

ENUMERATING A DIVERSE SET OF BUILDING DESIGNS USING DISCRETE OPTIMIZATION

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ABSTRACT

Numerical optimization is a powerful method for identifying energy-efficient building designs. Automating the search process facilitates the evaluation of many more options than is possible with one-off parametric simulation runs. However, input data uncertainties and qualitative aspects of building design work against standard optimization formulations that return a single, so-called optimal design.

This paper presents a method for harnessing a discrete optimization algorithm to obtain significantly different, economically viable building designs that satisfy an energy efficiency goal. The method is demonstrated using NREL's first-generation building analysis platform, Opt-E-Plus, and two example problems. We discuss the information content of the results, and the computational effort required by the algorithm.

INTRODUCTION

Building design is a multidisciplinary endeavor with qualitative and quantitative aspects. Input data, especially costs and long-term characteristics of equipment use and performance, are highly uncertain. Nonetheless, professional and governmental organizations provide design guidance (through standards and voluntary guides) to improve energy efficiency cost effectively.

Because of ambitious energy efficiency goals, this guidance is often informed by sophisticated design and analysis methods that rely on simulation engines. One approach, which has been used to provide guidance for Advanced Energy Design Guides at the 50% energy savings level, is numerical optimization, which automatically generates numerous candidate designs in search of a single optimal design (Hale et al. 2009; Leach et al. 2009). Candidate designs are evaluated using EnergyPlus, because it can be used in parallel on Linux cluster computers and has a comprehensive set of modeling features (Crawley et al. 2008).

However, it is widely appreciated that there is no single best design for a given building type in a given location. Different owners and occupants have different values and resources (Papamichael and Protzen 1993). Thus, the purpose of this paper is to present a method to harness numerical optimization to identify a number of good designs that can be evaluated by a project's stakeholders.

The core idea is to run multiple optimization searches, each with a significantly modified search space. The mod-

ifications are designed to extract information about which design strategies are necessary to reach the energy efficiency goal, which are optional, and which can be used to compensate for others that have been excluded from a given search. For instance, if plug and process loads cannot be reduced from their baseline values, perhaps a more efficient HVAC system can make up the gap. An important property of the generated solutions is that they are significantly different from one another.

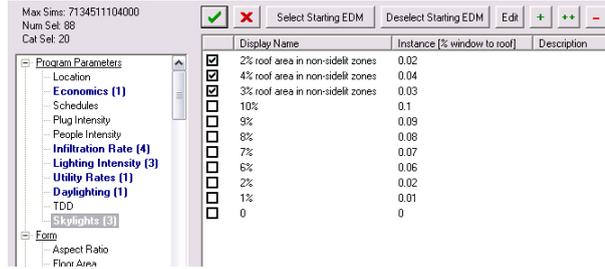
The optimization literature, including the subfield of optimal building design, focuses primarily on searches for a single optimal solution (Bouchlaghem and Letherman 1990; Christensen, Barker, and Horowitz 2004; Wetter 2001; Wright, Loosemore, and Farmani 2002). Significant deviations that come closer to what this work is trying to do include algorithms for finding feasible solutions (Bonami et al. 2009), and combinatorial optimization algorithms that find the k -best solutions to a given problem, that is, in the course of identifying the optimal solution, they keep track of and report the $k - 1$ runners-up as measured by objective function value (Hamacher and Queryranne 1985; Piper and Zoltners 1976). The work described in this paper differs from feasible solution algorithms in that solutions with better objective values are preferred, and from the k -best solution algorithms in that it further requires the solutions found to be significantly different from one another.

The algorithm presented in this paper, and the information that can be extracted from it, are highly influenced by the building energy analysis platform used by the authors. The optimization problem formulation embedded in the platform is described in the next section. An algorithm for finding multiple, diverse designs that all satisfy an energy efficiency (or other goal) follows in the ALGORITHM section. Numerical results, discussion, and conclusions follow.

SETTING

The analysis platform solves bi-objective building design problems over a set of discrete options called energy design measures (EDMs) (Ellis et al. 2006). Most often, the two objectives are to minimize an energy metric and an economic metric. The search starts at a baseline building and proceeds by constructing numerous candidate designs through the addition and removal of EDMs. By default, all designs are plotted on a graph of the two objective functions. The minimum cost designs at every level of en-

ergy use comprise the minimum cost curve. The portion of the curve containing designs that are more energy efficient than the minimum cost building is the Pareto front, which is the set of designs for which one objective cannot be improved without reducing performance with respect to the other objective.



Credit: Elaine T. Hale/NREL

Figure 1: EDM selection and editing

The set of EDMs to optimize over is chosen through a graphical user interface (GUI), a portion of which is depicted in Figure 1. The EDMs are grouped into functional categories that implicitly define a disjunctive constraint, that is, exactly one EDM from each category must be included in each design (although some EDMs may have no meaning in a given design, for instance, skylight construction in a design with no skylights installed). Every category includes a baseline selection that is either defined explicitly in the EDM tree or in the baseline building model, or is implicitly defined as null. Figure 1 shows the EDM selections for skylight amounts used in the full scale example. The baseline selection is 0% skylight coverage.

The design optimization problem embodied in the analysis platform can be expressed mathematically as:

$$x^* = \arg\{\min_x f(x), g(x) | x_i \in \{0, \dots, n_i\} \subset \mathbb{Z}\} \quad (1)$$

where x_i represents the EDM selection for category i , $x_i = 0$ is the baseline selection for category i , n_i is the number of non-baseline selections in category i , $f(x)$ is the economic objective function, and $g(x)$ is the energy efficiency objective function.

In what follows, it sometimes makes sense to group multiple EDM categories together to form a superset we call a design strategy. For instance, daylighting controls and skylights could together comprise a daylighting strategy. Such supersets fit into the framework of Equation 1 once all combinations of the EDMs in the individual categories are enumerated. There is still one baseline option with all categories set to baseline. All other possibilities (for instance, 2% skylight coverage and no daylighting controls) are assigned a nonzero integer.

ALGORITHM

We now present an algorithm that uses a discrete optimization solver to enumerate solutions that all (a) meet a single quantitative goal, and (b) are qualitatively different. The algorithm relies on the assumptions:

1. The decision variables are discrete,

$$x_i \in \{0, 1, \dots, n_i\} \subset \mathbb{Z}, i = 1, \dots, N, \quad (2)$$

with $x = 0$ representing a baseline decision. To obtain diverse solutions (qualitatively different), we further require each variable to represent a distinct type of decision.

2. There is a hard constraint

$$g(x) \leq 0, g(x) \in \mathbb{R}, \quad (3)$$

whose satisfaction indicates feasibility (for instance, meeting an energy efficiency goal).

3. There is a search algorithm

$$P(x^0, x^*, J, K) \in D \subset \mathbb{Z}^N \quad (4)$$

as described in Algorithm 1. Any combinatorial or discrete optimization solver that can handle inequality constraints and/or bicriteria problems should be satisfactory.

In our setting, solutions are building designs defined by EDM selections, $g(x)$ represents an energy savings goal, and our secondary objective $f(x)$ is a cost metric, usually a lifecycle cost that accounts for capital, maintenance, and energy costs accumulated over an analysis period.

Algorithm 1 Search Algorithm $P(x^0, x^*, J, K)$

Require: x^0 , starting point

Require: x^* , reference feasible point

Require: $J, K \subset \{1, \dots, N\}, J \cap K = \emptyset$

Require: $x_J^0 = 0, x_K^0 = x_K^* \neq 0$

return x

Ensure: $x_J = 0, x_K = x_K^*$

Ensure: $g(x) \leq 0$ if possible, otherwise, $x = \arg \min g(x)$

Ensure: $x = \arg \min f(x)$ s.t. $g(x) \leq \max\{0, \min g(x)\}$, that is, x minimizes a secondary objective

The enumeration of alternative designs begins after running the first search, $P(0, 0, \emptyset, \emptyset)$, and proceeds recursively. In short, for a given design x^* that satisfies the hard constraint, each strategy used in that design ($\{i | x_i^* > 0\}$) is iteratively removed from consideration (set to zero), and the search algorithm is warm-started from the resulting point. We thereby generate new candidate designs that

hopefully satisfy the hard constraint (energy efficiency goal) while minimizing a secondary objective (lifecycle cost) over a reduced search space, and are qualitatively different from the original feasible point in that the new point has $x_i = 0$ but the original point has $x_i > 0$. This idea is expressed in Algorithm 2.

Algorithm 2 runIteration(x^*, J, K, P)

Require: $x^* = P(\cdot, \cdot, J, K)$, $x_j^* = 0$, $x_K^* > 0$
for $j \in \{i | x_i^* > 0\} - K$ **do**
 $\hat{J} = J \cup \{j\}$
 $x^0 = x^*$
 $x_j^0 = 0$
 $\hat{x} = P(x^0, x^*, \hat{J}, K)$
 if $g(\hat{x}) \leq 0$ **then**
 queueIteration(\hat{x}, \hat{J}, P)
 else
 $K = K \cup \{j\}$
 end if
end for
setUninitializedKsInQueue(K)
(x^*, J, K, P) = popIterationQueue()
runIteration(x^*, J, K, P)

Algorithm 2 naturally leads to a tree structure of results, with the optimal decision over the full search space (the solution returned by $P(0, 0, \emptyset, \emptyset)$) as the root node. Each level in the tree is associated with a certain number of strategies turned off, that is, a certain cardinality in the set J . The root node has $J = \emptyset$, its children have $|J| = 1$, etc. However, there can be multiple paths to the same search, so our implementation does not queue searches whose J , K , x_K^* signature matches an existing search. This breaks the tree structure; see Figure 2. Future work includes restructuring the algorithm to more accurately reflect the structure of the enumerated set of solutions.

The solution returned by $P(0, 0, \emptyset, \emptyset)$ (Node 0 in Figure 2) is the optimal solution over the full search space, since for that search all elements are allowed to vary over their full domain. It is therefore guaranteed to have the best objective function value compared to all other solutions found by the algorithm, as subsequent searches explicitly reduce the size of the search space. Recall, however, that the objective function is not the absolute measure of goodness for a given design. Building designs are generally subject to multiple quantitative and qualitative criteria that are difficult, if not impossible, to capture in a single quantitative metric.

Running a search with decision variable x_i set to 0 necessarily leads to one of two conclusions (in the context of Algorithm 2): either strategy i is not required to reach the goal, and we can continue to look for more designs that have $x_i = 0$; or strategy i is required to meet the goal. In

the latter case, to make sure that each search results in a reduction of the subsequent search spaces, we add index i to the set K . Then grandchildren of the point x^* have x_i set equal to the value of that index in the parent search. This convention is overly rigid—the optimal decision for x_i will certainly be nonzero, but may not be equal to that of its parent. Nonetheless, this is the heuristic used in our current code.

Our implementation of Algorithm 2 saves all the items listed (x^*, x^0, \hat{x}, J, K) plus the objective function values and a pointer to even more information about each saved design. Keeping x^0 enables us to report perturbation information about individual strategies. In particular, we report the difference in key metrics between feasible designs (x^*), and those designs with one strategy removed (x^0). This provides the information needed to answer questions like: What happens if we take an energy efficient design and remove the daylighting infrastructure? What is the quantitative difference in energy efficiency? cost? This type of data tells us which strategies are valuable with regards to energy, power demand, and water savings per investment dollar. Example perturbation data are provided in Table 2.

NUMERICAL RESULTS

We now demonstrate Algorithm 2 in the context of energy-efficient building design.

ILLUSTRATIVE EXAMPLE

The first example is a 100 m² office building located in St. Louis, Missouri, subject to a few design strategies. The baseline building was generated using Opt-E-Plus, with ASHRAE Standards 90.1-2004, and 62.1-2004 automatically applied (ASHRAE 2004a; ASHRAE 2004b). A design is considered feasible if it has a net site energy savings of 20%. The net site energy use of the baseline building is 770 MJ/m²y, and the analysis period used to calculate life cycle cost is 10 years. We use Opt-E-Plus's sequential search algorithm as P (Andersen, Christensen, and Horowitz 2006; Ellis et al. 2006).

The strategies available to meet the energy efficiency goal are:

Plug load density reduction (PLD). One EDM category. Baseline and reduced value, respectively: 8.07 W/m², 2.69 W/m².

Lighting power density reduction (LPD). One EDM category. Baseline and reduced value, respectively: 14.0 W/m², 11.0 W/m².

Daylighting (DL). Two EDM categories. Daylighting controls can be installed with a set point of 400 lux. Tubular daylighting devices (TDDs) can be installed at a density of 18.58 m²/device.

Table 1: Alternative illustrative example designs. Strategies used and summary metrics. A strategy is “included” if at least one EDM category in the strategy is set to a non-baseline value.

Node	Included Strategies				PV	Lifetime	Capital	Peak	Energy
	PLD	LPD	DL	AR	Energy	Cost	Cost	Demand	Savings
					MJ/m ² y	\$/m ²	\$/m ²	kW	%
0	X	X	X		0.0	552	454	4.70	21.3
1		X	X		59.0	681	579	4.79	20.0
2	X		X		0.2	555	455	4.82	20.0
3	X	X			49.9	662	561	5.18	20.0
4			X		70.7	707	605	4.82	20.0
5		X		X	126.2	814	715	4.95	20.0
6	X			X	84.1	725	627	4.91	20.0
7					149.4	877	772	5.61	19.4
8					149.4	877	772	5.61	19.4
9		X			122.9	818	715	5.37	20.0
10	X				78.9	724	623	5.34	20.0
\bar{x}					81.0	727	625	5.10	
$100\sigma/\bar{x}$					65	16	18	6.6	

Aspect ratio (AR). One EDM category. The building can be stretched to an aspect ratio of 1:4 along either axis. The baseline building has an aspect ratio of 1:1.

In addition, up to 30% of the roof area not covered by TDDs can be covered with photovoltaic (PV) panels (10% efficient, 90% inverter efficiency). During sequential search, 30% PV coverage is simply another EDM. However, when alternative designs are being enumerated, if PV is required to reach the energy savings goal, the actual amount of PV is dialed back so that the resulting building just meets the goal. Also, PV is always available as a search option—it is not enabled and disabled like the other strategies.

The alternative designs found by the enumeration algorithm are depicted in Figure 2 and listed in Table 1. The initial point found by the search over the full space is represented by node 00. Three strategies are used in that point: plug load density reduction, lighting power density reduction, and daylighting. The algorithm removes those strategies in turn, and finds that none is necessary to reach the efficiency goal. However, reaching the goal now requires some PV. Going further down the tree, we can see, for instance, that with the lighting power density at baseline value, and no daylighting infrastructure available, Node 06 is able to meet the goal after it changes the aspect ratio and includes PV. The goal cannot be met if all three strategies used in the optimal point are removed from consideration.

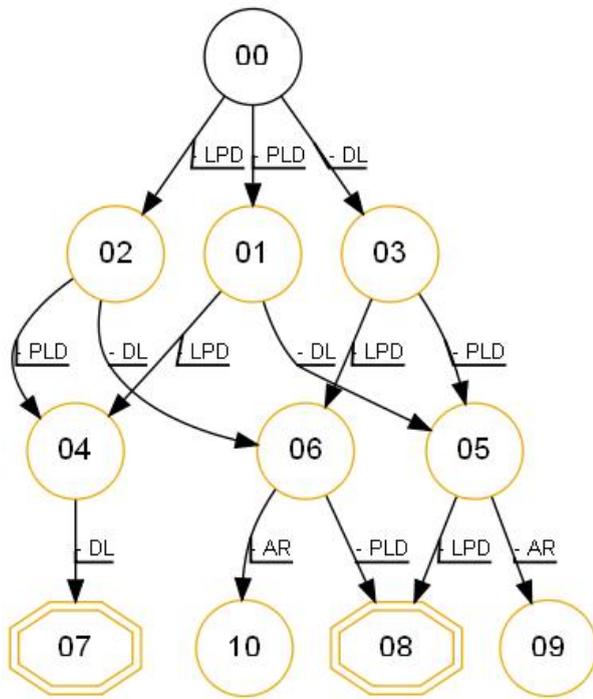
Table 1 provides additional summary information about

the alternative designs. In addition to a diversity of strategies used to meet the energy efficiency goal, other metrics vary significantly. For instance, the standard deviation of the capital cost intensities is 18% of the mean value, and the standard deviation of peak electricity demand is 6.6% of the mean value. Echoing comments made in the ALGORITHM section, node 00 has the lowest lifetime cost, that is, the lowest objective function value, but a given decision maker may be just as interested in reducing peak demand, or in keeping plug load levels at their baseline value.

For this small example, the computational impact of enumerating multiple feasible designs is modest. The original search required 71 EnergyPlus simulations, at about 30 s each on a Dell Latitude XT and a Linux cluster computer. Subsequent searches required fewer simulations total, and were able to reuse simulations from previous searches. On average, there were 14.7 total simulations and 6.4 new simulations per alternative design search. However, there were 11 additional searches, so the extra computational effort was about the same as doing the original search over the full space. In return, the enumeration provided nine new designs (nodes 7 and 8 are identical), of which eight met the energy efficiency goal.

FULL SCALE EXAMPLE

A partial run of Algorithm 2 was completed for an example taken from recent work on grocery store design (Leach et al. 2009). A fourteen zone prototypical grocery store model was instantiated in sixteen locations by applying ASHRAE Standard 90.1-2004 and ASHRAE



Credit: Elaine T. Hale/NREL

Figure 2: Diverse solutions for the illustrative example. Nodes represent designs, arcs represent the paths to get from the optimal design to the alternative designs. Designs marked in gold include photovoltaic (PV) panels. Designs marked with octagons were not able to reach the 20% energy efficiency goal, even with 30% of the roof covered with PV.

Standards 62.1-1999 (ASHRAE 2004a; ASHRAE 1999). Those baseline models were then optimized with an overall goal of 50% net site energy savings at minimum life cycle cost with an analysis period of five years. Here we optimize and then enumerate alternative designs for the baseline building in San Francisco, California. The EDMs are grouped into fourteen strategies:

Infiltration (IN). One EDM category that combines air barrier and vestibule EDMs. Four options including baseline.

Elec. Lighting (LPD). One EDM category that addresses lighting power density. Three options including baseline.

Daylighting (DL). Two EDM categories: daylighting controls and skylights. One daylighting control set point (500 lux) and three skylight amounts (2%, 3%, and 4%) available, for a total of eight options including baseline.

Window Area (WA). One EDM category that adjusts the

south-facing glazing amount. Two options: baseline and 50% less glazing than baseline.

Wall Insulation (WI). One EDM category. Eight options including baseline.

Roof Insulation (RI). One EDM category. Fourteen options including baseline. Four options include a cool roof membrane as the top layer.

Fenestration Types (FT). Two EDM categories. One for south-facing glazing type; the other for skylight type. There are eight options for south-facing glazing, and ten for skylights.

HVAC. One EDM category. Twelve options including baseline arise from varying the coefficient of performance (COP), fan efficiency, and economizers.

Demand Control Ventilation (DCV). One EDM category. Two options including baseline.

Energy Recovery Ventilation (ERV). One EDM category. Three options including baseline: no ERV, 50% effective ERV, and 70% effective ERV.

Frozen Food Cases (FFC). One EDM category providing six types of frozen food case, including baseline.

Ice Cream Cases (ICC). One EDM category providing six types of ice cream case, including baseline.

Meat Cases (MC). One EDM category providing eight types of refrigerated meat cases, including baseline.

Dairy/Deli Cases (DDC). One EDM category providing five types of refrigerated dairy/deli cases, including baseline.

As in the illustrative example, PV is treated separately from the other EDMs—it is always available, and is used to make up the difference to the energy efficiency goal when necessary.

For this large-scale example, the additional computational burden of enumerating a diverse set of designs was significant. Each EnergyPlus run required about 12 min of simulation time (averaged over runs completed on a Dell Precision, 4-core desktop and a Linux cluster computer), and 5.5 MB of hard drive space. We were unable to run Algorithm 2 to completion because the memory requirements became too large for one desktop computer as we approached 75,000 EnergyPlus simulations. The original search (to find the overall optimal point) required 2,938 EnergyPlus simulations. The enumeration algorithm finished ten iterations, started another, and ran 73 searches to completion. Eleven searches derived from the root node; 63 searches had two strategies set to zero; just one search

in the third level ($|J| = 3$) ran to completion. On average, each additional search required 962 new simulations. Thus, the effort of one full-space search is approximately equivalent to that of three reduced-space searches, a result significantly worse than the ratio of 11:1 seen in the previous example.

The goal of design enumeration was largely accomplished despite the computational drawbacks just described. A total of 69 feasible designs were generated. The optimal point over the full search space used eleven of the fourteen energy efficiency strategies. The remaining three (roof insulation, fenestration type, and DCV) came into play in subsequent searches, once one or more of the original eleven strategies were eliminated.

Table 2 summarizes the energy efficiency strategies used in this example from the perturbation perspective. Each search for an alternative design produces a starting point x^0 that is the same as the initial feasible point x^* except that one strategy has been removed (set to the baseline value). Therefore, differences in the performance of x^0 and x^* shows the value or lack thereof of the strategy. The metrics reported in Table 2 are calculated by subtracting the metric for x^* from that of x^0 . In other words, we report $\text{metric}(\text{without the strategy}) - \text{metric}(\text{with the strategy})$. The first entry in the table, Dairy/Deli Cases, is therefore a strategy that always saves energy and lifetime cost, but requires more up front capital. On the other hand, Electric Lighting is a winner all around—it cost less up front and saves energy.

The last two columns of Table 2 are an estimate of how much PV is required to match the energy savings of the given strategy. In one sense it is a simple unit conversion of the EUI Savings columns. However, it does provide a way to visualize the value of an energy efficiency measure, as in, “I can daylight my store”, or “I could add 24 to 369 m² of PV panels to achieve similar carbon dioxide emission reductions.”

DISCUSSION

Overall, the proposed algorithm accomplishes its goal: it is able to identify multiple energy-efficient building designs that all satisfy a specific energy efficiency goal, and are significantly different from one another. In addition, perturbation data help identify high-value design strategies. Research continues on how to communicate these results to potential users.

One communication problem that is tied up with the organization of the algorithm is EDM hiding. In particular, because the algorithm operates on EDM categories as a whole (or even bundled with other categories) it is not immediately clear which EDMs are present in each design. In some sense it is an easy matter to remedy this situation—simply create larger tables that list all the EDMs. On the other hand, such verbose information be-

comes hard to absorb. Ideally, decisions in a given category would be grouped into meaningful levels. Then a strategy could be implemented at the baseline level, level 1, level 2, etc. We expect to investigate this idea after a more natural data structure for the algorithm as a whole is identified.

The computational issues encountered for the full-scale example could be mitigated by stopping sequential searches soon after the energy efficiency goal is met. For illustration purposes, see Figure 3, which depicts the full space search’s Pareto front in black, and the reduced space searches’ Pareto fronts in purple. The search highlighted by the orange circle didn’t really need to run—the starting point meets the 50% energy savings goal. In other cases, when some search is required because the starting point is not feasible, it may be worth the computational time and memory savings to stop the search early even though this may preclude finding the actual optimal point for that search. An even better solution to this problem would be to use an optimization algorithm that directly solves the problem at hand, $\{\min f(x) | g(x) \leq 0\}$, rather than adapting a bi-objective solver for the purpose, as such an algorithm should be equipped with better stopping criteria.

CONCLUSION

An algorithm for generating a number of feasible solutions to a combinatorial optimization problem in approximate rank order of objective function value was presented. If each decision variable represents a distinct aspect of the problem, the set of decisions so generated will be diverse in that they will be qualitatively different from one another. One-off perturbations of feasible points are also generated, and provide valuable information concerning the individual strategies that compose the overall problem.

Numerical results for energy-efficient building design problems suggest that the additional computational effort is modest for small problems. For larger problems, the additional effort can be considerable. Stopping searches short once a feasible solution has been found, or using a more appropriate search algorithm, should remedy the situation to a large extent.

ACKNOWLEDGMENTS

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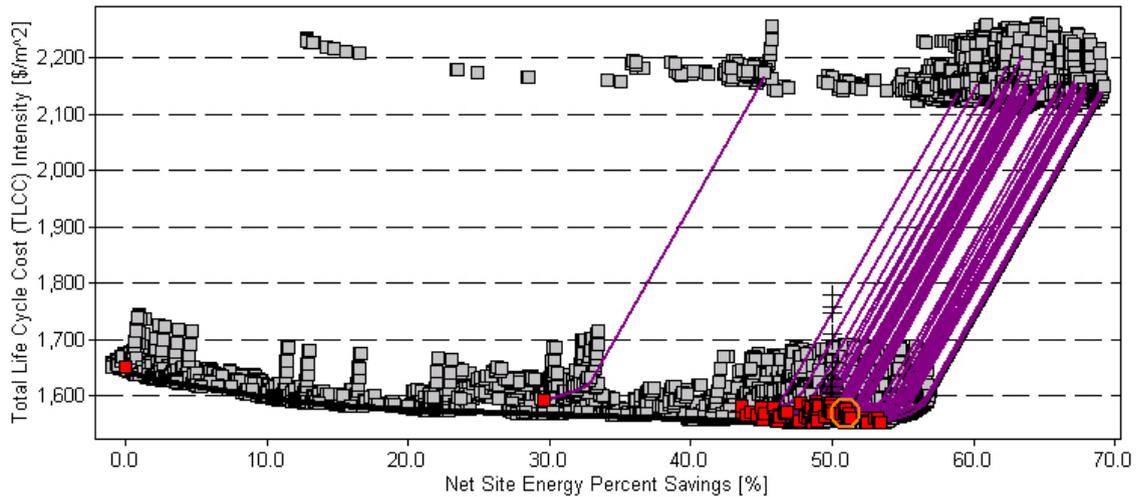
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Table 2: Grocery store perturbation data

Strategy	Count	EUI Savings (MJ/m ² yr)		Lifetime Cost Savings (\$/m ²)		Capital Cost Savings (\$/m ²)		Equivalent PV (m ²)	
		min	max	min	max	min	max	min	max
Dairy/Deli Cases	1	626.2	626.2	40.56	40.56	-9.87	-9.87	4501	4501
Daylighting	8	3.4	51.4	-14.90	1.27	-21.12	0.11	24	369
DCV	1	33.0	33.0	-5.37	-5.37	-4.66	-4.66	238	238
Elec. Lighting	5	30.7	64.9	9.94	14.6	0.97	2.31	221	467
ERV	9	118.6	193.8	-1.60	3.94	-7.42	-7.27	853	1393
Fenestration Type	2	4.3	10.1	-2.33	-0.48	-1.12	-0.92	31	72
Frozen Food Cases	1	232.3	232.3	13.75	13.75	-12.23	-12.23	1670	1670
HVAC	9	46.9	193.1	3.82	14.88	-3.68	-0.87	337	1388
Ice Cream Cases	9	57.5	60.7	8.91	9.24	2.87	2.9	413	436
Infiltration	4	24.6	40.7	0.95	5.27	-0.75	2.1	177	292
Meat Cases	9	118.5	132.2	0.09	1.01	-8.14	-8.08	852	950
Wall Insulation	8	-0.1	42.6	-0.34	1.33	-1.23	0	-1	306
Window Area	7	-3.6	35.3	5.25	10.06	5.58	8.41	-26	254

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Credit: Elaine T. Hale/NREL

Figure 3: Demonstration of possible computational savings. The search to which the highlighted starting point belongs did not need to run since the initial point already meets the energy savings goal. The cross hairs are x^* designs that include PV. The red points denote search start points (x^0 's).

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NOMENCLATURE

\emptyset	the empty set
\in	the entity on the left is an element of the set on the right
\subset	the set on the left is a subset of the set on the right
\mathbb{R}	the set of all real numbers
\mathbb{Z}	the set of all integers
$f(x)$	$\in \mathbb{R}$, objective function over x
$g(x)$	$\in \mathbb{R}$, sometimes objective function, sometimes hard constraint over x
n_i	$\in \mathbb{Z}$, number of non-baseline decisions available for variable x_i
x	$\in D \subset \mathbb{Z}^N$, vector of integer decision variables
x^*	feasible point used as the seed for an alternative design iteration
\hat{x}	best point found by a reduced space search, may or may not be feasible
x^0	starting point for a reduced space search
x_i	i^{th} element of x
x_J	components of x whose indices are in J
COP	coefficient of performance
EDM	energy design measure
D	the domain of x , $D_1 \times D_2 \times \dots \times D_N$
D_i	the domain of x_i , $\{0, 1, \dots, n_i\}$
GUI	graphical user interface
J	set of indices, a subset of $\{1, \dots, N\}$
$ J $	the cardinality of J
K	set of indices, a subset of $\{1, \dots, N\}$
N	number of integer decision variables
P	search algorithm
PV	photovoltaics
TDD	tubular daylight device