



Methodology for an Analytical Abstraction and calibration of Solar Heat Gain Using Dynamic SHGCs

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Abstract

This manuscript presents a validated approach for the abstraction of building glazing for analytical building performance assessment. The proposed dynamic analytical calculation uses a series of pre-generated adjustment coefficients, which are correction factors based on the solar angle of incidence (AOI), for calibrating a given window's solar heat gain coefficient (SHGC). The validation study has shown that the proposed methodology is reusable for solar gain assessments of similar window types under different orientations and climatic conditions. The goal is to achieve consistency in predicting solar gains through glazing between the different building performance assessment methodologies ranging from early-stage design methods to full-building detailed simulation.

Key words: analytical analysis; steady-state; dynamic; SHGC; calibration; solar gain through windows;

Introduction

At an early stage of a building design, designers and consultants rely on accessible tools that provide instant feedback on the potential of various design strategies. Early-design phase analytical methods that use fast and lightweight computations compared to detailed building performance simulation engines such as EnergyPlus play a crucial role in designing low-carbon, low-energy buildings.

Solar gains and heat conductions leading to heat losses or gains through building glazing are among the most crucial building performance parameters. For example, *Figure 1* shows the various sources of heat gains and losses in a free-floating building that should be considered when maintaining indoor air temperature per occupants' comfort requirements.

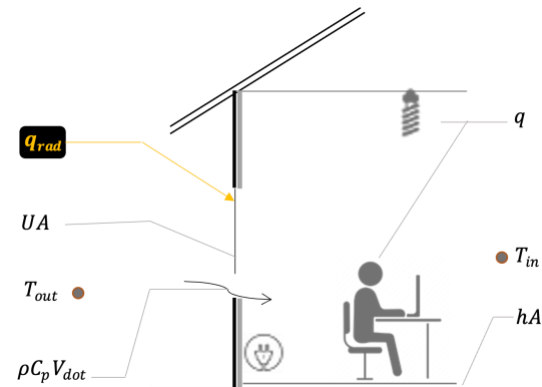


Figure 1 An example of a non-geometric representation of building parameters as performance analysis inputs. The image is adapted from an earlier publication by the authors (Arsano, 2022). The highlighted q_{rad} is solar gain; q represents internal gains from artificial lighting and plug loads; UA represents envelope conduction; $\rho C_p V_{dot}$ represents natural ventilation and infiltration; and hA represents convection heat transfer from thermal mass surfaces.

A steady-state analytical methodology is an excellent first-step approach for getting a rule-of-thumb estimation of solar gains, as discussed below with *equation (1)*. In addition, a simple annual or point-in-time radiation simulation during a massing study or readings of cumulative radiations from a hemispherical radiation map may allow a designer to estimate solar radiation incident on a surface or a particular orientation.

However, this approach tends to oversimplify the predictions of solar gains through glazing, primarily when the analyses are conducted not just for a representative hour but for extended periods in transient analysis, ranging from days to a full year. A dynamic analytical analysis is instrumental when estimating the total solar gains through windows over time, for example, in the peak summer or the peak winter season.

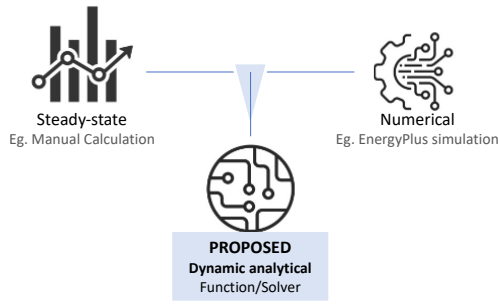


Figure 2 The proposed analytical approach uses a dynamic SHGC compared to the default steady-state approach.

This manuscript proposes a dynamic analytical approach that is a calibrated version of the commonly used steady-state analytical approach using a detailed full-building numerical simulation engine, namely EnergyPlus, as a reference. The validation of the proposed approach is presented, showcasing a comparison of the window's solar heat gain predictions using the calibrated, dynamic analytical method against EnergyPlus simulations.

Background

In steady-state analytical methodologies for early-stage building design, the glazing area (A) of a building or a thermal zone, the solar heat gain coefficient (SHGC), total solar radiations calculated from hourly weather data (Rad_{tot}), the thermal transmittance (U) of the glazing, and temperature difference between the inside and outside (T_{delta}) are crucial parameters.

A commonly simplified approach uses the following equation to estimate hourly solar gains, $q_{rad,i}$, (1) through glazing, where i stands for glazing in a particular orientation.

$$q_{rad_{tot},i} = A_{glazing,i} * Rad_{tot,i} * SHGC \quad (1)$$

Window Calculation and Heat Balance

Understanding the optical properties of glazing is the first step in understanding the role of windows in building performance. Solar radiation through windows, either absorbed or transmitted, influences the thermal environment inside buildings as described below (U.S. Department of Energy, 2023a).

- The *absorbed* solar radiation affects the glazing heat balance, which determines the inside window temperature, affecting the heat gain to the zone.
- The *transmitted* solar radiation through windows is absorbed by interior surfaces, contributing to the zone heat balance.

Programs such as the National Fenestration Rating Council (NFRC) test, certify, and label windows based

on their energy performance, providing a reliable way to determine a window's energy properties and to compare products. As a simplified approach, the NFRC rating and many other specifications base their qualification on the U-factor and SHGC. The U-factor is about the non-solar heat flow, while the SHGC is the fraction of solar radiation admitted through a window – either transmitted directly or absorbed and subsequently released as heat inside a building (energy.gov, n.d.).

Steady-state analytical method and its limitation

However, such a simplified approach to building performance analysis is just an approximation, and the following significant limitations are identified (EnergyPlus-Engineering Reference, 2016). First, SHGC combines directly transmitted solar radiation and radiation absorbed by the glass that flows inward. Figure 3 shows the energy distribution when solar radiation is incident on a window, which has different implications for space heating/cooling. In addition, different window orientations with the same SHGC often have different ratios of transmitted to absorbed solar radiation.

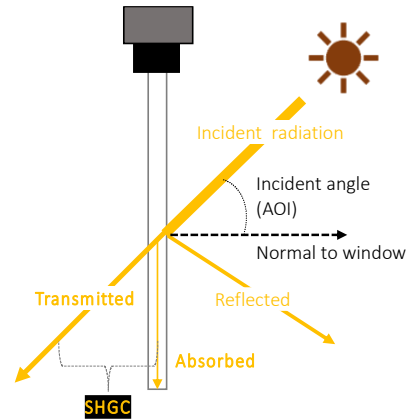


Figure 3 When solar radiation is incident on a glazing surface, it will be reflected, absorbed, and transmitted. SHGC represents the percentage of the solar radiation absorbed and transmitted through the window.

Second, SHGC is determined at normal incidence; angular properties of glazing vary with the number of layers, tints, and coatings. Products with the same SHGC can have different angular properties. Third, the simple analytical approach described in equation (1) uses a steady-state SHGC for all incident radiation, as with a uniform sky model in Figure 4. Unlike a clear sky or all-weather sky, the standard uniform sky is characterized by a uniform luminance that does not change with altitude and azimuth.

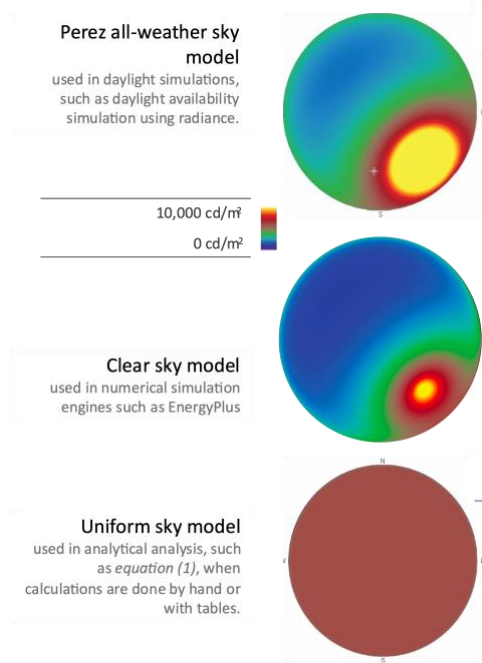


Figure 4 Sky models comparison between EnergyPlus and analytical approaches. Sky images are adapted from (Reinhart, 2018)

Furthermore, it is essential to note that the energy transferred by the direct and the diffused components of solar radiation is different. Direct radiation is measured as direct solar beams incident on the glazing surface, and the energy delivered is affected by the angle of incidence. In contrast, diffuse radiation is redirected by atmospheric conditions and indirectly reaches the surface. The sum of the two solar radiation types is the global solar radiation. This manuscript presents a dynamic analytical approach by addressing the limitations of the simplified steady-state approach discussed in this section.

Discrepancy between steady-state analytical and numerical simulation

SHGCs are handy not only for labeling windows' energy performance but also for quick, analytical calculations, as shown in equation (1). However, this leads to a significant discrepancy in solar gain estimations through windows when used for a transient, analytical building performance study. Figure 5 shows the discrepancy observed between the steady-state analytical model and the numerical simulation of EnergyPlus. It shows that the diurnal peak solar gain calculated using the steady-state analytical model (SA) is 2X larger than the diurnal peak of the EnergyPlus simulation. The S.A. given in equation (1) overestimates solar gains through windows.

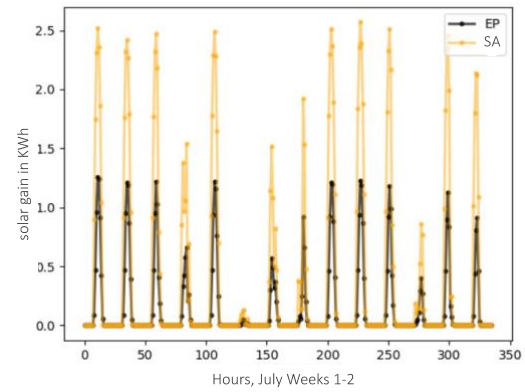


Figure 5 Comparison of Boston's diurnal solar heat gain outputs from EnergyPlus (E.P., black) and the steady-state analytical approach (S.A., yellow) for two weeks in summer. The S.A. uses non-angle-dependent SHGC to calculate hourly solar gains.

The numerical simulation in EnergyPlus offers two alternative approaches for model inputs on window types: *detailed* and *simple* glazing models. In the detailed window model, optical properties – Transmittance (T) and reflectance (R) – of individual glass layers are given at normal incidence. On the other hand, the *simple* glazing model uses SHGC and U-factor to define a glazing type, simplifying the required inputs by users.

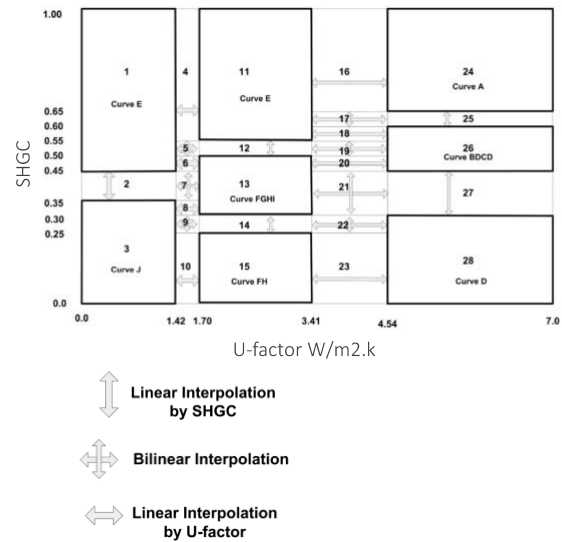


Figure 6 Diagram of Transmittance and Reflectance Correlations used in EnergyPlus based on U and SHGC.

However, additional algorithms are used in the *simple* glazing model to translate the definition with just SHGC and U-factor into a detailed window model. For this purpose, 28 different solar transmittance and reflectance correlation curves are implemented based on U-factor

and SHGC, as shown in *Figure 6*. It is important to note that window U-factors vary with temperatures, unlike the single number assumption in the simplified steady-state analytical approach, where lower exterior temperatures correspond to higher U-factors (Banionis et al., 2021).

Proposed Methodology

The proposed methodology presents a simple approach that bridges the gap between the steady-state analytical approach and the numerical building simulation approach discussed in the upper sections.

The proposed approach calibrates the simple method in equation (1) using *Angle of Incidence (AOI) dependent SHGCs* and *correction factors* for hourly direct and diffused radiations. The following sub-sections present how these *correlation factors* are calculated and implemented in the proposed dynamic analytical approach (*equations 3, 4, and 5*).

Angle of Incidence (AOI)

As illustrated in *Figures 3 and 7*, the angle of incidence (also known as incident angle) is the angle between a ray incident on a surface and the line perpendicular to the surface at the point of incidence, in this case, the window.

The solar angle of incidence on a given surface changes every hour with the sun's position, and it is defined using two angles: solar azimuth and solar altitude (Page, 2012).

Using a Python-based library called pvlib (F. Holmgren et al., 2018), hourly solar azimuth, solar altitude, and the solar angle of incidence are calculated for every hour (*equation 2*). *Figure 7* shows the relationship between these parameters for a given surface.

$$\nu(\beta, \alpha) = \cos^{-1}(\cos \gamma_s \cos \alpha_F \sin \beta + \sin \gamma_s \cos \beta) \quad (2)$$

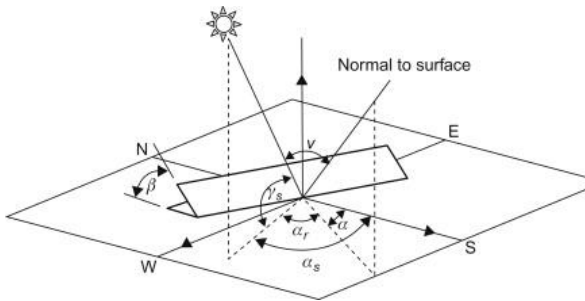


Figure 7 Definition of angles used to describe the solar position (γ_s , solar altitude and α_s , solar azimuth) and the orientation and tilt of the irradiated plane (α , tilt angle of the plane and β , slope of the plane). For vertical windows, the tilt or β is 90° while α gives the window orientation. The angle of incidence (AOI) is shown as ν in the figure above (CIBSE, 2002).

The extended version of the Python code can be referenced here (*GitHub - Pvlib/Pvlib-Python: A Set of Documented Functions for Simulating the Performance of Photovoltaic Energy Systems.*, n.d.).

Reference EnergyPlus Model

A single-zone thermal model with a 30% glazing area is modelled in EnergyPlus. We used ANSI/ASHRAE Standard 140 *Standard Method of Test for the Evaluation of Building Energy Analysis Computer Programs* as a reference to define the single-zone model shown in *Figure 8*. Many simulation engines, including EnergyPlus, have been validated according to this standard through comparison to specific testing criteria (ANSI/ASHRAE, 2017).

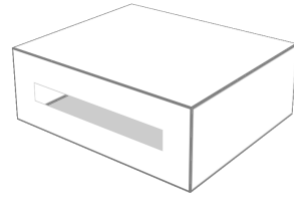


Figure 8 Single zone office space has the dimensions 6m by 8m and with 12 m² south-facing window.

AOI dependent Correction Factors (f_{AOI})

Two types of AOI dependent correction factors are proposed: f_{AOI_dir} and f_{AOI_dif} for direct and diffused solar gains. The widely used *EnergyPlus Weather Data (EPW)* for a given location includes normalized hourly data on global horizontal radiation, normal direct radiation, and diffuse horizontal radiation in Wh/m² (*EnergyPlus Weather File (EPW) Data Dictionary: Auxiliary Programs — EnergyPlus 8.3*, n.d.).

The first step is generating a dataset of the hourly direct solar gains ratios of EnergyPlus and the AOI agnostic steady-state *equation (1)*, shown as EP/SA in *Figures 9 and 10*. Each point in the dataset is visualized with the black dots.

Then, a clustering algorithm known as K-means clustering is implemented by initializing k different centroids $\{\mu(1), \dots, \mu(k)\}$ to different values, then alternating between two steps until convergence (Goodfellow et al., 2016):

- (i) each training example is assigned to cluster i , where i is the index of the nearest centroid $\mu^{(i)}$
- (ii) each centroid $\mu^{(i)}$ is updated to the mean of all training examples $x^{(j)}$ assigned to cluster i .

The authors used the *scikit-learn* Python library on the solar gain ratio dataset, a machine learning package in Python (*Scikit-Learn: Machine Learning in Python —*

Scikit-Learn 1.3.2 Documentation, n.d.). After the iterative cluster assignments have been completed, the result is a set of centroids, the AOI dependent correction factors, shown with the orange dots in Figures 9 and 10.

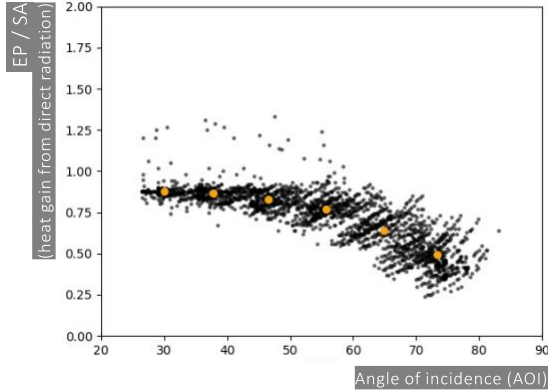


Figure 9 Clustering algorithm used to identify angle dependent correction factors for direct radiation. Centroids are marked with orange dots. This figure is for a double clear window with SHGC 0.70.

One angle dependent correction factor is associated with a range of AOI, as illustrated in Figure 9. For example, for hours (i) when AOI is between 40° and 50°, the correction factor for direct radiation $f_{AOI_dir_i}$ can be described as the representation of the ratio of $q_{dir_rad_i}$ from equation (1) divided by $q_{dir_rad_i}$ from EnergyPlus.

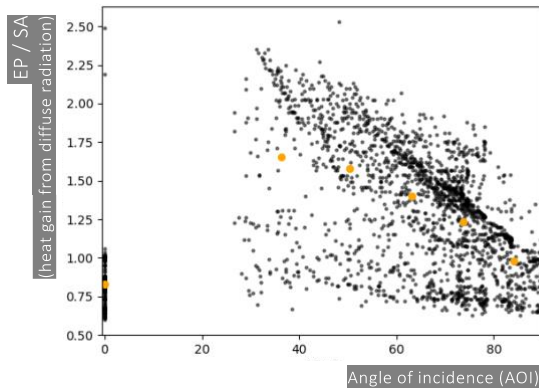


Figure 10 K-Means Clustering algorithm used to identify angle dependent correction factors for diffuse radiation. Centroids are marked with orange dots. This figure is for a double clear window with SHGC 0.70.

The optimum number of centroids (K) has been mathematically determined using the *Within Cluster Sum Squares* (WCSS), the sum of the squared deviations from each observation, and the cluster centroid. It measures the variability of the observations within each cluster, and the optimum number of centroids (K) is

when WCSS flattens, also known as the elbow method. The elbow method plots the explained variation, the distortion score on the y-axis, as a function of a number of clusters shown on the x-axis (Figure 11). The optimum number of centroids is found at the "elbow," the point of inflection of the curve (Scikit-Learn: Machine Learning in Python — Scikit-Learn 1.3.2 Documentation, n.d.).

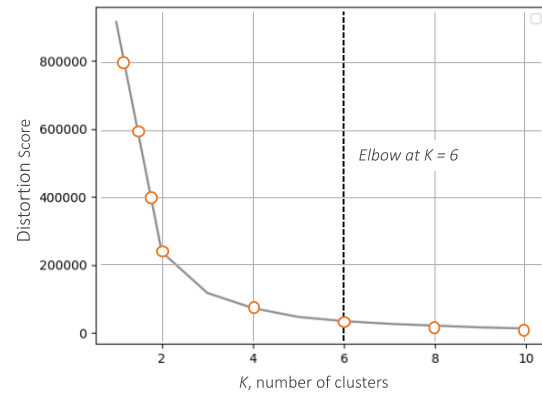


Figure 11 Distortion score elbow for K-Means clustering on the solar gain dataset.

In contrast to traditional supervised machine learning algorithms, K-Means clustering is an unsupervised machine learning algorithm that can classify data without first being trained with labelled data. Once the algorithm has been run and the groups are defined, new data can be easily assigned to the most relevant group. This approach has a great advantage for calibrating early design stage analytical models where sets of correction factors are identified based on example datasets as presented in this section, with the possibility for a broader application to a wide range of scenarios based on window orientation, window types, and climates. Additional discussion is presented in the validation section below.



Figure 12 Different types of windows with different performance parameters are considered in the study.

The methodology is repeated for different window types, as shown in Figure 12, and the authors found that adjusting the AOI dependent correction curves for different SHGCs is crucial. The authors simulated the single-zone office space with four different window types. The more significant the difference in the SHGC, for example, the two types of double pane windows shown in Figure 12, the wider the gap between the correction curves, as shown in Figure 13.

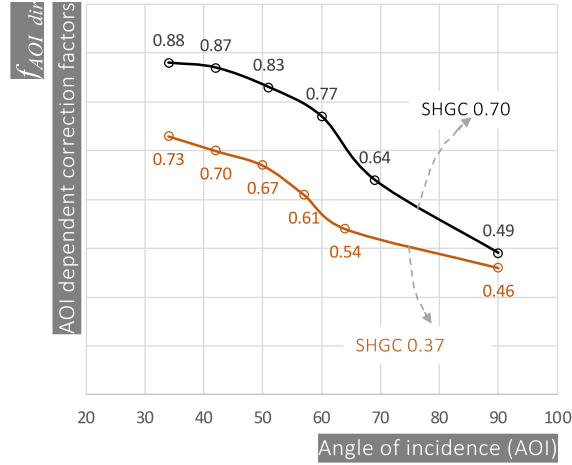


Figure 13 AOI dependent correction factor curves for two different double pane windows with a significant difference in SHGC.

Implementation: Dynamic Analytical Calculation

The AOI dependent correlation factors for different glazing types and both direct and diffused radiation are compiled as a series of decimal numbers, each associated with a given range of AOI, as shown in equations (3) and (4). The total hourly solar gain is calculated as a summation of the adjusted direct ($q_{dir_rad_i}$) (3) and diffused ($q_{dif_rad_i}$) (4) solar gains.

$$q_{tot_rad_i} = q_{dir_rad_i} + q_{dif_rad_i} \quad (3)$$

$$q_{dir_rad_i} = Rad_{dir_i} * SHGC * f_{AOI_dir} \quad (4)$$

$$q_{dif_rad_i} = Rad_{dif_i} * SHGC * f_{AOI_dif} \quad (5)$$

Validation

The EnergyPlus single zone model presented in the methodology section (see Figure 8) is used for a direct comparison analysis. As per the ANSI/ASHRAE 140 recommendation, Denver's cold and dry climate is used for the hourly comparative study shown in Figures 14 and 15.

Predicted hourly solar gains for different orientations are compared between the proposed methodology and the EnergyPlus output. For a detailed direct comparison, two weeks during the peak of the winter and summer seasons are analyzed (U.S. Department of Energy, 2023b).

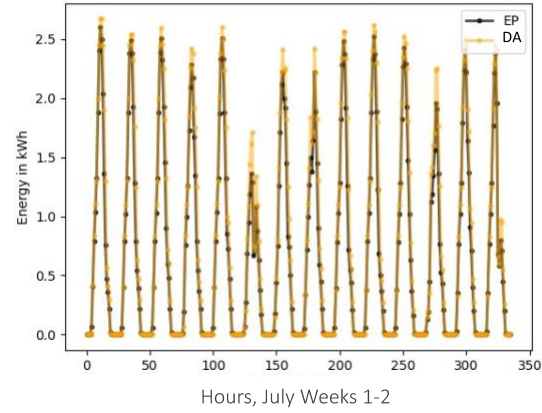


Figure 14 Comparison of Denver's diurnal solar heat gain outputs from EnergyPlus (E.P., black) and the dynamic analytical approach (DA, yellow) for two weeks in summer.

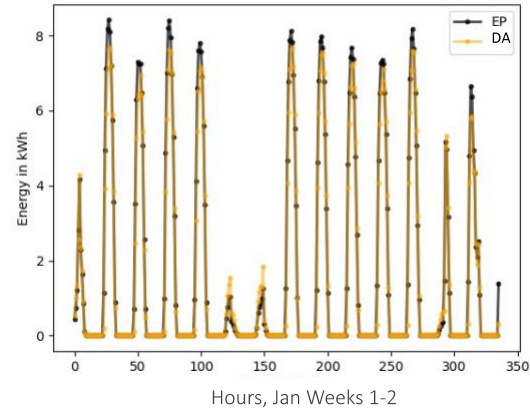


Figure 15 Comparison of Denver's diurnal solar heat gain outputs from EnergyPlus (E.P., black) and the dynamic analytical approach (DA, yellow) for two weeks in winter.

The validation result shows that the hourly solar gains of the proposed methodology trail very closely to the hourly solar gains in EnergyPlus with a root mean square error (RMSE) of 0.07 and an annual solar gain difference of < 2%. Figures 14 and 15 show hourly solar gain for selected two weeks during summer and winter. The DA

uses angle dependent, adjusted SHGC to calculate hourly solar gains.

Figure 16 shows the second part of the validation. When re-running the K-means clustering algorithm on the corrected hourly direct solar gain ratios EP/DA, the centroids shown with the orange dots suggest a correction factor of 1 – the average direct solar gains in EnergyPlus (EP) are similar to those in the dynamic analytical model (DA).

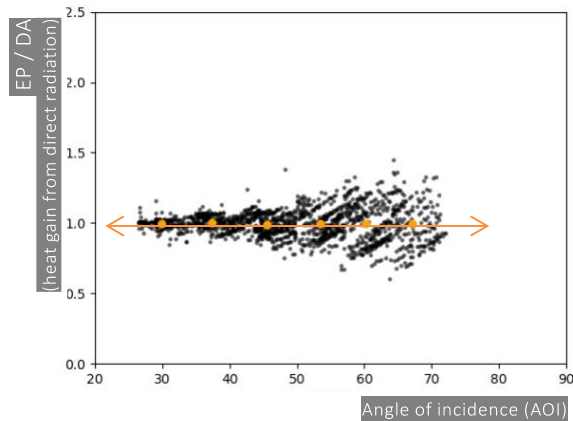


Figure 16 Testing the direct solar gain predictions from the dynamic analytical methodology using the K-Means clustering algorithm.

The proposed dynamic analytical approach with AOI dependent SHGC has been evaluated for Phoenix, a hot and dry climate, and San Francisco, a temperate climate. A RMSE < 0.1 has been recorded for two weeks in winter and two weeks in summer. It is also observed that the proposed dynamic SHGCs can be applied for any window orientation because they are based on the incident angle of solar radiation on a given plane (in this case, windows), which already considers orientation.

Discussion and Future Work

As discussed in the background section, AOI is not the only determinant factor influencing the amount of solar radiation absorbed and transmitted through glazing. Glazing with different combinations of SHGC and U-factor have different performance curves (Carmody & Haglund, 2012). A multi-objective calibration approach will be explored in future work to identify multiple adjustment factors that combine defined window types and environmental scenarios.

Additionally, as shown in Figure 10, unlike solar gains from direct radiation, the steady-state analytical prediction of solar gains from diffuse radiation requires additional steps for multi-dimensional calibration.

The methodology will be further improved by incorporating an interpolation algorithm for the AOI dependent factors identified using the K-Means clustering algorithm. Rather than associating a range of AOI with a single factor, each AOI will have a corresponding adjustment factor by linear interpolation.

The proposed methodology is reusable for solar gain assessments of similar window types under different orientations and climatic conditions. To support this argument, the validation study will be expanded, to showcase a broader range of climates in addition to Denver, Boston, Phoenix, and San Francisco. Additionally, the implication and potential impact of the calibration approach will be further explored for other building performance analyses such as daylighting and visual comfort.



Figure 17 Illustrates a design process from understanding outdoor climate conditions to evaluating multiple possible building strategies to maintain occupants' comfort requirements while reducing energy use and associated carbon emissions.

The proposed methodology will be valuable in improving early-design stage analytical approaches used in simple design web apps and teaching tools for educators, designers, and consultants. Figure 17 illustrates an example of such processes. Improved prediction of solar heat gains in buildings is crucial when evaluating and comparing different sustainable building strategies, including window areas and types, building orientations, solar shading strategies in hot climates, and strategies to maximize solar gain in cold climates. A well-calibrated prediction of solar heat gains is also essential when designing active systems by playing a significant role in the predictions of heating and cooling loads.

Conclusion

This manuscript presents a dynamic analytical approach that enables a calibrated prediction of solar gains through windows for early-design stage building-strategy explorations. The goal is to consistently predict solar gains between the different building performance assessment methodologies ranging from early-stage design methods to full-building detailed simulation.

Acknowledgment

Acknowledgment will be added in the final version of the paper.

Nomenclature

ANSI/ASHRAE Standard 140 – *Standard Method of Test for the Evaluation of Building Energy Analysis Computer Programs*

AOI – *Angle of Incidence*

DA – *Dynamic analytical*

E.P. – *EnergyPlus*

EPW – *EnergyPlus Weather Data*

f_{AOI} – *Correction Factors*

f_{AOI_dif} – *Correction Factors for direct radiation*

f_{AOI_dir} – *Correction Factors for diffuse radiation*

S.A. – *Steady-state analytical*

SHGC – *Solar Heat Gain Coefficient*

WCSS – *Within Cluster Sum Squares*

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