REDUCED-ORDER BUILDING MODELING FOR APPLICATION TO MODEL-BASED PREDICTIVE CONTROL

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ABSTRACT

This paper presents methods and results for representing the complex thermal network of a building envelope and interior in the form of reduced-order state-space equations that can more easily be applied in model-based predictive and other advanced control approaches. The complexities of heat transfer phenomena through glazings and long wavelength exchanges among walls make the representation difficult. The model employs the net radiosity method for long-wave interaction, one-dimensional transient conduction through walls, conductive and convective coupling between zones, etc. Model order reduction is applied to simplify the state-space representation and case study results are presented.

INTRODUCTION

The application of Model Predictive Control (MPC) to buildings could have high computational requirements depending on the number of control variables, the time horizon for the optimization, the discretization for control decisions and model solution, and the complexity of the models. Reducing model complexity and computational requirements, while retaining prediction accuracy, is the goal when considering reduced-order building modeling.

In general, models for control can be grouped into two types: forward and data-driven models. Forward models start from first principles such as conservation equations of mass, energy and momentum. Most of the existing energy simulation programs, such as EnergyPlus, employ forward models for building load requirements. They can be highly accurate but may have a high computational burden especially for application to online control. On the other hand, data-driven models rely on experimental data or simulation results from a physical model to train an empirical or black-box model and can sometimes yield unrealistic and non-physical results (J. Casillas et al. 2003), especially when employed outside of the range where they were trained. One of the most important and difficult problems in the system identification procedure is choosing the “best” model among a set of candidate models (L. Ljung 1999). It necessarily requires “model validation” to choose the “best” model.

The main purpose of the study described here is to develop a general methodology for converting the complex thermal network of a multi-zone building into a reduced-order linear time invariant state-space representation that will more readily enable implementation and assessment of advanced control concepts. This approach also makes it possible to perform frequency response analysis for control design and to obtain some characteristic parameters of the building envelope system, such as eigenvalues that are not available with the current models.

The complexities of heat transfer phenomena through glazings and long wavelength exchanges among walls make the representation difficult. In the detailed reference model used to determine a reduced-order model (ROM), the net radiosity method for long-wave interaction and a simple window model are adopted to construct a general form which includes heat transfer through windows, long-wave interaction among surfaces, conduction through walls, distributed thermal capacitance within walls, conductive heat transfer between the rooms and so on. After constructing a state-space representation for the reference model, a model order reduction method is applied to the forward model to construct a reduced-order model .

The performance of a ROM for a building envelope was tested for case a study with respect to both open-loop and closed-loop responses in both the time and frequency domain. Comparisons with results and computational requirements for TRNSYS are provided.

MATHEMATICAL MODELING

State-Space Representation of a Thermal Building Model

The terminology of a zone that is used in this section is assigned to a unit or module of the thermal network of a room. A multi-zone representation is a group of thermal network units. The state-space model of a building system starts from a module (network unit) and extends to constructing more complex multi-zone models. A detailed formulation is described in this section. Some of the important assumptions used to construct the network unit are:

- The temperature of each surface or surface segment and of its cross section is uniform.
- A room is well-stirred.
- Each wall emits or reflects diffusely and is gray and opaque.
- Air is nonparticipating media to radiation.
- Heat transfer is one dimensional.
- Conduction between window and window frame is neglected (1-D assumption).

The main approach to treating the complex thermal network is to linearize long-wave radiation exchange and group the states (temperatures) and fundamental equations in the form of a state-space representation.

Conduction through Walls

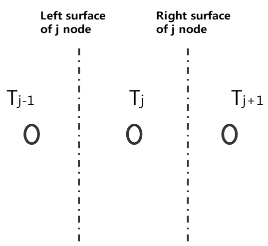

Figure 1: Notation for conduction through walls.

A finite volume formulation is used to describe the heat conduction through walls and is depicted in Figure 1. For any j^{th} node in a wall except the first and last nodes, an energy balance leads to

$$\rho_j^i C_j^i w_j^i \frac{dT_j^i}{dt} = h_{cd}^L \,\big|_j^i\, T_{j-1}^i - (h_{cd}^L \,\big|_j^i + h_{cd}^R \,\big|_j^i) T_j^i + h_{cd}^R \,\big|_j^i\, T_{j+1}^i + q_{gen\,j}^{\;i} \quad (1)$$

where $h_{cd}{}^L \,\big|_j^i = \frac{k^L \,|_j^i}{w^L \,|_j^i}$ and $q_{gen\,j}^{\;i}$ is an energy source [W/m^2] inside the j^{th} finite control volume that belongs to the i^{th} wall.

By using the following matrix notation,

$$\vec{q}_j \equiv \begin{bmatrix} q_{gen\,j}^{\;1} \\ q_{gen\,j}^{\;2} \\ .. \\ q_{gen\,j}^{\;Nw} \end{bmatrix}, \; \vec{T}_j \equiv \begin{bmatrix} T_j^1 \\ T_j^2 \\ .. \\ T_j^{Nw} \end{bmatrix},$$

$$\tilde{H}_j^L \equiv \begin{bmatrix} h_{cd}^L \,|_j^1 & & & 0 \\ & h_{cd}^L \,|_j^2 & & \\ & & .. & \\ 0 & & & h_{cd}^L \,|_j^{Nw} \end{bmatrix} \text{ and}$$

$$\tilde{C}_j \equiv \begin{bmatrix} \rho C w \,|_j^1 & & & 0 \\ & \rho C w \,|_j^2 & & \\ & & .. & \\ 0 & & & \rho C w \,|_j^{Nw} \end{bmatrix}$$

Equation (1) can be expressed in the following matrix form.

$$\tilde{C}_j \frac{d\vec{T}_j}{dt} = \tilde{H}_{cd,j}^L \vec{T}_{j-1} - (\tilde{H}_{cd,j}^L + \tilde{H}_{cd,j}^R)\vec{T}_j + \tilde{H}_{cd,j}^R \vec{T}_{j+1} + \vec{q}_j \quad (2)$$

Note that $\vec{T}_j$ is a group of all of the wall temperature nodes belonging to an individual zone.

Heat Balance at Outside Surface

For any i^{th} outside wall (connected to the external ambient) belonging to an individual zone, the heat balance equation at the surface is

$$\rho_1^i C_1^i w_1^i \frac{dT_1^i}{dt} = h_{cv,ex}^i (T_a - T_1^i) + h_{cd}^R \,\big|_1^i\, (T_2^i - T_1^i) + \alpha_1^i q_{SWR}^i + q_{LWR}^i \quad (3)$$

where T_1^i represents the wall temperature of the first node which is set to be an outside surface of the wall.

With the assumptions that the outside surface is gray and diffuse and the air is a non-participating radiation media, net long wavelength interactions with the environment can be expressed as

$$q^i_{LWR} = \sigma\epsilon^i_1 F^i_{sky}(T^4_{sky} - T^{i4}_1) + \sigma\epsilon^i_1 F^i_{grd}(T^4_{grd} - T^{i4}_1)$$

Using a linear approximation of the long-wave heat exchange term gives

$$\rho^i_1 C^i_1 w^i_1 \frac{dT^i_1}{dt} = -(h^i_{cv,ex} + h^R_{cd}|^i_1 + h^i_{rad,ex})T^i_1 + h^R_{cd}|^i_1 T^i_2 + q^i_1 \quad (4)$$

where

$$h^i_{rad,ex} = 4\sigma\epsilon^i_1(F^i_{sky}\overline{T}^3_{sky} + F^i_{grd}\overline{T}^3_{grd}),$$

$$q^i_1 = h^i_{cv,ex}T_a + 4\sigma\epsilon^i_1(F^i_{sky}\overline{T^3_{sky}}T_{sky} + F^i_{grd}\overline{T^3_{grd}}T_{grd}) + \alpha^i_1 q^i_{SWR}$$

$$\overline{T}_{sky} = \frac{T_{sky} + T_1}{2},\ \overline{T}_{grd} = \frac{T_{grd} + T_1}{2}$$

The mean temperatures for long-wave exchange between the surface and sky and surface and ground are assumed to be the same for all outside surfaces.

Equation (4) can be generalized and written in compact matrix form as

$$\tilde{C}_1 \frac{d\vec{T}_1}{dt} = -(\tilde{H}^R_{cd,1} + \tilde{H}_{rad,ex} + \tilde{H}_{cv,ex})\vec{T}_1 + \tilde{H}^R_{cd,1}\vec{T}_2 + \vec{q}_1 \quad (5)$$

where

$$(\vec{q}_1)_i = h^i_{cv,ex}T_a + 4\sigma\epsilon^i_1(F^i_{sky}\overline{T}^3_{sky}T_{sky} + F^i_{grd}\overline{T}^3_{grd}T_{grd}) + \alpha^i_1 q^i_{SWR}$$

Heat Balance at Inside Surface

For the I[th] zone and i[th] wall, the energy balance equation for the inside surface is

$$\rho^i_n C^i_n w^i_n \frac{dT^i_n}{dt} = h^i_{cv,in}(T_z - T^i_n) + h^L_{cd}|^i_n (T^i_{n-1} - T^i_n) - q^i_{net,rad} \quad (6)$$

where $q_{net,rad}$ is net radiative flux out of the inside wall.

The radiosity method is utilized to express the net flux under the assumption that the walls are gray, diffuse and opaque. The same linearization method used in Equation (4) is employed leading to

$$\vec{q}_{net,rad} = \tilde{A}^{-1}[\tilde{B}'\vec{T}_n - \vec{h}_o] \quad (7)$$

where,

$$\tilde{A}_{ij} \equiv \frac{\delta_{ij}}{\epsilon_j} - \frac{\rho_j}{\epsilon_j}F_{ij}$$

$$\tilde{B}'_{ij} = 4\sigma(\delta_{ij} - F_{ij})\overline{T}^3$$

Radiosity does not appear explicitly in the above expression, which is convenient for building simulation. Since h^i_o represents an external radiative source acting on the i[th] surface, the effects of internal sources and transmitted solar energy though windows are treated in a consistent manner. For any shaped room, the net radiative flux can be explicitly calculated as a function of surface temperatures if the view factors and the external radiative sources are known.

By letting $\tilde{H}_{rad,in} \equiv \tilde{A}^{-1}\tilde{B}'$ and $\vec{q}_n \equiv \tilde{A}^{-1}\vec{h}_o$

$$\tilde{C}_n \frac{d\vec{T}_n}{dt} = \tilde{H}^L_{cd,n}\vec{T}_{n-1} - (\tilde{H}^L_{cd,n} + \tilde{H}_{rad,in} + \tilde{H}_{cv,in})\vec{T}_n + \vec{h}_{cv,in}T_z + \vec{q}_n \quad (8)$$

State-Space Representation of Thermal Network Module

Gathering the system of equations that represent heat balance equations for external to internal wall elements, i.e. Equations (2), (5) and (8),

$$\tilde{C}_w\dot{\vec{T}}_w = \tilde{H}_{ww}\vec{T}_w + \tilde{H}_{wz}T_z + \vec{q}_w \quad (9)$$

where $\tilde{C}_w = \begin{bmatrix} \tilde{C}_1 & & & \\ & \tilde{C}_2 & & \\ & & .. & \\ & & & \tilde{C}_n \end{bmatrix}$,

$$\tilde{H}_{ww} = \begin{bmatrix} -\tilde{H}_1 & \tilde{H}^R_{cd,1} & & \\ \tilde{H}^L_{cd,2} & -(\tilde{H}^L_{cd,2} + \tilde{H}^R_{cd,2}) & \tilde{H}^R_{cd,2} & \\ & .. & .. & .. \\ & & \tilde{H}^L_{cd,n} & -\tilde{H}_n \end{bmatrix},$$

$$\tilde{H}_1 = (\tilde{H}^R_{cd,1} + \tilde{H}_{cv,ex} + \tilde{H}_{rad,ex}),\quad \tilde{H}_n = (\tilde{H}^L_{cd,n} + \tilde{H}_{cv,in} + \tilde{H}_{rad,in})$$

$\tilde{H}_{wz} = \begin{bmatrix} \mathbf{0}^T & \mathbf{0}^T & .. & (\vec{h}_{cv,in})^T \end{bmatrix}^T$ and

$$\vec{q}_w = \begin{bmatrix} \vec{q}^T_1 & \vec{q}^T_2 & .. & \vec{q}^T_n \end{bmatrix}^T.$$

As expected, a tri-diagonal block matrix is formed with parameters that characterize heat transfer due to conduction in the walls (from node number 2 to node number n-1) and radiative/convective heat transfer at the boundaries (first and last nodes only). The terms $\vec{q}_j$ where j is from 2 to n-1 vanish if there are no heat flux sources inside the wall such as embedded radiant heating or cooling systems.

Because a multi-zone representation is a group of the thermal network units, it can be easily developed based on Equation (9). The form is

$$\tilde{C}_W \dot{\vec{T}}_W = \tilde{H}_{WW}\vec{T}_W + \tilde{H}_{WZ}\vec{T}_Z + \vec{q}_W \tag{10}$$

The coefficient matrices, $\tilde{C}_W$, $\tilde{H}_{WW}$ and $\tilde{H}_{WZ}$, have block diagonal forms consisting of the coefficient matrices for each zone, $\tilde{C}_w, \tilde{H}_{ww}$ and $\tilde{H}_{wz}$. Each variable also represents the set of all individual variables in all zones. For example, $\vec{T}_W$ is the set of all temperature nodes of walls in multi-zones, i.e., $\vec{T}_W = \left[(\vec{T}_w^1)^T \quad .. \quad (\vec{T}_w^{Nz})^T \right]^T$.

State-Space Representation of Zone Air Balance

To complete the state-space representation of a multi-zone building, the dynamics of zone air temperatures need to be included. Heat fluxes to an air control volume within a zone can be categorized as follows.

$\dot{Q}^I_{vent}$ = Ventilation

$\dot{Q}^I_{inf}$ = Infiltration

$\dot{Q}^I_{cpl}$ = Air mixing among rooms

$\dot{Q}^I_S$ = Convective internal source due to computer, human action and so on,

$\dot{Q}_{cv}$ = Convective heat exchange between a zone air and its surrounding walls

The balance equation for the Ith zone is

$$\rho^I C_V^I V^I \frac{\partial T_z^I}{\partial t} = \dot{Q}_{cv} + \dot{Q}^I_{cpl} + \dot{Q}^I_{inf} + \dot{Q}^I_S + \dot{Q}^I_{vent} \tag{11}$$

A linear state-space form was formulated based on Equation (11) under the assumption that the inter-zone mixing and infiltration terms are negligible.

$$\tilde{C}_Z \dot{\vec{T}}_Z = \tilde{H}_{ZW}\vec{T}_W + \tilde{H}_{ZZ}\vec{T}_z + \vec{Q}_S + \vec{Q}_{vent} \tag{12}$$

Due to the limit of pages, the definition of each term is omitted, but one can easily derive the form.

Final State-Space Representation of Thermal Network

Based on Equations of (10) and (12), the state-space representation of the thermal network can be written as

$$\begin{bmatrix} \tilde{C}_W & 0 \\ 0 & \tilde{C}_Z \end{bmatrix} \begin{bmatrix} \dot{\vec{T}}_W \\ \dot{\vec{T}}_Z \end{bmatrix} = \begin{bmatrix} \tilde{H}_{WW} & \tilde{H}_{WZ} \\ \tilde{H}_{ZW} & \tilde{H}_{ZZ} \end{bmatrix} \begin{bmatrix} \vec{T}_W \\ \vec{T}_Z \end{bmatrix} + \begin{bmatrix} \vec{q}_W \\ \vec{Q}_Z \end{bmatrix} \tag{13}$$

The matrices $\tilde{C}$ and $\tilde{H}$ represent the thermal capacitance and heat transfer coefficients of all wall nodes and zone nodes, respectively. The subscript W and Z refer to wall and zone.

When constructing the thermal network in the form of $\dot{x} = Ax + Bu$ from Equation (13), the size of the B matrix will be the same as the size of A. However the matrix size B can be dramatically reduced by introducing a simple transformation denoted by $\tilde{T}'$ in Equation (14), since the heat sources inside the wall are zero except for the case of radiant floor heating and cooling.

The final standard form is

$$\begin{aligned} \begin{bmatrix} \dot{\vec{T}}_W \\ \dot{\vec{T}}_Z \end{bmatrix} &= \begin{bmatrix} \tilde{C}_W & 0 \\ 0 & \tilde{C}_Z \end{bmatrix}^{-1} \begin{bmatrix} \tilde{H}_{WW} & \tilde{H}_{WZ} \\ \tilde{H}_{ZW} & \tilde{H}_{ZZ} \end{bmatrix} \begin{bmatrix} \vec{T}_W \\ \vec{T}_Z \end{bmatrix} \\ &+ \begin{bmatrix} \tilde{C}_W & 0 \\ 0 & \tilde{C}_Z \end{bmatrix}^{-1} \tilde{T}'\vec{u} \\ \vec{y} &= \tilde{C} \begin{bmatrix} \vec{T}_W \\ \vec{T}_Z \end{bmatrix} \end{aligned} \tag{14}$$

Model Order Reduction (MOR) of Thermal Building Network

The purpose of model order reduction (MOR) is to derive a lower order model from a high order system preserving the dominant dynamics of the original high order model. The problem definition of MOR for a linear time invariant system is to construct lower order matrices for A_r, B_r, C_r, D_r, Equation (15), from the original system of Equation (16).

$$G_r : \begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r u(t) \\ y_r(t) = C_r x_r(t) + D_r u(t) \end{cases} \tag{15}$$

$$G_o : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{16}$$

where the state $x(t) \in \mathbb{R}^n$, $x_r(t) \in \mathbb{R}^r$ and $r << n$.

Note that we want to reduce the state order from n to r while keeping the response error $\| G_o - G_r \|_\infty$ small.

($\| \cdot \|_\infty$ is the Hardy infinity norm and G_o and G_r are transfer functions of the original and reduced-order systems, respectively.)

Several model order reduction methods are available, but one representative method employed in the current study is the balanced truncation technique originally proposed by B.C. Moore (1981).

The main idea of the balanced truncation approach is to truncate states which make little contribution to input and output behavior. The importance of states can be measured by the singular values of the Hankel operator defined by

$$(\Gamma v)(t) = \int_0^\infty C e^{A(t+\tau)} B v(\tau) d\tau \qquad (17)$$

In other words, if one can find the Hankel singular values, denoted by Σ, one can judge the contribution of each of the states.

The first step of the state-space transformation technique is to find a transformation which balances the observability and controllability gramians. The controllability gramian and observability gramian are defined by

$$W_c = \int_0^\infty e^{A\tau} B B^\dagger e^{A^\dagger \tau} d\tau, W_o = \int_0^\infty e^{A^\dagger \tau} C^\dagger C e^{A\tau} d\tau ,$$

where the superscript $\dagger$ denotes conjugate transpose.

After balancing the gramians, one can obtain the Hankel singular value via the well known formula of $W_c = W_o = \Sigma$.

The second step is to construct and to perform a Galerkin projection based on the singular values of the balanced gramians (S. Lall et al. 2000). Important features of the approach are the reduced-order model maintains most of the original model properties such as dynamic behavior, observability, controllability and stability (M. Green, D.N.Limebeer, 1995), thereby providing a more reliable model. Furthermore the truncation of error, $\| G_o - G_r \|_\infty$, is bounded by twice the sum of the truncated singular values.

In this study, the balanced truncation method was adopted for constructing a reduced-order model based on the system of Equation (14). An algorithm to compute the state-space balancing transformation is presented by A.J. Laubi (1987).

CASE STUDY RESULTS

For testing the validity of the detailed (full-order) model representation, both steady and dynamic responses were compared for various cases and good agreement was found. The direct stiffness method (FEM) with the exact radiosity solution for long-wave radiation was used to check the steady-state behavior. TRNSYS Version 16 was used to compare dynamic response. In this paper, a case study for the Purdue Living Lab #1 is presented to demonstrate the reduced-order modeling accuracy and computational requirements as compared with the full-order model. Figure 5 shows the building and the specific zone under consideration. Some parameters employed in the modeling include:

- The size of the zone is 32[ft] for width and depth and 14.5[ft] for height. The south window area is 130[ft^2].
- The materials for wall construction consist of concrete, insulation board, stucco, gypsum board, and double glazed windows.
- The east wall and floor are adjacent to other rooms that are assumed to be at a fixed air temperature of 22ºC.
- TMY2 weather data in Indianapolis for the summer season (July 1 to 31) was employed.
- Values of 17.77[W/m^2-K] and 3.05[W/m^2-K] were used for convective heat transfer coefficients at the outside and inside surfaces, respectively.

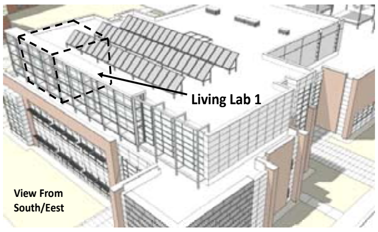

Figure 2: Sketch of Purdue Living Lab

In order to construct a linear time invariant system, average values of T_{sky}, T_{grd} and T_w are needed for linearization of the radiation heat transfer coefficient and are determined prior to simulation. The average sky and air temperature were determined from the TMY2 weather file, whereas the mean external wall temperature was set to 15ºC. The full-order model had a total of 297 nodes for the zone. A reduced-order model was found having 10 states.

Model output comparisons between TRNSYS, the full-order and the reduced-order model (ROM) are shown in Figure 3 for the case of a floating zone temperature for several days during the month of July. The black solid line (T_{Trn}), blue solid line (T_o) and red dashed line (T_r) represent zone air temperature profiles generated by TRNSYS, full-order and reduced-order models, respectively. Overall, the agreement between the three models is very good. The full-order model, which had 297 states, required a simulation time for a month of 3.320 sec with a 1.4GHz laptop computer. On the other hand, the reduced-order model, which had 10 states, had a simulation time of 1.351sec.

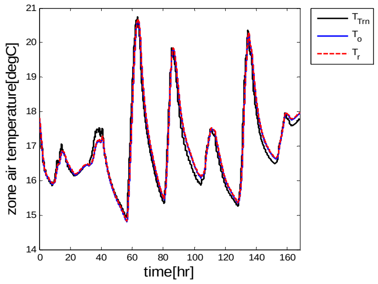

Figure 3: Model comparisons between TRNSYS, full-order, and reduced-order models

Note that computational time for solution of differential equations is highly dependent on the differential or difference equation solver algorithm. The computational times are based on an ODE (ordinary differential equation) solver of MATLAB Version 7. Performance judgment with respect to simulation time relative to TRNSYS is difficult. Furthermore one of the most important differences between the state-space representation and TRNSYS is that the former is continuous, but the latter is represented using discrete equations.

In order to provide reasonable computational comparisons between the ROM and TRNSYS, first-order hold discretization was performed for the ROM with results shown in Table 1 for a year simulation. The time required for preprocessing in both models was not counted in the results. The ROM provides an order of magnitude faster computation.

Table 1: Computational time comparison for ROM and TRNSYS with one-year simulation

TIME STEP [MIN]	TRNSYS [SEC]	ROM [SEC]	MEAN ERROR [ºC]	RMS ERR [ºC]	REL ERR [%]
10	7.780	0.583	0.166	0.246	1.040
30	2.600	0.263	0.170	0.228	0.960
60	1.200	0.187	0.177	0.259	1.100

$$MEAN = \frac{1}{N}\left(\sum_{k}^{N} T_r[k] - T_{Trn}[k]\right)$$

$$RMS = \sqrt{\frac{1}{N}\sum_{k}^{N}\left(T_r[k] - T_{Trn}[k]\right)^2}$$

$$REL = \sqrt{\sum_{k}^{N}\left(T_r[k] - T_{Trn}[k]\right)^2 / N} \Big/ \sqrt{\sum_{k}^{N} T_{Trn}^2[k] / N}$$

In order to test closed-loop response of the ROM, the responses under a PID controller were determined in a SIMULINK environment with a 22ºC zone air temperature setpoint and arbitrarily chosen controller gains. Results are shown in Figure 4 for the full-order and reduced-order models.

To further test the validity of the lower-order model compared to the full-order model, frequency responses were generated and are shown in Figures 5 and 6. The heat extraction rate associated with mechanical cooling by ventilation was selected as a manipulated input for the responses shown in Figures 5. External heat flux acting on the roof due to solar radiation was chosen as a disturbance input for the responses shown in Figures 6. In all cases, the output is zone air temperature and all the variables are scaled from a nominal condition.

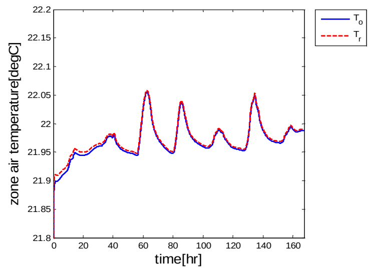

Figure 4: Comparison of dynamic zone air temperature behavior between full-order and reduced-order models

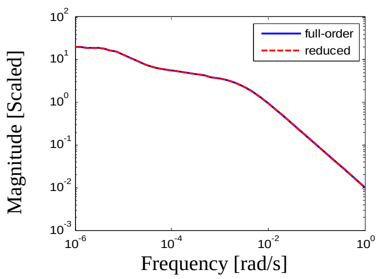

Figure 5: Frequency response of zone air temperature to mechanical cooling

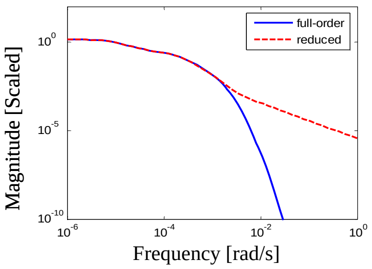

Figure 6: Frequency response of zone air temperature to solar disturbance

DISCUSSION

Compared to the full-order model, the ROM resulted in about a factor of 30 reduction in state variables (from around 300 to 10 states) with a factor of 3 reduction in computation for a specific case study. Compared to TRNSYS, the computation was reduced by about a factor of 10. As a result, the ROM is more readily utilized in optimization, optimal control or any high level control than existing modeling approaches.

Furthermore the results in Figure 5 and 6 demonstrate that the reduced-order model accurately matches the full-order model outputs over a wide range of frequencies. Even though discrepancies appear in the frequency responses of the disturbance input at high frequencies, the magnitude of the errors is negligible and does not significantly affect the accuracy. The linearized reduced-order modeling approach maintains the input-output dynamic response while reducing the simulation time. Greater computational savings would be expected for more complicated case studies.

CONCLUSION AND FUTURE WORK

A mathematical representation to describe the thermal network of a building was formulated and applied to generate a reliable reduced-order model for advanced control purposes. A case study was carried out for a single zone space. The model gave results that are close to those predicted by TRNSYS with about one-tenth the computational requirements. The model has been successfully applied to a Model Predictive Control case study for this same building system by D.Kim and J.E. Braun (2012). A more detailed case study is being developed for a more complicated multi-zone building system.

The physics-based, reduced-order building model lends itself to application of control theory to investigate system properties and to evaluate control performance with the help of control toolkits such as Matlab/Simulink. For example, the system time constant, DC gain under various control inputs, frequency response, stability, controllability and observability can easily be investigated for design of controller for building system.

The modeling approach is general for any shaped building (i.e., not restricted to rectangular shapes of walls) because it is developed using the radiosity method that incorporates view factors that can be determined for any geometry. This aspect allows individual wall elements to be divided into separate smaller elements to facilitate coupling to a CFD model for the indoor environment. This spatial discretization on walls is appropriate when the surface temperatures vary significantly due to solar or other non-uniform inputs. A case study for coupling a reduced-order CFD indoor air model with the reduced-order building envelope has been carried out by E. Cliff et al. (2012).

ACKNOWLEDGMENTS

The authors wish to acknowledge Dr. George Chiu, Dr. Jianghai Hu, Jie Cai, Vamsi Putta and Guangwei Zhu for their valuable suggestions. This work was supported by the Department of Energy through the Energy Efficient Buildings Hub.

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NOMENCLATURE

Nz = number of zones (or rooms)
Nw = number of walls in a zone (or a room)
$nnod$ = number of nodes in a wall
$\vec{T}_W$ = set of all temperatures nodes of walls in multiple zones, $\vec{T}_W = \left[(\vec{T}_w^1)^T \quad .. \quad (\vec{T}_w^{Nz})^T \right]^T$
$\vec{T}_w^{\,I}$ set of all node temperatures of walls and windows in I^th zone, $\vec{T}_w = \left[\vec{T}_1^{\,T} \ \vec{T}_2^T \ \ . \ \ . \ \ \vec{T}_n^{\,T} \right]$
$\vec{T}_j$ = set of all jth temperature nodes of walls in a single zone, $\vec{T}_j = \left[T_j^1 \quad T_j^2 \quad .. \quad T_j^{Nw} \right]^T$
T_j^i = temperature at j^th node in ith wall
$\vec{T}_Z$ = set of all zone air temperatures nodes in multiple zones
$T_z^{\,I}$ =I^th zone air temperature [K]
ρ_j^i = density at j^th node in ith wall
w_j^i = width of control volume of j^th node in ith wall [m]
$k^L \,|_j^i$ = thermal conductivity at left surface of the jth node in ith wall
$w^L \,|_j^i$ = distance from the "j-1"th node to the jth node in ith wall [m]
q_{LWR} = net long wavelength radiation exchange with environment [W/m2]
q_{SWR} = short wavelength solar irradiation [W/m2]
q_{gen} = energy source term [W/m2]
$(\vec{q})_i$ = i^th component of a vector $\vec{q}$
$h_{cv,in}$ = convective heat transfer coefficient at an internal wall [W/m2K]
$h_{cv,ex}$ =convective heat transfer coefficient at the outside surface of a wall[W/m2K]
$q_{net,rad}$ = net radiative heat flux out of an internal surface [W/m2]
$\overline{T}$ = mean temperature [K]
δ_{ij} = Kronecker delta
$\tilde{A}_{ij}$ = (i,j) components of a matrix $\tilde{A}$
$\tilde{A}^T$ = transpose of a matrix $\tilde{A}$
$\tilde{I}$ = identity matrix
h_o^i = external radiative source term acting on ith surface [W/m2K]
ρ^I = I^th zone air density [kg/m3]
C_V^I = I^th zone air thermal capacity(constant volume specific heat) [J/kg-K]
V^I = I^th zone air volume [m3]
$\dot{Q}$ = heat flow rate [W]
Γ = Hankel operator
Σ =Hankel singular value
W_c =Controllability gramian
W_o =observability gramian
$\|\cdot\|_\infty$ =Hardy infinity norm (or H_∞ norm)