.

Based on this study, future work that should be done includes the simulation of the model based on the energy demands of real buildings and the economic assessment of SOFC systems.

## NOMENCLATURE

$A_{ra}$ : surface area of the reforming reaction ($m^2$)
$C_{p,A}$ : specific heat of the air stream (J/g•K)
$C_{p,F}$ : specific heat of the fuel gas (J/g•K)
$E_{CH4}$ : 82 kJ/mol
$E_N$ : Nernst Potential (V)
$E^o$ : Gibbs potential (V)
$i$ : cell current (A)
$k_{CH4}$ : 4274 mol•m$^{-2}$•s$^{-1}$•bar$^{-1}$
$p_{CH4}$ : partial pressure of methane (Pa)
$P_{cell}$ : electricity power output (W)
$p_{H2}$ : partial pressures of hydrogen (Pa)
$p_{O2}$ : partial pressures of oxygen (Pa)
$p_{H2O}$ : partial pressures of steam (Pa)
$Q_{rad,A}$ : radiation between the solid cell and the air (W)
$Q_{rad,F}$ : radiation between the solid cell and fuel side interconnect (W)
$Q_{conv,A}$ : convection between solid cell and the air (W)
$Q_{conv,F}$ : convection between the solid cell and fuel channel (W)
$Q_{conv,A-I}$ : convection between the interconnect and the air stream (W)
$Q_{conv,F-I}$ : convection between the interconnect and the fuel gas (W)
$r_{CH4}$ : reaction rate of steam reforming (mole/s)
$r_{ws}$ : reaction rate of water-gas shifting reaction (mole/s)
$r_E$ : reaction rate of electrochemical oxidation (mole/s)
$R_{con}$ : voltage loss due to concentration polarization (Ω)
$T$ : cell temperature (K)
$V_{cell}$ : cell voltage (V)
$V_A$ : volume of the air stream ($m^3$)
$V_F$ : volume of the fuel gas ($m^3$)
$\rho_A$ : density of the air stream (kg/$m^3$)
$\rho_F$ : density of the fuel gas (kg/$m^3$)
$\Omega_{ohm}$ : voltage loss due to ohmic resistance (Ω)
$\Omega_{pol}$ : voltage loss due to ohmic resistance (Ω)

## REFERENCES

E. Achenbach, 1994, Three-dimensional and Time-dependent Simulation of a Planar Solid Oxide Fuel Cell Stack, Journal of Power Sources, 49, pp. 333-348

P. Costamagna and K. Honegger, 1998, Modeling of Solid Oxide Heat Exchanger Integrated Stacks and Simulation at High Fuel Utilization, Journal of Electrochemical Society, 145, pp. 3995-4007

N. F. Bessette Ⅱ, W. J. Wepfer and J. Winnick, 1995, A Mathematical Model of a Solid Oxide Fuel Cell, Journal of Electrochemical Society, 142, pp. 3792-3800

A. F. Massardo and F. Lubelli, 2000, Internal Reforming Solid Oxide Fuel Cell-Gas Turbine Combined Cycles, Journal of Engineering for Gas Turbines and Power, 122

X. Xue, J. Tang, N. Sammes and Y. Du, 2005, Dynamic Modeling of Single Tubular SOFC Combining Heat/mass Transfer and Electrochemical Reaction Effects, Journal of Power Sources, 142, pp. 211-222

R. Bove and S. Ubertini, 2006, Modeling Solid Oxide Fuel Cell Operation: Approaches, Techniques and Results, Journal of Power Sources, 159, pp. 543-559

G. C. Karvountzi, 2005, Modeling of High Temperature Fuel Cells and Their Integration with Turbines, Ph.D. Thesis, Columbia University, USA

R. Bove, P. Lunghi and N. M. Sammes, 2005, SOFC Mathematical Model for Systems Simulations – Part 2: Definition of an Analytical Model, International Journal of Hydrogen Energy, 30, pp. 189-200

A. M. Murshed, B. Huang and K. Nandakumar, 2007, Control Relevant Modeling of Planar Solid Oxide Fuel Cell System, Journal of Power Sources, 163, pp. 830-845

E. Riensche, J. Meusinger, U. Stimming and G. Unverzagt, 1998, Optimization of a 200 kW SOFC Cogeneration Power Plant. Part 2: Variation of the Flowsheet, Journal of Power Sources, 71, pp. 306-314

The U.S. Energy Information Administration Web-page, www.eia.doe.gov, 2007