AUTOMATIC MODEL REDUCTION IN ARCHITECTURE: A WINDOW INTO BUILDING THERMAL STRUCTURE

Justin R. Dobbs and Brandon M. Hencey
Cornell University, Ithaca, NY

ABSTRACT

Reducing building model complexity is currently a heuristic process requiring experience and skill; little progress has been made toward a numerical framework to automate this process. This paper proposes integrating automatic model reduction into the design process using a transparent model abstraction. Resistor-capacitor network analysis, mapped appropriately to a building information model, can provide the architect with insight into thermal coupling among building elements. Experimental results demonstrate the use of model aggregation to reduce simulation time without significant loss of accuracy.

INTRODUCTION

The specter of high energy prices has fueled renewed interest in energy efficient buildings. Beyond component-level improvements, further efficiency gains require insight into a building's thermal performance during early design. An iterative process using contemporary integrated design and simulation tools (Figure 1a) produces large amounts of raw simulation data but relatively little insight. Thermal network model reduction techniques provide such insight but are incompatible with object-oriented building information models (BIM). This paper proposes the use of an abstraction layer to incorporate model reduction techniques into the design workflow. The process (Figure 1b) gives the architect direct information about the building's thermal coupling.

Although many model abstractions are feasible, the resistor-capacitor (RC) network is considered here. Progress has been made toward proving and optimizing the accuracy of RC building models, as in Davies (2003). RC networks have also been used to simulate building dynamics for HVAC control simulations (Goyal and Barooah 2010) and have been proposed for for improving hierarchical building system control design (Mehta, Dorobantu, and Banaszuk 2006) and real-time prediction and control (Deng et al. 2010). While the application of RC networks to modeling existing designs has received extensive treatment, their use in early design has received little attention.

This paper proposes a way to integrate model reduction

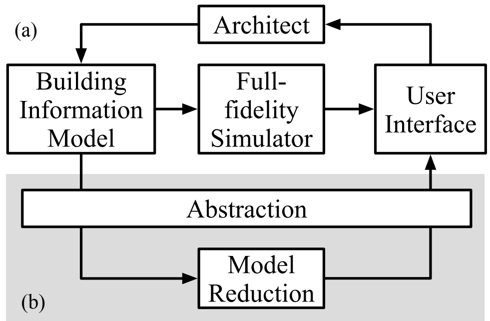


Figure 1: Existing design workflow (a) and proposed process using automatic model abstraction (b)

techniques into the architectural workflow for the purpose of analyzing thermal relationships and accelerating simulations. First, it outlines a resistor-capacitor network reduction algorithm adopted from the literature. Second, it proposes a novel abstraction framework between the network reduction algorithm and the geometric building model, including a way to map model reduction results back to the BIM. After validating the abstraction, the paper details the model reduction procedure and demonstrates how the results can be visually presented to the architect. Finally, experimental results demonstrate the potential simulation speed improvements attainable using model order reduction. A key feature of the proposed method is that the abstraction remains transparent to the user at all times.

BACKGROUND

The algorithm described here is based on resistor-capacitor networks treated analogously to Markov chains as described in Deng et al. (2010). The purpose of the algorithm is to cluster the capacitors in the RC network by thermal coupling, reducing the size of the network while minimizing accuracy degradation. Here are some key concepts of the method:

1. *Nodes* are individual capacitors in the full-size RC network. All nodes are members of a vector $[C_k]$.

2. *Supernodes* are collections of nodes. A supernode contains at least one node and can be divided if it contains two or more nodes. Supernode j is denoted $\bar{C}_j$.

3. A *partition function* $\phi_\gamma(k)$ maps a node k to the supernode number to which it belongs. The inverse $\phi_\gamma^{-1}(j)$ maps a supernode number j to the set of capacitors belonging to it. A new partition function is produced at each aggregation step γ.

A supernode behaves dynamically like the sum of its member nodes:

$$\bar{C}_j = \sum_{k \in \phi^{-1}(j)} C_k \tag{1}$$

and is interconnected to other supernodes via computed equivalent resistances $[\bar{R}]_{jk}$. Simulations of the full-order RC model operate on the full set of nodes, but simulations on reduced-order models operate on supernodes, which are fewer and therefore simulate faster. When a supernode $\bar{C}_j$ is at a temperature T, its constituent nodes are also at that temperature, allowing reduced-model simulation results to be mapped to the full-size state space:

$$T(\bar{C}_j) = T(C_k)\ \ \forall k \in \phi^{-1}(j) \tag{2}$$

Model aggregation is achieved by grouping the entire system into one supernode and then dividing the supernode recursively along lines of lowest thermal coupling. The optimal supernode division produces two or more child supernodes which are each strongly internally coupled but weakly coupled to each other. The method in Deng et al. (2010) has been extended to produce a tree structure that can be applied to the original object-oriented building model.

RC NETWORK ABSTRACTION

The method described in this paper differs from previous work in that the RC network serves as an abstraction of the original building and not as the core platform. The purpose of the model abstraction is to convert the object-oriented model into a form amenable to numerical methods. The BIM consists of geometric data, material composition, and the environmental setting of the building. The goal is to provide the architect with insight that can be applied to the original model and preserve the ability to use full-fidelity simulation tools.

Generating the RC Network

A resistor-capacitor network is generated from the BIM in a three-step process consisting of:

1. Generating capacitances for interior air volumes

2. Producing the RC sub-circuit for each wall

Symbol	Units	Description
Δ	m	Thickness
ρ	$\mathrm{kg \cdot m^{-3}}$	Density
A	$\mathrm{m^2}$	Surface area
c_p	$\mathrm{J \cdot kg^{-1} K^{-1}}$	Specific heat
k	$\mathrm{W \cdot m^{-1} K^{-1}}$	Thermal conductivity

Table 1: Layer properties used to construct RC circuit

3. Connecting walls to interior air volumes and to the outdoors

The BIM contains two types of objects: zones (rooms) and enclosing wall surfaces. The RC network represents each room air volume as a single capacitance. The capacitance value follows directly from the room volume and assumed properties of air:

$$C_{\text{room}} = \rho_{\text{air}}\, c_p V_{\text{room}} \tag{3}$$

where ρ_{air} and c_p are chosen for typical indoor conditions.

Although modeled as a flat polygon, a wall object may include any arbitrary stack of material layers of various thicknesses. For instance, an exterior wall may have bricks, studs, insulation, and drywall. The material properties in Table 1 are used to obtain the resistance and thermal capacitance of each layer according to the following elementary relations. (The area A is inherited from the polygon.)

$$R = \frac{\Delta}{kA} \qquad C = c_p \rho A \Delta \tag{4}$$

Because resistance and capacitance are distributed properties (Figure 2a), a discrete-component approximation is necessary to form the RC circuit. The optimal RC configuration for a wall has been shown to depend on the simulation conditions (Davies 2003); EnergyPlus, for example, uses between six and eighteen nodes per layer with its conduction transfer function (CTF) algorithm (D.O.E. 2011). To limit model complexity and resulting computational load, two capacitors per layer are used in the RC network (Figure 2b).

Each wall circuit must be terminated to a zone air volume or to the outdoor environment. An interior wall consists of two surface objects with common geometry, each facing a different room and exchanging heat with its corresponding capacitance. An exterior wall has only one surface object marked with an exterior boundary condition, such as whether it touches outside air or the ground. Convective transfer from walls to interior air volumes and to outdoor air is approximated with

$$R_{\text{convection}} = \frac{1}{hA} \tag{5}$$

Convection Type	**Coefficient** $(\mathrm{W \cdot m^{-2} K^{-1}})$	**Includes Radiant**
Rough exterior surface (no wind)	12.49	✓
Smooth exterior surface (no wind)	10.22	✓
Horizontal interior surface, reduced convection	0.948	
Horizontal interior surface, enhanced convection (warm floor or cold ceiling)	4.040	
Vertical interior surface	3.076	

Table 2: Air-to-surface heat transfer coefficients used in the RC network abstraction and by EnergyPlus under no-wind conditions and the simple convection algorithm

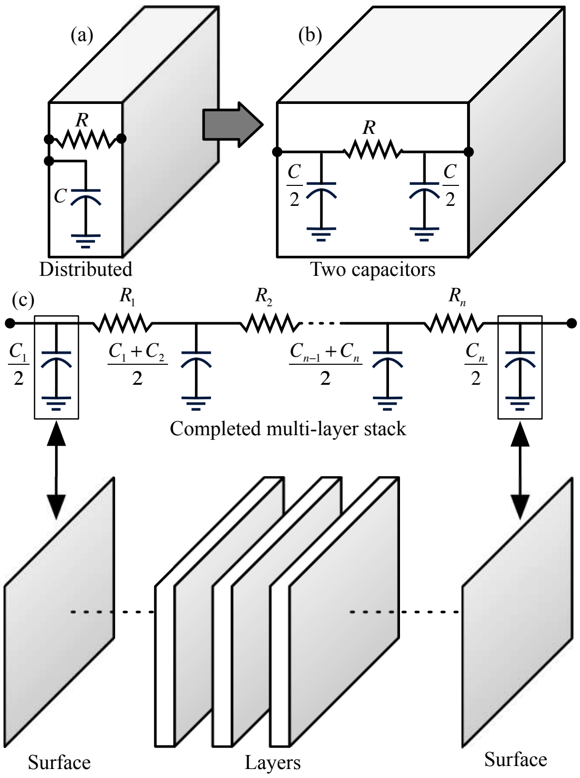


Figure 2: Generating RC circuit for a multi-layer wall

where h depends on surface roughness and orientation. The coefficients, given in Table 2, are those used by EnergyPlus under no-wind conditions and, for exterior walls, include radiant exchange with the sky (D.O.E. 2011). Because the RC network is a time-invariant linear model, coefficients must be fixed at the time of synthesis, whereas EnergyPlus changes coefficients on-the-fly based on conditions; thus differences inevitably arise between RC and EnergyPlus simulations. Interior radiant heat transfer, although not included in this analysis, could be linearized using computed wall view factors and rolled into the RC network.

For surfaces touching the ground, heat conduction is estimated with the relation (D.O.E. 2011)

$$R_{\text{conduction}} = \frac{R_{\text{ground}}}{A} = \frac{2.592 + 10.736\Delta_{\text{soil}}}{A} \ (\mathrm{K/W}) \quad (6)$$

where Δ_{soil} is the soil depth specified in the BIM.

The finished RC network consists of a capacitance vector $[C_i]$ and a resistance matrix $[R_{ij}]$ where

$$R_{ij} = \begin{cases} r \in \mathbb{R}^+ & i \neq j,\ C_i \text{ and } C_j \text{ connected by } r \\ \infty & i \neq j,\ C_i \text{ and } C_j \text{ not adjacent} \\ 0 & i = j \end{cases} \quad (7)$$

The outdoor air and ground temperatures drive the temperature evolution of the building during simulation. These would normally be connected to the system as inputs, but the model reduction method in Deng et al. (2010) requires a fully self-contained system; thus, the exogenous conditions are modeled as very large capacitances inside the system and heat is conserved. The system evolves according to $\dot{x} = Ax$ where x is a the vector of temperatures, and the transition rate matrix is

$$A_{ij} = \begin{cases} 1/C_i R_{ij} & i \neq j \\ -\sum_{k \neq j} A_{ik} & i = j \end{cases} \quad (8)$$

with row sums of zero. For simulation, a conventional driven state space model $\dot{x} = Ax + Bu$ can be formed by moving the entries of A that refer to the external capacitances to an input gain matrix B.

Mapping RC Results to the Building Information Model

Because each wall may have an arbitrary number of layers, the RC network has many more capacitors than the original model has objects. To be meaningful in a design context, the RC network results must be mapped back to the original model. The mapping for room air volumes is straightforward because rooms are represented as single objects in both frameworks; thus the map is bijective. Wall capacitances, on the other hand, are mapped selectively as shown in Figure 2c. Inner wall layers, ground, and outdoor air have no representation in the geometric model and do not pass through the map.

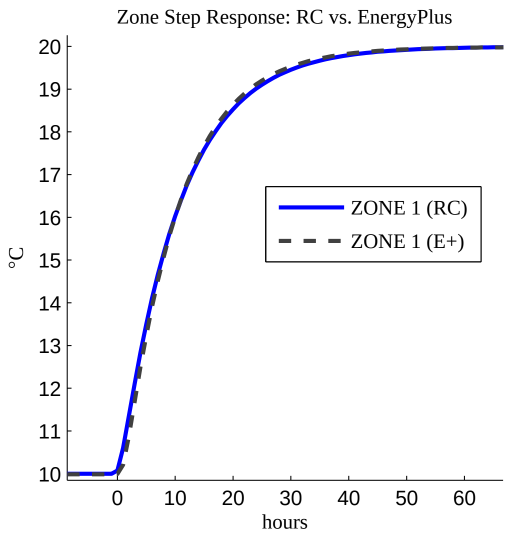


Figure 3: Zone temperature step response of a two-room building under RC network and EnergyPlus simulation

RC Network Validation

Before aggregating the RC network, it is important to demonstrate that it plausibly approximates the building dynamics. The goal is not to exactly match full-fidelity simulation, but rather to show that the RC network represents the dominant processes to a first approximation. The building used for validation has two rooms, eleven walls, and 43 capacitors. The simulation conditions are:

- No sun or wind
- 1% relative humidity
- EnergyPlus settings: Conduction transfer functions and simple convection
- Step change in air and ground temperatures at $t = 0$ from 10°C to 20°C with a matching sky infrared temperature profile
- Time step: 15 minutes

The time responses of Zone 1 for the RC network and EnergyPlus simulation are virtually identical (Figure 3). Wall exterior surfaces, not shown in the plot, also exhibit very similar thermal behavior, suggesting that the RC network adequately conveys the dynamics.

MODEL AGGREGATION

We have extended the RC network aggregation scheme in Deng et al. (2010) to produce a tree structure. The algorithm first combines all capacitors into a single large cluster, or supernode, forming the top tier (Figure 4a). It then computes the transition matrix (8) and converts it to discrete-time using $P = e^{A\tau}$, where τ is inverse to the least negative eigenvalue of A. Because the system described by A is self-contained with no inputs, the matrix P is stochastic and can be considered a Markov transition probability matrix (Deng et al. 2010). The second eigenvector of P reveals how best to divide the system into two pieces: if a capacitor's component of the second eigenvector is less than zero, that capacitor is placed into a new cluster (Figure 5). The resulting two supernodes are shown in Figure 4b. This bipartition scheme minimizes the erosion of simulation accuracy with reduced model size (Mehta, Dorobantu, and Banaszuk 2006).

This process repeats recursively on each supernode. When two or more supernodes are present, each will yield its own optimal bipartition. Among the supernodes available to divide, however, one yields the lowest overall modeling error, as given by the Kullback-Leibler divergence rate; that supernode is chosen for division while the others are left unchanged for the iteration and pass to the next lower level of the tree unchanged.

Sometimes a proposed division introduces disconnected nodes within one of the new supernodes. Such unreachable subnetworks cause the algorithm to fail and must be prevented. In evaluating a division point, the algorithm checks the resulting supernodes to make sure no enclosed portions are disjoint. Figure 6 shows an example where the optimal division results in two mutually unreachable sets to the right of the cut. In this case a third division is necessary such that the original supernode must be divided into three parts, rather than two (Figure 4c). The algorithm favors divisions where this does not happen, because such divisions increase model order by more than necessary.

The division procedure described above repeats with each successive system configuration until no more divisible supernodes remain (Figure 4d). Each level of the tree represents a snapshot of the system at a given aggregation step γ. The bottom tier represents the full-order system, and higher levels in the tree contain aggregated systems. Only one supernode division occurs at any one level, so

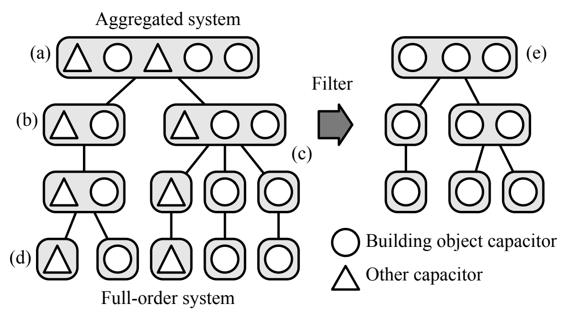


Figure 4: Filtering the tree to include building objects

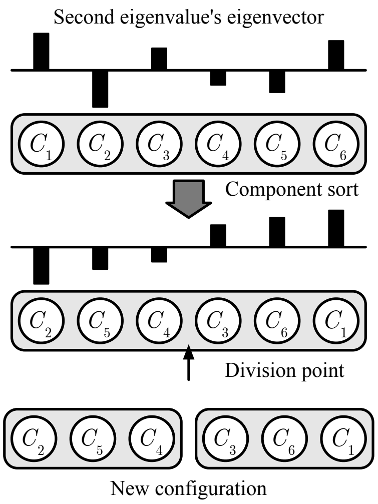


Figure 5: Supernode division

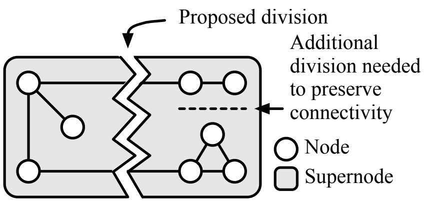


Figure 6: Preventing unreachable nodes

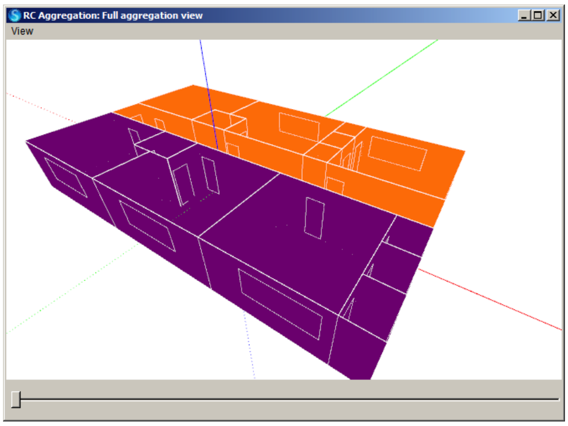
traversing the tree from the bottom up yields a distinct aggregation sequence.

The aggregation tree reveals how capacitors in the RC network are dynamically coupled, but the challenge remains to map the results back to the building information model so that their architectural significance can be realized. The map from Figure 2 transforms the tree into direct references to surfaces, and the interior air volumes are mapped directly from their respective zone capacitances. Figure 4e shows the result of the filtering process. Notice that the building object tree is smaller than the RC network tree; objects not mappable to building objects are discarded by the filter.

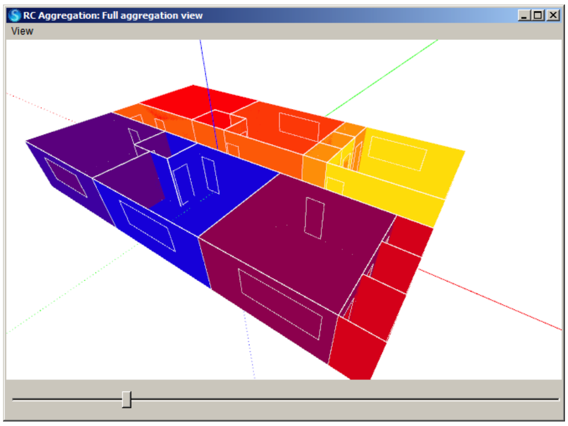
Visualization

The viewer program shown in Figure 7 is integrated into *Sustain*, a new modeling and simulation framework under development at Cornell University (Greenberg et al. 2012). The viewer colors building elements by supernode membership at a user-specified aggregation level. Sections of the building can be hidden to reveal interior walls and room air capacitances, which are shown as spheres centered in the rooms.

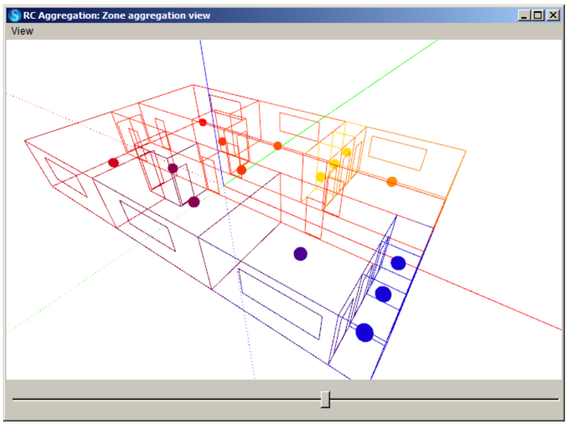
The building shown contains only 15 rooms but 454 RC



(a)



(b)



(c)

Figure 7: Viewing the model aggregation in Sustain. All building objects are shown in the full aggregation view (a,b). The wireframe view (c) shows only zone air volumes, represented by colored spheres.

Surface	**Composition**	**R-value** ($K \cdot m^2 W^{-1}$)
Ext. wall	Brick, insulation, drywall	16.6
Int. wall	Insulated aluminum	4.0
Floor	Concrete slab	7.9
Roof	Plywood, insulation, gypsum	10.9

Table 3: Composition of the fifteen-room building with R-values

network capacitors. The ground and exterior air capacitors are omitted from the rendering. At the coarse end of the control range (Figure 7a), the entire building is shown in just two colors separated along the line of weakest thermal coupling. At finer levels of aggregation (Figure 7b), subtler thermal relationships among smaller sections of the building become apparent: for instance, the three foreground closets (shown in red) are more strongly coupled to each other than to the adjacent large room. At much finer aggregation levels, air volumes and windows decouple from their adjacent walls. Although these relationships could be inferred by an experienced designer in this case, they may be less obvious in more complex designs, yet would be revealed automatically by this method. The zone-only view (Figure 7c) displays zone air volumes in a wireframe and may be useful, for example, in planning HVAC zones.

IMPROVING SIMULATION SPEED

The size of the RC network for the modestly sized fifteen room building shows that simple geometry can mask a surprising degree of complexity. Features such as windows and doors, which have a significant impact on a building's thermal performance and are thus often included in models, introduce additional resistor-capacitor chains in parallel with the walls. Furthermore, if the object-oriented building model represents curved surfaces using large collections of polygons, each facet represents additional equations that increase simulation time. Such explosion in complexity can be overcome by aggregating these surfaces before simulation. The following discussion examines the speed/accuracy trade-off introduced by various degrees of model reduction.

Measuring Model Error

Evaluating simulation accuracy of reduced-order models compared to the full-order model requires a performance metric. To compute the error, the reduced-order simulation output must be mapped to the full-order state space. When a supernode $\bar{C}_i$ is at some temperature T, its member nodes are all considered to be at the same temperature, thus completing the map to the full-order state space. For a given aggregation step γ, the total error is the sum, across all the full-order capacitances, of the root mean square difference between each full-order temperature and its equivalent from the reduced-order model:

$$E_{\text{total}} = \sum_{k=1}^{N} \sqrt{\frac{1}{M} \sum_{j=1}^{M} \left(\tilde{y}_\gamma\left(\phi_\gamma(k), j\right) - y(k, j)\right)^2} \tag{9}$$

The index j is the time step number (of M total time points), k is the node index along the full-order system state space containing N capacitors, and $\tilde{y}(\phi_\gamma(k), j)$ is the temperature at time j of the supernode containing node k. ($\phi_\gamma(k) = i$ means that, at aggregation step γ, the capacitor C_k belongs to supernode $\bar{C}_i$.) The error could also be taken over a subset of the state space, such as those capacitors that map to building objects.

Complex Building Simulation Example

Model reduction can be used to accelerate simulations directly, as shown here, or indirectly, via improvements to the geometric model. The direct approach is shown here using the fifteen-room building in Figure 7. A method described by Deng et al. (2010) to find equivalent resistances between supernodes was used to compute equivalent resistances for each aggregated system. Each reduced network was then run through a full-year simulation using the same time step and weather data as the full-order RC network simulation. The simulation conditions were:

- Weather data: EnergyPlus file for Elmira/Corning Regional Airport (New York)
- Simulation period: one year
- Time step: one hour

Figure 8 compares simulation time versus E_{total}. $\gamma = 2$ indicates a nearly fully aggregated building. Because the simulation time increases with the square of the model size, even a small degree of aggregation can yield much faster simulation. The error reaches a plateau of 15°C near $\gamma = 270$ with a corresponding 60% reduction in simulation time. Such flat spots in the error curve occur when extremely small capacitances are combined with adjacent large capacitances, for instance within wall layers, and their presence is dependent on the particular model.

CONCLUSION

This paper has shown that automated model reduction, previously applied to control and monitoring, can be applied to the architectural design process. Producing a suitable abstraction automatically from the object-oriented model allows bidirectional linkage to be maintained, eliminating the step of manual model translation. This linkage enables the use of numerical methods while providing a means to convey results in a visual context. This augmentation to the design workflow provides the designer

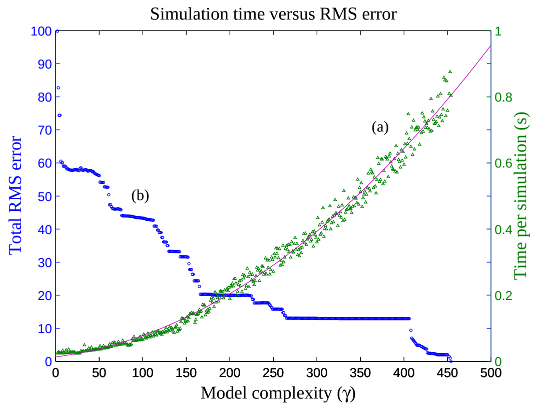


Figure 8: Simulation time (a) and summed RMS error (b) versus model complexity (γ)

with building performance information that might not otherwise be evident from simulation data. Insight gained from this technique may be used to adjust building layout or control system design. Furthermore, this paper has demonstrated how model reduction can dramatically reduce simulation time with relatively little degradation in accuracy. Restricting the aggregations based on spatial relationships and automating the application of these results are left for future work.

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DISCLAIMER

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NOMENCLATURE

$[C_k]$	Vector of capacitances on the full-order state space
$[\bar{C}_i]$	Vector of supernode capacitances
$\phi_\gamma(k)$	The supernode to which capacitor C_k belongs before aggregation step γ
$\phi_\gamma^{-1}(j)$	The set of capacitors belonging to supernode j before aggregation step γ