ASHRAE and IBPSA-USA SimBuild 2016
Building Performance Modeling Conference
Salt Lake City, UT
August 8-12, 2016

A NEW POLYNOMIAL BASED MODEL FOR DETERMINING COOLING TOWER EVAPORATION WATER LOSS

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ABSTRACT

Water conservation has become a critical issue in recent years. Evaporation in cooling towers can be a significant source of water consumption in commercial buildings; however, most cooling tower models used in whole building simulation programs are either prohibitively time-consuming or overly simplistic with respect to water consumption. Therefore, it is difficult to determine the effects of control strategies on water consumption using these models. This paper presents a new polynomial based model which defines the water loss by evaporation as a function of tower inlet conditions. The polynomial coefficients have been derived through a multi-variable regression performed on data generated by a detailed finite difference model. Test cases show improved agreement with the detailed finite difference model compared to several other existing models. In addition, the execution time of the new model has been found to be much faster than the detailed model and comparable to those of other simplified models.

INTRODUCTION

Cooling towers are widely used in commercial buildings for heat rejection. A typical counterflow tower is shown schematically in Figure 1. Hot water enters the tower through a set of spray nozzles and exchanges both sensible and latent heat with ambient air drawn through the tower. As a result, the water exits the tower at a lower temperature, and the air exits at a higher specific enthalpy. Water is lost in the process due to a combination of evaporation, drift, and blowdown. The mass flow rate of water at the tower outlet is approximately equal to that of the entering water mass flow rate due to the introduction of makeup water.

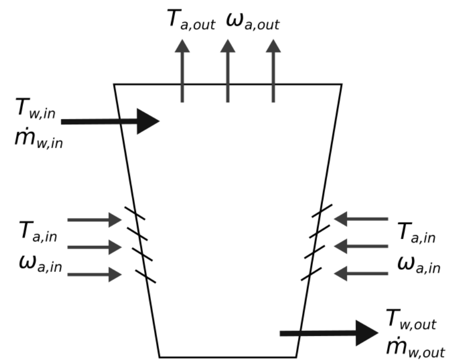

Figure 1 Cooling tower schematic

There are two primary reasons for incorporating cooling tower models into whole building simulation programs. One is to determine the effects that the thermal behavior will have on building energy consumption. The other is to quantify the total makeup water consumption.

Due to the complexities involved with the simultaneous heat and mass transfer in cooling towers, various simplifications are typically employed before a model is incorporated into whole building simulation programs. Merkel (1925) introduced one of the first theories on cooling towers and developed a model which greatly reduced the computational complexity associated with tower analysis. A version of this model is included in the ESP-r simulation package (ESP-r, 2015). Braun (1988) introduced an effectiveness model which allows for an analytic solution to Merkel's equations by assuming a linear saturation enthalpy with respect to temperature. In addition, Braun provided a means for estimating the evaporation water loss by introducing an effective saturation humidity ratio. Braun's model has been implemented in the TRNSYS energy simulation program (TRNSYS 17, 2009). A similar effectiveness model is included in the EnergyPlus engine with the reintroduction of Merkel's assumption that the water mass flow rate is constant through the tower (EnergyPlus, 2015). Polynomial based curve fits to empirical data are among the most widely used thermal models for cooling towers in whole building simulation programs. Examples can be found in both the DOE2 and EnergyPlus engines (DOE2.2, 2014; EnergyPlus, 2015). These models are notable for being very computationally efficient; however, they are primarily concerned with the thermal performance of cooling towers and do not provide a means for estimating water loss. Two simplified water loss models are included in the EnergyPlus engine (EnergyPlus, 2015). The first assumes the outlet air is always fully saturated, and the second assumes a linear relationship between water loss and the tower range.

These existing models have been widely used to influence cooling tower and control system design by providing an estimate of annual energy consumption; however, they have had little influence on designs which minimize both energy and water consumption. The ability to minimize both energy and water consumption in HVAC systems will continue to grow in importance as water conservation becomes more of an issue. To bridge this gap, new cooling tower models are needed which can account for the full set of cooling tower boundary conditions and are computationally efficient enough to be included in whole building simulation programs.

This paper introduces a new polynomial based model which relates the difference between the inlet and outlet air humidity ratios as a function of inlet water and air conditions. The coefficients for the model were developed in a three step process. Fist, a prototype tower was modeled from first principle using energy and mass conservation equations. Second, this model was used to generate outlet tower condition data over a wide range of inlet conditions. Finally, a least squares analysis was used to fit the new curves to the data. Coefficients have been generated for both counterflow and crossflow configurations.

DETAILED MODEL DESCRIPTION

The main elements of this model have been described in a number of publications (Li et. al., 2010; Braun, 1988; Sutherland, 1983). The model is presented here in detail because it provides the basis for the newly proposed polynomial based model. The model employs a number of assumptions:

1. The mass fraction of water vapor in moist air can be approximated by the humidity ratio.
2. The film between water droplets and moist air is saturated at the water droplet temperature.
3. Heat transfer to the tower walls is negligible.
4. Water particles in supersaturated air are treated as water vapor.
5. Air velocities are high enough to prevent precipitation in supersaturated air.
6. Drift losses are ignored on the interior of the tower.

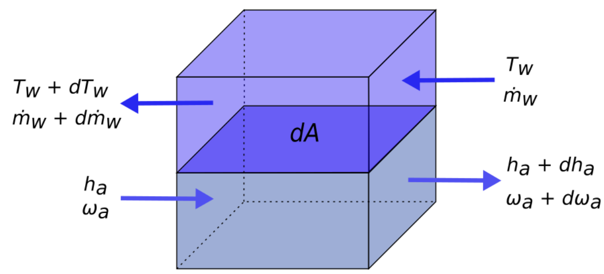

Figure 2 Differential tower element

With these assumptions, the model can be developed from a differential analysis on the interface between air and water particles in a cooling tower (Figure 2). The enforcement of energy and mass conservation on a differential element leads to the following set of equations.

$$\dot{m}_a d\omega_a = h_D dA(\omega_s - \omega_a) \quad (1)$$

$$\dot{m}_a dh_a = h_Q dA(T_w - T_a) + h_{fg} h_D dA(\omega_s - \omega_a) \quad (2)$$

$$c_{p,w} d(\dot{m}_w T_w) = -h_Q dA(T_w - T_a) - h_{fg} h_D dA(\omega_s - \omega_a) \quad (3)$$

$$d\dot{m}_w = -h_D dA(\omega_s - \omega_a) \quad (4)$$

The differential Number of Transfer Units for mass transfer and the Lewis factor are defined as

$$Ntu = \frac{h_D dA}{\dot{m}_a} \quad (5)$$

$$Le_f = \frac{h_Q}{h_D c_{p,m}} \quad (6)$$

The Lewis factor is calculated according to Bosnjakovic (1965).

$$Le_f = 0.865^{2/3}\left[\left(\frac{\omega_s + 0.622}{\omega_s + 0.622} - 1\right) / \ln\left(\frac{\omega_s + 0.622}{\omega_s + 0.622}\right)\right] \quad (7)$$

The Merkel Number is related to the ratio of inlet mass flows according to Baker and Shryock (1963).

$$Me^* = \frac{h_D A}{\dot{m}_{w,in}^*} = c\left(\frac{\dot{m}_{w,in}^*}{\dot{m}_a^*}\right)^n \quad (8)$$

where c and n are empirical parameters which depend on tower and fill design. The superscript $*$ is used to denote non-differential quantities. This equation can be modified to relate the overall Ntu to the ratio of inlet mass flow rates.

$$Ntu^* = c\left(\frac{\dot{m}_a^*}{\dot{m}_{w,in}^*}\right)^{n+1} \quad (9)$$

With the assumption that the heat and mass transfer coefficients are uniform throughout the tower, the differential Ntu can be related to the total tower Ntu by

$$Ntu = Ntu^*\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \quad (10)$$

With these additional definitions, the model can be recast to the following set of equations.

$$d\omega_a = Ntu^*(\omega_s - \omega_a)\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \quad (11)$$

$$dh_a = Ntu^*\left[Le_f c_{p,m}(T_w - T_a) + h_{fg}(\omega_s - \omega_a)\right]\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \quad (12)$$

$$c_{p,w} d(\dot{m}_w T_w) = -\dot{m}_a Ntu^*\left[Le_f c_{p,m}(T_w - T_a) + h_{fg}(\omega_s - \omega_a)\right]\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \quad (13)$$

$$d\dot{m}_w = -\dot{m}_a Ntu^*(\omega_s - \omega_a)\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \quad (14)$$

An examination of Eq. (11) through (14) reveals the primary computational difficulties which have prevented versions of this model from being incorporated into whole building simulation programs.

1. The system cannot be solved analytically.
2. The equations are nonlinear.
3. The equations are coupled and must be solved simultaneously.

For the present study, the system was solved by approximating derivatives with first order finite differences on a uniform mesh. The discretized versions of Eq. (11) through (14) were then solved in sequence from the air inlet to outlet with iterations as required by the tower configuration.

NEW POLYNOMIAL MODEL

The new model was created in a three step process. First, prototype counterflow and crossflow towers were modeled according to the theory presented in the previous section. The design specifications of the prototype towers are shown in Table 1. Second, the models were used to generate tower outlet condition data over a wide range of inlet conditions (Table 2). Finally, multi-variable regressions were used to fit the data to polynomial curves. The new model is of the following form.

$$\begin{aligned}\omega_{a,out} - \omega_{a,in} = {} & c_1 + c_2\Delta\omega + c_3\Delta T + c_4 LGR \\ & + c_5 X + c_6\Delta\omega^2 + c_7\Delta T^2 \\ & + c_8\Delta\omega LGR + c_9\Delta T \cdot LGR \\ & + c_{10}\Delta\omega X + c_{11}\Delta T \cdot X \\ & + c_{12}\Delta\omega T_{w,in} + c_{13}\Delta\omega LGR^2 \\ & + c_{14}\Delta T \cdot LGR^2\end{aligned} \quad (15)$$

where

$$\Delta\omega = \omega_{s,in} - \omega_{a,in} \quad (16)$$

$$\Delta T = T_{w,in} - T_{a,in} \quad (17)$$

$$LGR = \frac{\dot{m}_{w,in}}{\dot{m}_a} \quad (18)$$

$$X = \left(\frac{\dot{m}_a}{\dot{m}_{w,in}}\right)^{n+1} \quad (19)$$

and n has the same meaning as in Eq. (9). Thus the new model relates the water loss by evaporation to the tower inlet conditions and the tower characteristic exponent. The regressions resulted in an $R^2 = 0.999$ for both the counterflow and crossflow models. The model coefficients are shown in Table 3. These coefficients apply when temperatures are specified in units of degrees Fahrenheit.

Table 1 Prototype tower design specifications

Item	Value	Units
$\dot{m}_a$	1800	$\frac{lbs}{hr}$
$\dot{m}_{w,in}$	2000	$\frac{lbs}{hr}$
$T_{w,in}$	95	°F
$T_{w,out}$	85	°F
$T_{a,in}$	95	°F
$\omega_{a,in}$	0.01678	$\frac{lbs\ H_2O}{lbs\ Dry\ Air}$

Table 2 Range of conditions used to generate tower outlet data

Item	Min Value	Max Value	Units
$\frac{\dot{m}_a}{\dot{m}_{a,des}}$	0.4	1.0	-
$\frac{\dot{m}_{w,in}}{\dot{m}_{w,in,des}}$	0.4	1.0	-
$T_{w,in}$	65	100	°F
$T_{w,out}$	40	100	°F
$\omega_{a,in}$	0.004	0.023	$\frac{lbs\ H_2O}{lbs\ Dry\ Air}$
n	-1.1	-0.35	-

Table 3 Model coefficients

Coeff.	Counterflow	Crossflow
c_1	4.55483584492e-04	7.30509496544e-04
c_2	1.03671184500e+00	1.03730061482e+00
c_3	-6.16981746792e-05	-8.27070387249e-05
c_4	-3.07203825490e-04	-3.80567257433e-04
c_5	-2.25762975948e-04	-4.59493908423e-04
c_6	5.22961317540e-01	6.56604597285e-01
c_7	-1.17741227844e-07	-1.22686953610e-07
c_8	2.28790313521e-01	2.71330831321e-01
c_9	3.49728877836e-05	4.11867673515e-05
c_{10}	-2.44472224600e-01	-1.81947834762e-01
c_{11}	8.41058054702e-06	1.51681065449e-05
c_{12}	-4.47274433862e-03	-5.43301935910e-03
c_{13}	-3.99692984435e-02	-4.84400808134e-02
c_{14}	-7.30082887497e-06	-8.57716165513e-06

COMPARISON TO OTHER MODELS

The performance of the new model was compared to several other models. These models were selected based on their current use in whole building simulation programs. These methods are briefly described below.

Merkel's Model

Merkel's model can be derived by introducing two simplifications to the detailed model. First, the Lewis factor is assumed to be unity. Second, the water mass flow rate is assumed to be constant throughout the tower. With these assumptions, the governing equations can be reduced to

$$d\omega_a = Ntu^*(\omega_s - \omega_a)\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \tag{20}$$

$$dh_a = Ntu^*\left[c_{p,m}(T_w - T_a) + h_{fg}(\omega_s - \omega_a)\right]\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \tag{21}$$

$$c_{p,w}\dot{m}_w dT_w = -\dot{m}_a Ntu^*\left[c_{p,m}(T_w - T_a) + h_{fg}(\omega_s - \omega_a)\right]\left(\frac{\dot{m}_a^* dA}{\dot{m}_a A}\right) \tag{22}$$

When written in terms of specific enthalpies only, Eq. (21) and (22) are commonly referred to as Merkel's Equations. Merkel's model introduces the obvious computational efficiency of reducing the size of the system by one dimension. A closer look reveals that Eq. (20) has also been decoupled from the others. In fact, a complete thermal analysis can be performed by solving only Eq. (21) and (22) simultaneously. Eq. (20) can then be solved individually to determine the water loss. For this study, these equations were solved using first order finite differences on a uniform mesh.

Effectiveness Model

Braun introduced an effectiveness model which allowed for an analytic solution to Eq. (21) by assuming an average saturation specific heat. In addition, Braun provided a means for estimating the evaporation water loss by introducing an effective saturation humidity ratio.

$$\omega_{a,out} = \omega_{s,e} + \left(\omega_{a,in} - \omega_{s,e}\right) exp(-Ntu) \qquad (23)$$

The effective saturation humidity ratio corresponds to the humidity ratio at an effective saturation enthalpy determined by the effectiveness model.

$$h_{s,e} = h_{a,in} + \frac{h_{a,out} - h_{a,in}}{1 - \exp(-Ntu)} \qquad (24)$$

$$h_{a,out} = h_{a,in} + \varepsilon_a\left(h_{s,in} - h_{a,in}\right) \qquad (25)$$

where ε_a is computed based on the tower configuration. Although it re-couples the heat and mass transfer equations, the effectiveness model has two distinct advantages over Merkel's model:

1. The coupled equations are algebraic rather than differential, so computation time will generally be faster.
2. Braun's effectiveness model accounts for the decreasing water flow through the tower which improves the accuracy.

EnergyPlus Models

Two models are included in the EnergyPlus engine for determining the evaporation water loss in cooling towers. The first model assumes that the air exits the tower fully saturated at the specific enthalpy determined by a thermal model.

$$\omega_{a,out} = \omega\left(h_{a,out}, \phi = 1\right) \qquad (26)$$

This model will be referred to as EnergyPlus1.

The second EnergyPlus model assumes a linear relationship between the evaporation water loss and the tower range.

$$\dot{m}_{w,in} - \dot{m}_{w,out} = F_{loss}\left(T_{w,in} - T_{w,out}\right)\dot{m}_{w,in} \qquad (27)$$

F_{loss} was set to the EnergyPlus default of $0.2°K^{-1}$ for this study. This model will be referred to as EnergyPlus2.

Both EnergyPlus models require a thermal model to compute the tower water loss. For the purposes of this study, the EnergyPlus emprical thermal model was used with the YorkCalc coefficients.

Results

Models of the prototype tower were simulated over the range of conditions shown in Table 2 (with the exception that *n* was set to a constant of -0.6). The percent water loss predicted by each of the models was compared to the percent water loss predicted by the detailed water loss. The error is defined as

$$err = \left|\frac{\dot{m}_{loss,modeled} - \dot{m}_{loss,detailed}}{\dot{m}_{loss,detailed}}\right| \qquad (28)$$

The mean error and execution time normalized to the detailed model over all simulations for each model are shown in Table 4. Plots are also shown for the counterflow configuration in Figures 3 through 7.

Table 4 Results vs detailed model

Model	Counterflow		Crossflow	
	$\overline{err}$	Time	$\overline{err}$	Time
Merkel	0.112	0.15	0.044	0.49
Effectiveness	0.024	5.0e-3	0.017	0.02
EnergyPlus1	0.068	9.3e-4	0.052	1.5e-3
EnergyPlus2	0.296	8.4e-4	0.293	1.5e-3
New	0.010	6.7e-5	0.012	4.9e-4

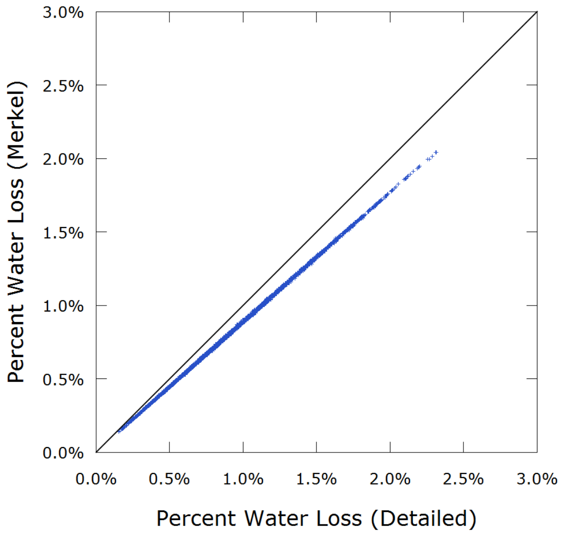

Figure 3 Merkel vs detailed model

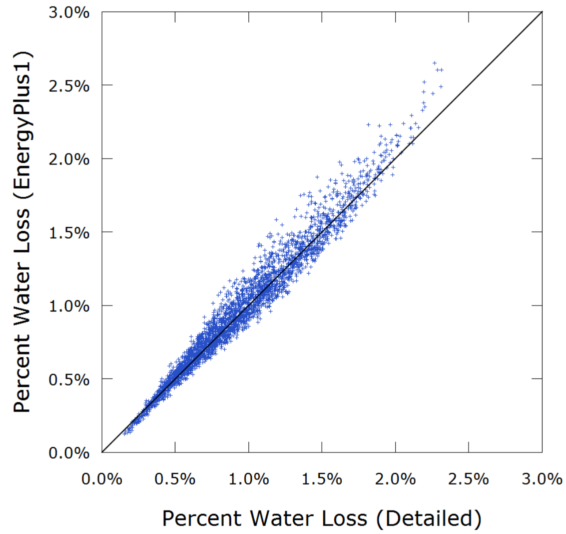

Figure 5 EnergyPlus1 vs detailed model

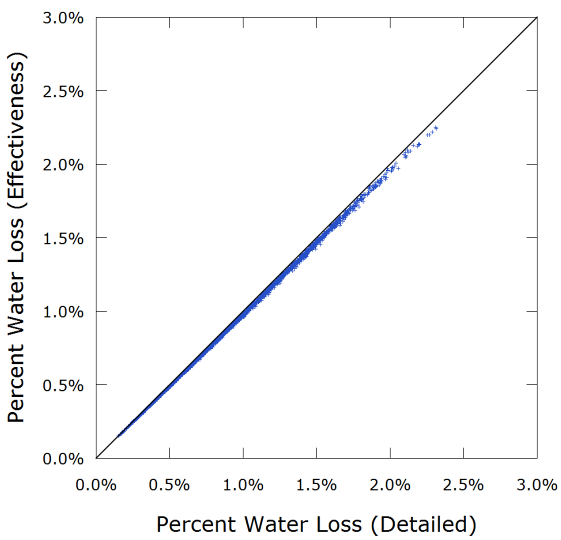

Figure 4 Effectiveness vs detailed model

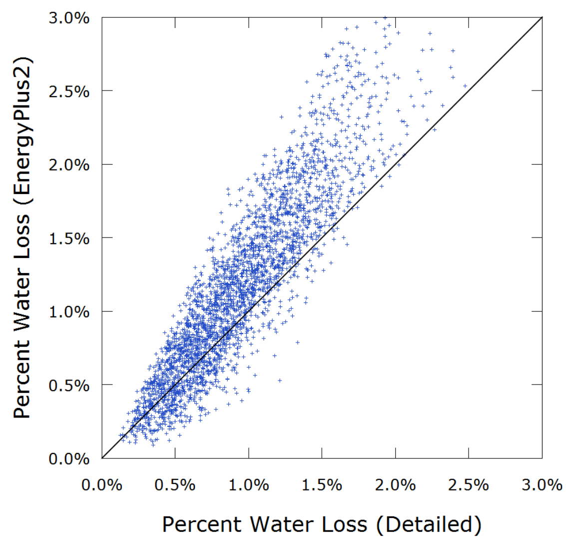

Figure 6 EnergyPlus2 vs detailed model

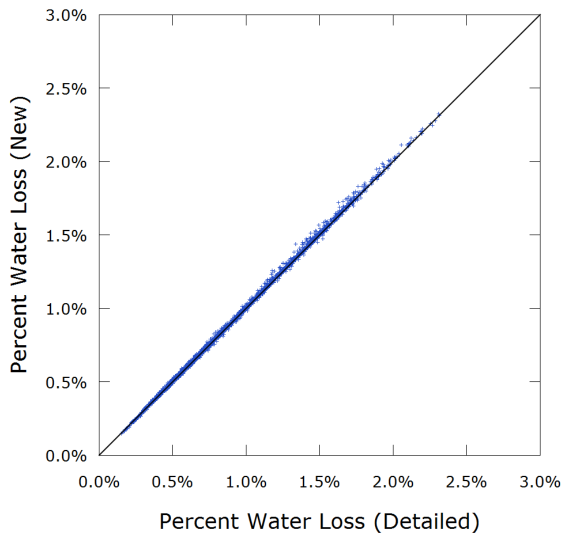

Figure 7 New vs detailed model

As might be expected, Merkel's Model tends to underpredict the water loss. The effectiveness model shows a better agreement, although it tends to slightly underpredict the water loss. This is similar to the results found by Braun who showed that the effectiveness model tended to underpredict the water loss compared to the experimental data from Simpson and Sherwood (Braun, 1988). The EnergyPlus1 model tended to overpredict or underpredict depending on the inlet conditions. The model tended to underpredict the water loss when the inlet conditions were favorable for supersaturated outlet conditions (i.e. low drybulb temperature and high relative humidity). Similarly, the model tended to overpredict the water loss when the inlet conditions were not favorable for supersaturated outlet conditions (i.e. high drybulb and low relative humidity). The EnergyPlus2 model showed the highest mean error of all the models. This model tended to overpredict the water loss when the inlet drybulb was less than the inlet water temperature and underpredict when the inlet drybulb was greater than the inlet water temperature. The new model showed the best agreement to the detailed model with the lowest mean error. The model performed well across all inlet conditions. Execution times of simulations using the new model were faster than those using the detailed, Merkel, and effectiveness models and were comparable to those using the EnergyPlus models.

COMPARISON TO MEASURED DATA

The percent water loss predicted by the new model was compared to experimental data calculated based on inlet and outlet humidity measurements from Simpson and Sherwood (1946). The results show a good agreement with the measured data (Figure 8). Some of the larger errors occurred when the new model predicted supersaturated conditions at the tower outlet. This was expected as the experiments by Simpson and Sherwood used humidity sensors at the tower outlet which would not have sensed the small entrained water droplets in the supersaturated air.

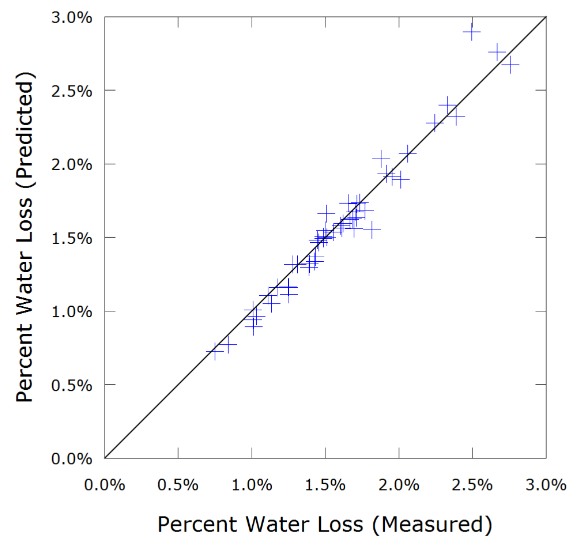

Figure 8 Comparison to measured data

CONCLUSION

A new polynomial based model was developed for this paper which relates the water loss by evaporation in cooling towers as a function of inlet water and air conditions. The purpose of the new model is to provide a method for estimating cooling tower water loss which is detailed enough to account for varying inlet conditions and also fast enough to be incorporated into whole building simulation. The coefficients for the new model were generated through a multi-variable regression on data generated by a detailed finite difference model. The new model shows an improved agreement with the detailed model compared to several other models currently used in whole building programs. The execution time of simulating the new model was shown to be much faster than the detailed model and comparable to the fastest methods currently in use.

NOMENCLATURE

A : total water to air contact area

$c_{p,m}$: specific heat of moist air

$c_{p,w}$: specific heat of water

F_{loss}: loss factor
h_a : specific enthalpy of moist air
h_D : mass transfer coefficient
h_{fg} : latent heat of vaporization for water
$h_{s,e}$: effective saturation specific enthalpy
h_Q : heat transfer coefficient for convection
Le_f: Lewis factor
$\dot{m}_a$: mass flow rate of air
$\dot{m}_w$: mass flow rate of water
Me: Merkel number
Ntu: Number of transfer units for mass
T_a : temperature of air
T_w : temperature of water

Greek Symbols

ε_a : air-side effectiveness
ϕ: relative humidity
ω_a : air humidity ratio
ω_s : saturation humidity ratio at water temperature
$\omega_{s,e}$: effective saturation humidity ratio

Subscripts

a: air
des: design
in: inlet to the cooling tower
out: outlet from the cooling tower
w: water

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